

Risk Management: Coordinating Corporate Investment and Financing Policies

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ABSTRACT

This paper develops a general framework for analyzing corporate risk management policies. We begin by observing that if external sources of finance are more costly to corporations than internally generated funds, there will typically be a benefit to hedging: hedging adds value to the extent that it helps ensure that a corporation has sufficient internal funds available to take advantage of attractive investment opportunities. We then argue that this simple observation has wide ranging implications for the design of risk management strategies. We delineate how these strategies should depend on such factors as shocks to investment and financing opportunities. We also discuss exchange rate hedging strategies for multinationals, as well as strategies involving “nonlinear” instruments like options.

CORPORATIONS TAKE RISK MANAGEMENT very seriously—recent surveys find that risk management is ranked by financial executives as one of their most important objectives.¹ Given its real-world prominence, one might guess that the topic of risk management would command a great deal of attention from researchers in finance, and that practitioners would therefore have a well-developed body of wisdom from which to draw in formulating hedging strategies.

Such a guess would, however, be at best only partially correct. Finance theory does do a good job of instructing firms on the implementation of hedges. For example, if a refining company decides that it wants to use options to reduce its exposure to oil prices by a certain amount, a Black-Scholes type model can help the company calculate the number of contracts needed. Indeed, there is an extensive literature that covers numerous practical aspects of what might be termed “hedging mechanics,” from the computation of hedge ratios to the institutional peculiarities of individual contracts.

Unfortunately, finance theory has had much less clear cut guidance to offer on the logically prior questions of hedging strategy: What sorts of risks

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¹ See Rawls and Smithson (1990).

should be hedged? Should they be hedged partially or fully? What kinds of instruments will best accomplish the hedging objectives? Answering these questions is difficult because, paradoxically, the same arbitrage logic that helps the refining company calculate option deltas also implies that there may be no reason for it to engage in hedging activity in the first place. According to the Modigliani-Miller paradigm, buying and selling oil options contracts cannot alter the company's value, since individual investors in the company's stock can always buy and sell such contracts themselves if they care to adjust their exposure to oil prices.

It is not that there are no stories to explain why firms might wish to hedge. Indeed, a number of potential rationales for hedging have been developed recently, by, among others, Stulz (1984), Smith and Stulz (1985), Smith, Smithson, and Wilford (1990), Stulz (1990), Breeden and Viswanathan (1990), and Lessard (1990). However, it seems fair to say that there is not yet a single, accepted framework which can be used to guide hedging strategies.² In part, this gap arises precisely because previous work has focused on why hedging can make sense, rather than on how much or what sort of hedging is optimal for a particular firm. Indeed, much of the previous work has the extreme implication that firms should hedge fully—completely insulating their market values from hedgeable risks.

In this paper, we illustrate how optimal risk management strategies can be designed in a variety of settings. To do so, we build on one strand of the previous work on hedging—that which examines the implications of capital market imperfections. Broadly speaking, this work argues that if capital market imperfections make externally obtained funds more expensive than those generated internally, they can generate a rationale for risk management.

The basic logic can be understood as follows. If a firm does not hedge, there will be some variability in the cash flows generated by assets in place. Simple accounting implies that this variability in internal cash flow must result in either: (a) variability in the amount of money raised externally, or (b) variability in the amount of investment. Variability in investment will generally be undesirable, to the extent that there are diminishing marginal returns to investment (i.e., to the extent that output is a concave function of investment). If the supply of external finance were perfectly elastic, the optimal ex post solution would thus be to leave investment plans unaltered in the face of variations in internal cash flow, taking up all the slack by changing the quantity of outside money raised. Unfortunately, this approach no longer works well if the marginal cost of funds goes up with the amount raised externally. Now a shortfall in cash may be met with some increase in outside financing, but also some decrease in investment. Thus variability

² This gap in knowledge is illustrated in the most recent edition of Brealey and Myers's (1991) textbook. Brealey and Myers do devote an entire chapter to the topic of "Hedging Financial Risk," but the chapter focuses almost exclusively on questions relating to hedging implementation. Less than one page is devoted to discussing the potential goals of hedging strategies.

in cash flows now disturbs both investment and financing plans in a way that is costly to the firm. To the extent that hedging can reduce this variability in cash flows, it can increase the value of the firm.

A prominent example of this line of reasoning is Lessard (1990).³ Lessard writes: "...the most compelling arguments for hedging lie in ensuring the firm's ability to meet two critical sets of cash flow commitments: (1) the exercise prices of their operating options reflected in their growth opportunities (for example, the R & D or promotion budgets) and (2) their dividends... The growth options argument hinges on the observation that, in the case of a funding shortfall relative to investment opportunities, raising external capital will be costly."

The model that we develop below is very much in the spirit of this verbal argument. However, it takes the argument a couple of steps farther: rather than simply demonstrating that there is a role for hedging, we are able to show how a firm's optimal hedging strategy—in terms of both the amount of hedging and the instruments used—depends on the nature of its investment and financing opportunities. Or put differently, we illustrate how a well-designed risk management program can enable a firm to optimally coordinate its investment and financing policies.

The plan of the paper is as follows. In Section I, we briefly sketch several other explanations of corporate risk management that have been offered. In Section II, we present our model in its most elemental form, and use it to demonstrate the basic rationale for hedging. We then examine a series of practical applications of our framework. In Section III, we extend the model to show how optimal hedge ratios can be calculated as a function of shocks to investment and financing opportunities. Section IV considers the question of optimal currency hedging by multinationals that have investment opportunities in more than one country. Section V examines "nonlinear" hedging strategies that make use of options and other complex hedging instruments. Section VI briefly outlines a few further extensions. Section VII examines the empirical implications of the theory, and Section VIII concludes.

I. Other Rationales for Corporate Risk Management

A. Managerial Motives

Stulz (1984) argues that corporate hedging is an outgrowth of the risk aversion of managers. While outside stockholders' ability to diversify will effectively make them indifferent to the amount of hedging activity undertaken, the same cannot be said for managers, who may hold a relatively large portion of their wealth in the firm's stock. Thus managers can be made strictly better off (without costing outside shareholders anything) by reducing the variance of total firm value.

³ Closely related rationales for hedging include Froot, Scharfstein, and Stein (1989), Smith, Smithson, and Wilford (1990), and Stulz (1990). These papers are discussed in detail below.

One weakness of the Stulz theory is that it implicitly relies on the assumption that managers face significant costs when trading in hedging contracts for their own account—otherwise, they would be able to adjust the risks they face without having to involve the firm directly in any hedging activities. At the same time, unless one also introduces transactions costs to hedging at the corporate level, the Stulz theory makes the extreme prediction that firms will hedge as much as possible—that is, until the variance of stock prices is minimized.

A very different managerial theory of hedging, based on asymmetric information, is put forward by Breeden and Viswanathan (1990) and DeMarzo and Duffie (1992). In both of these models, the labor market revises its opinions about the ability of managers based on their firms' performance. This can lead some managers to undertake hedges in an attempt to influence the labor market's perception.

B. Taxes

Smith and Stulz (1985) argue that if taxes are a convex function of earnings, it will generally be optimal for firms to hedge. The logic is straightforward—convexity implies that a more volatile earnings stream leads to higher expected taxes than a less volatile earnings stream. Convexity in the tax function is quite plausible for some firms, particularly those who face a significant probability of negative earnings and are unable to carry forward 100 percent of their tax losses to subsequent periods.

C. Costs of Financial Distress and Debt Capacity

For a given level of debt, hedging can reduce the probability that a firm will find itself in a situation where it is unable to repay that debt. Thus if financial distress is costly, and if there is an advantage to having debt in the capital structure (say due to taxes or agency problems associated with "free cash flow") hedging may be used as a means to increase debt capacity. The simplest variant of this argument, put forth by Smith and Stulz (1985), simply assumes that bankruptcy involves some exogenous transactions costs.

D. Capital Market Imperfections and Inefficient Investment

A more sophisticated version of the argument invokes Myers's (1977) "debt overhang" underinvestment effect to endogenize the costs of financial distress. This rationale for hedging (or equivalently, for using debt indexed to exogenous sources of risk) is given by Froot, Scharfstein, and Stein (1989) in the context of highly indebted less developed countries. The same basic point is made in a corporate finance setting by Smith, Smithson, and Wilford (1990). Stulz (1990) also argues that hedging can add value by reducing the investment distortions associated with debt finance.⁴

⁴ A somewhat related paper is Diamond (1984). In his model of financial intermediation, "hedging" (actually diversification) mitigates incentive problems associated with debt finance.

We view these debt overhang explanations for hedging to be very close cousins of those presented both in Lessard (1990) and in our model below. Although the exact mechanism is somewhat different, all the theories rely on the basic observation that, without hedging, firms may be forced to underinvest in some states of the world because it is costly or impossible to raise external finance.

II. The Basic Paradigm

A. A Simple Model of the Benefits to Hedging

As stated above, hedging is beneficial if it can allow a firm to avoid unnecessary fluctuations in either investment spending or funds raised from outside investors. To illustrate this point, it is best to begin with a very simple and general framework. Afterwards, we demonstrate how this simple framework corresponds to a well-known optimizing model of costly external finance.

Consider a firm which faces a two-period investment/financing decision. In the first period the firm has an amount of liquid assets, w . At this time the firm must choose its investment expenditures and external financing needs. In the second period, the output from the investment is realized and outside investors are repaid.

On the investment side, let the net present value of investment expenditures be given by

$$F(I) = f(I) - I, \quad (1)$$

where I is investment, $f(I)$ is the subsequent expected level of output, $f' > 0$ and $f'' < 0$.⁵ For notational simplicity we assume the discount rate is equal to zero.

As will become clear, the company prefers to finance investment with internal funds first before turning to external sources. Therefore, the company will raise from outside investors an amount e , so that

$$I = w + e. \quad (2)$$

Given the discount rate of zero, outside investors require an expected repayment of e in the second period.

We assume, however, that there are additional (deadweight) costs to the firm of external finance, which we denote by C . (Per dollar raised, these funds therefore cost C/e above the riskless rate.) These costs could arise from a number of sources. First, they could originate in costs of bankruptcy and financial distress, which include direct costs (e.g., legal fees) as well as

⁵ The most natural interpretation of the concavity of $f(I)$ is that there are technological decreasing returns to scale. However, if the corporate tax system is progressive, then $f(I)$ will be concave even with constant technological returns to scale. Of course, taxes will impact the hedging decision in other ways since they affect not only the returns on *new* investment ($f(I)$), but also the returns on existing assets; see the discussion in Section I.B above.

indirect costs (e.g., decreased product-market competitiveness and underinvestment). Second, such costs could arise from informational asymmetries between managers and outside investors. Or, to the extent that managers are not full residual claimants, there may be agency costs associated with motivating and monitoring managers who resort to certain types of outside finance. Finally, managers may obtain private benefits from limiting their dependence on external investors. Thus even if there are no observable costs to external finance, management may *act* as though external financing has real economic costs.⁶

Regardless of which interpretation one chooses, the deadweight costs should be an increasing function of the amount of external finance. We represent these costs as $C = C(e)$ and note that $C_e \geq 0$.⁷

The issue of hedging arises when first-period wealth, w , is random. To the extent that there are marketable risks that are correlated with w , the firm may attempt to alter the distribution of w by undertaking hedging transactions in period zero. For simplicity, we make the extreme assumption that all the fluctuations in w are completely hedgeable, and furthermore that hedging has no effect on the expected level of w .⁸ Given this assumption, complete hedging will clearly be beneficial if and only if profits are a concave function of internal wealth.⁹

To explore the impact of hedging on optimal financing and investment decisions, we solve the model backwards, starting with the firm's first-period investment decision. The firm enters the first period with internal resources of w and chooses investment (and thereby the amount of external financing, $e = I - w$) to maximize net expected profits:

$$P(w) = \max_I F(I) - C(e). \quad (3)$$

The first-order condition for this problem is

$$F_I = f_I - 1 = C_e, \quad (4)$$

⁶ On costs of external finance, see e.g., Townsend (1979), Myers and Majluf (1984), Jensen and Meckling (1976), and Myers (1977) among many others.

⁷ A more general formulation of these costs would allow them to depend also on the scale of the investment project undertaken, $C = C(I, e)$. This would make it possible for a firm to lower its per dollar costs of external finance by undertaking larger investment projects. The qualitative nature of our results is unaffected (although the exposition is somewhat complicated) by using this more general formulation. As we discuss below, either formulation can be rationalized in an optimal contracting framework.

⁸ In order for fluctuations in w to be completely hedgeable (with default-free contracts) we need to assume that w is costlessly observable and verifiable. For example, w might represent a firm's exposure to gold price risk because the firm holds 100 bricks of gold. In this case, the exposure can be hedged if market participants can verify that the firm actually owns the bricks. For a discussion of how credit risks could interfere with hedging transactions, see footnotes 19, 28, and 31. The additional assumption that hedging does not affect the expected future level of w would follow from risk neutrality on the part of investors. It is straightforward to extend our analysis to the case where systematic risk is priced in equilibrium.

⁹ Concavity of the profit function is clearly a necessary condition for *any* model in which hedging raises value.

where we have used the fact that, in the second period when w is given, $de/dI = 1$. Equation (4) implies that there is underinvestment—the optimal level of investment, I^* , is below the first-best level, which would set $f_I = 1$.

Moving to period zero, the firm chooses its hedging policy to maximize expected profits. As noted above, random fluctuations in w reduce expected profits if $P(w)$ is a concave function. Using the first-order condition in (4), the second derivative of profits is given by

$$P_{ww} = f_{II} \left(\frac{dI^*}{dw} \right)^2 - C_{ee} \left(\frac{dI^*}{dw} - 1 \right)^2, \quad (5)$$

where f_{II} and C_{ee} are evaluated at $I = I^*$. If this expression is globally negative, then hedging raises average profits. Equation (5) can be rewritten by applying the implicit function theorem to (4) to yield¹⁰

$$P_{ww} = f_{II} \frac{dI^*}{dw}. \quad (6)$$

Equation (6) clarifies the sense in which hedging activity is determined by the interaction of investment and financing considerations. If hedging is to be beneficial, two conditions must *both* be satisfied: (i) marginal returns on investment must be decreasing, and (ii) the level of internal wealth must have a positive impact on the optimal level of investment. The latter condition is a ubiquitous feature of models of external finance in the face of information and/or incentive problems. Furthermore, there is substantial empirical evidence suggesting that corporate investment is indeed sensitive to levels of internal cash flow.¹¹

Two simple examples may help to further develop the intuition behind equations (5) and (6). In the first, assume that a company has no access at all to financial markets. In this case, C is always equal to zero in equilibrium, and any variation in w is reflected one-for-one in changes in investment, $dI^*/dw = 1$. Equations (5) and (6) then tell us that $P_{ww} = f_{II}$: the concavity of the profit function comes solely from the concavity of the production technology.

In the second polar example, investment is completely fixed (e.g., the company has only one indivisible investment project with high returns). Now any fluctuations in internal funds translate one-for-one into fluctuations in the amount of external funds that must be raised, $dI^*/dw = 0$. Equation (5)

¹⁰ The first-order condition (4) and the implicit function theorem together imply that I^* , satisfies

$$\frac{dI^*}{dw} = \frac{-C_{ee}}{f_{II} - C_{ee}},$$

at $I = I^*$. We assume that the second-order conditions with respect to investment are satisfied, so that the denominator of this expression is always negative.

¹¹ See, for example, Fazzari, Hubbard, and Petersen (1988), and Hoshi, Kashyap, and Scharfstein (1991).

then says that the concavity of the profit function comes exclusively from the convexity of the C function, i.e., $P_{ww} = -C_{ee}$.

Clearly, for intermediate cases—those in which $0 < dI^*/dw < 1$ —the concavity of the profit function will come from both the concavity of the investment technology and the convexity of the financing cost function. Another way to see this is to substitute out dI^*/dw from equation (5), yielding

$$P_{ww} = \frac{-f_{II}C_{ee}}{f_{II} - C_{ee}}. \quad (7)$$

Equation (7) illustrates again that hedging is driven by an interaction between investment and financing considerations (as represented by f_{II} and C_{ee} , respectively).

Thus far we have used an arbitrary specification for the C function to establish conditions under which hedging is value increasing. However, it is unclear whether those conditions (i.e., the requirement that $C_{ee} \geq 0$) would emerge naturally if we derived the C function from an optimizing model with rational agents. Next, we examine an important class of such models, and demonstrate that the required convexity in C obtains under a wide range of parameterizations.

B. Hedging in an Optimal Contracting Model

The model we adopt is a variant of the costly-state-verification (CSV) approach developed by Townsend (1979) and Gale and Hellwig (1985). As we shall see, the prescription that companies should hedge takes the form of a simple and fairly weak restriction on the specification of this CSV model. Moreover, we are able to rewrite the $C(e)$ function explicitly in terms of parameters of the CSV model.

As before, we assume that in the first period a firm can invest an amount I , which yields a gross payoff of $f(I)$ in the second period. Also in the second period, the firm generates *additional* random cash flows of x from its preexisting assets. The cumulative distribution and density of x are given by $G(x)$ and $g(x)$, respectively.

As in the Townsend and Gale-Hellwig models, we assume that cash flows are costlessly observable to company insiders, but are observable to external creditors only at some cost. In particular, we suppose that the cash flows from the *existing* assets can be observed at a cost c , but that it is infinitely costly to observe the cash flows from the new investment project. As is well known, when $c > 0$, the optimal contract between outside investors and the company will be a standard debt contract. In return for receiving e in the first period, the company is required to repay in the second period a state-invariant amount D . If the company fails to perform, creditors pay the monitoring costs, then observe—and keep for themselves—company profits. States in which monitoring occurs can be interpreted as bankruptcy.

Our formulation of the CSV model is slightly different from that in Townsend and Gale-Hellwig: we suppose that a set of preexisting assets

entirely determines the firm's capacity for external finance, so that this capacity is unaffected by the current investment spending. This parallels our setup in Section II.A above, where we assume that new investment spending has no independent effect on deadweight costs for a given level of external finance. That is, in both models C can be represented simply as $C(e)$. This assumption simplifies our analysis, but does not affect the basic results.¹²

Under these circumstances, the company chooses investment and outside financing to maximize

$$L \equiv \max_{I, D} f(I) + \int_D^\infty (x - D)g(x)dx, \quad (8)$$

subject to a nonnegative profit constraint for outside investors:

$$\int_{-\infty}^D (x - c)g(x)dx + \int_D^\infty Dg(x)dx \geq I - w. \quad (9)$$

The first-order conditions for this constrained optimization problem are

$$\frac{\partial L}{\partial D} = (\lambda - 1)(1 - G(D)) - \lambda cg(D) = 0, \quad (10)$$

$$\frac{\partial L}{\partial I} = f_I - \lambda = 0, \quad (11)$$

where λ is the Lagrange multiplier on constraint (9).

Equations (10) and (11) together imply that the firm sets I^* such that

$$f_I = \frac{1 - G(D)}{1 - G(D) - cg(D)} \geq 1. \quad (12)$$

If there are no deadweight costs ($c = 0$) the firm sets investment efficiently ($f_I = 1$). However, if $c > 0$, then the firm underinvests, setting $f_I > 1$.¹³ Underinvestment occurs in this model because an increase in I necessitates an increase in D , which raises the probability of bankruptcy. At the optimum, the firm reduces investment from the first-best level in order to economize on deadweight costs.

In this setup, there is a direct correspondence between expected deadweight costs of external finance and the probability of bankruptcy:

$$C(e) = cG(D), \quad (13)$$

where equation (9) implicitly defines the function $D = D(e)$.

¹² One way to rationalize this assumption would be to suppose that the assets in place are comprised of physical capital that has some value in liquidation, whereas the new investment is in intangible assets (e.g., R & D, market share, etc.) that have no value in liquidation.

¹³ This analysis assumes that there exists an optimally chosen D such that $1 - G(D) - cg(D) > 0$ and that investors' zero-profit constraint (9) is satisfied. Otherwise, there would be no solution to the problem in (8), and no investment would take place.

One can verify that the first-order condition, $F_I = C_e + 1$, derived in Section II.A, is identical to (12) above. From equation (11), it is clear that the expected shadow value of an additional dollar of internal wealth ($L_w = \lambda$) is equal to the marginal return on investment, which is given by f_I .

As before, hedging raises the value of the company if profits are concave in internal wealth, i.e., $L_{ww} = d\lambda/dw = F_{II} dI^*/dw < 0$. (Note that this is the same condition we derived in equation (6) for our reduced form model.) Totally differentiating equations (9) through (11) and solving for dI^*/dw , we can show that a sufficient condition for $dI^*/dw > 0 \forall x$ is that the hazard rate $g(x)/1 - G(x)$ is strictly increasing in x . This is a fairly weak condition, and is satisfied for the normal, exponential, and uniform distributions, among others.¹⁴ Thus, when $f_{II} < 0$ and the hazard rate of $G(\cdot)$ is increasing, hedging is optimal in this CSV framework.

III. Optimal Hedging with Changing Investment and Financing Opportunities

So far our results create a very simplistic picture of optimal hedging policies—firms with increasing marginal costs of external finance should always fully hedge their cash flows. In this section, we extend our analysis to incorporate randomness in both investment and financing opportunities. As will be seen, these considerations lead to a richer range of solutions to the optimal hedging problem.

A. Changing Investment Opportunities

In the discussion above, we have assumed that a firm's investment opportunities were nonstochastic, and thus independent of the cash flows from its assets in place. In many cases, however, this assumption is unrealistic. For example, a company engaged in oil exploration and development will find that both its current cash flows (i.e., the net revenues from its already developed fields) and the marginal product of additional investments (i.e., expenditures on further exploration) decline when the price of oil falls. For such a company, hedging against oil price declines is less valuable—even without hedging, the supply of internal funds tends to match the demand for funds.

It is straightforward to extend the analysis of the previous section to address the question of the optimal hedge ratio in a world of changing investment opportunities. If we focus for the moment on linear hedging strategies (i.e., forward sales or purchases), the hedging decision can be modelled by writing internal funds as¹⁵

$$w = w_0(h + (1 - h)\epsilon), \quad (14)$$

¹⁴ The same restriction on the hazard rate also implies that $C_{ee} > 0$. This can be seen by twice differentiating equation (13), and then by noting that equation (9) implicitly defines $D = D(e)$.

¹⁵ In Section V below, we consider alternative, nonlinear hedging strategies that involve instruments such as options.

where h is the "hedge ratio" chosen by the firm, and ϵ is the primitive source of uncertainty.¹⁶ To keep things simple, we assume that ϵ —the return on the risky asset—is distributed normally, with a mean of 1 and a variance of σ^2 .¹⁷

To model changing investment opportunities, we redefine profits as

$$F(I) = \theta f(I) - I, \quad (15)$$

with $\theta = \alpha(\epsilon - \bar{\epsilon}) + 1$. In this formulation, α is a measure of the correlation between investment opportunities and the risk to be hedged.

In period zero, the firm must choose h to maximize expected profits:

$$\max_h E[P(w)], \quad (16)$$

where the expectation is taken with respect to ϵ . The first-order condition for this problem is

$$E\left[P_w \frac{dw}{dh}\right] = 0. \quad (17)$$

Equation (17) simplifies to

$$E[P_w(1 - \epsilon)] = 0, \quad (18)$$

which can be written as

$$\text{cov}(P_w, \epsilon) = 0. \quad (19)$$

Equation (19) says that the optimal hedge ratio insulates the marginal value of internal wealth (P_w) from fluctuations in the variable to be hedged. Notice that this is *not* necessarily the same as insulating the total value of the firm, P , from such fluctuations.

To simplify the covariance term, we use a second-order Taylor series approximation (which is exact if the asset's return, ϵ , is normally distributed) with respect to h around $\epsilon = 1$.¹⁸ Equation (19) and a little algebra then yield the optimal hedge ratio

$$h^* = 1 + \alpha \frac{E[f_I P_{ww} / \theta f_{II}]}{w_0 \bar{P}_{ww}}, \quad (20)$$

where a bar over a variable implies that an expectation has been taken with respect to ϵ , e.g., $\bar{P}_{ww} = E[P_{ww}]$.

¹⁶ To see what (14) implies for actual futures positions and prices, define x_0 as the current futures price and q_1 as the future spot price of the variable in question. The variable ϵ then corresponds to $\epsilon = (q_1/x_0)$ and a hedging position of h corresponds to selling $h(w_0/x_0)$ futures contracts.

¹⁷ Assuming that the mean of ϵ is one implies, as before, that the expected level of wealth is unaffected by the amount of hedging.

¹⁸ If x and y are normally distributed, and $a(\cdot)$ and $b(\cdot)$ are differentiable functions, then $\text{cov}(a(x), b(y)) = E_x[a_x]E_y[b_y]\text{cov}(x, y)$. See Rubinstein (1976) for a proof. Note that if we were to assume that ϵ is log-normally distributed (with the same mean and variance as above), we would arrive at results very similar to those given throughout the paper.

The last term in equation (20) takes account of the direct effect of ϵ on output. Clearly, if $\alpha = 0$ (i.e., there is no correlation between investment opportunities and the availability of internal funds), it is optimal to hedge fully (i.e., $h^* = 1$), as in Section II above.

If $\alpha > 0$, the firm will not want to hedge as much. To see why, note that when ϵ is low, the firm may be low on cash, but doesn't need much, since it has few attractive investment opportunities. Conversely, when ϵ is high, the firm has good investment opportunities and therefore needs the additional cash generated internally. This logic implies that there is less to be gained from a hedge which transfers funds from high ϵ states to low ϵ states. Thus, the more sensitive are investment opportunities to ϵ , the smaller is the optimal hedge ratio.

It should be emphasized that in this case ($\alpha > 0$), the firm chooses *not* to insulate fully either its cash flows or market value from fluctuations in ϵ . In the example of the oil company mentioned above, the optimal hedging strategy would involve leaving the stock price exposed to oil price fluctuations. This conclusion differs from that of many other papers, which often imply complete insulation.

It should also be noted that according to equation (20), h^* need not necessarily be between zero and one. The possibility of $h^* < 0$ arises when investment opportunities are extremely sensitive to the risk variable. In that case it may make sense for a firm to actually *increase* its exposure to the variable in question, so as to have sufficient cash when ϵ is high and very large investments are required. Conversely, optimal hedge ratios greater than one will arise when investment opportunities are negatively correlated with current cash flows. In this case it makes sense to "overhedge," so as to have more cash when ϵ is low.¹⁹

To build some further intuition for why companies with different investment opportunities might implement different hedging strategies, consider the following example. Suppose there are two companies engaged in natural resource exploration and extraction. Company g is a gold company. It currently owns developed mines which produce 100 units of gold in period one at zero marginal cost. Thus company g 's period one cash flows are $100\tilde{p}_g$, where $\tilde{p}_g$ is the random price of gold.

¹⁹ Note that while $h^* < 0$ or $h^* > 1$ may (according to equation (20)) be optimal for the firm, such positions may implicitly leave the firm with negative first-period resources in some states. As a consequence, the capital market may no longer charge default-free prices for futures contracts, because these contracts can now involve credit risk. For example, a firm with initial wealth consisting of nothing but 100 gold bricks may not be able to buy more *on net*, because it has no nongold collateral. (That firm would have no resources to pay for the additional purchases if the price of gold were to fall to zero.) Similarly, a firm that sells futures contracts for more than the equivalent of 100 gold bricks might be unable to make good on its position when gold prices rise sufficiently. This entire credit risk issue disappears, however, if we are willing to assume that the investment function satisfies the Inada conditions, i.e., that the marginal product of investment is infinite at $I = 0$. In this case the optimal hedge ratio in equation (20) endogenously ensures that firm resources (and hence investment) are positive in all states.

Company g also has the opportunity to invest in additional exploration activities in period one. If it spends an amount I on exploration, it discovers undeveloped lodes containing $f_g(I)$ units of gold. Before the gold can be extracted, however, a further *per unit* development cost of c_g must be paid in period two. Thus, the net returns to an exploration investment of I are given by $(\bar{p}_g - c_g)f_g(I) - I$.

Company o is an oil company. In most respects it is very similar to company g . Its period one cash flows are $100\bar{p}_o$, and it is assumed that $\bar{p}_o$ has the same distribution as $\bar{p}_g$. Thus, both companies face exactly the same risks with regard to the nature of their period one cash flow.

Company o also can uncover undeveloped reserves containing $f_o(I)$ units of oil by spending an amount I on exploration in period one. Company o 's development costs are higher than company g 's—it must pay $c_o > c_g$ in period two to develop the new reserves before they can be extracted. Thus, the net returns to an exploration investment of I are given by $(\bar{p}_o - c_o)f_o(I) - I$. To preserve comparability across the two companies, it is further assumed that $f_o(I) = (\bar{p} - c_g / \bar{p} - c_o)f_g(I)$, where $\bar{p}$ is the mean of both price distributions. This implies that in the “base case” where commodity prices equal their means, both companies have the same marginal product of capital at any given level of investment.

The key difference between company o and company g is the fact that higher development costs make company o 's investment opportunities *more leveraged* with respect to commodity prices. For example, if $c_g = 0$ and $c_o = 50$, the marginal product of capital for the gold company falls by 10 percent when gold prices fall from 100 to 90. However, the marginal product of capital for the oil company falls by 20 percent when oil prices fall from 100 to 90.

In the terminology of the above model, this difference in technology can be represented as a higher value of the parameter α for the oil company. Thus, the two companies should pursue different hedging strategies, with company g hedging more than company o . In other words, company o should leave its market value more exposed to fluctuations in oil prices than company g because its investment opportunities are more sensitive to the price of oil.

B. Changing Financing Opportunities

Up to now, we have assumed that the supply schedule for external finance—given by the $C(e)$ function—is exogenously fixed and insensitive to the risks impacting the firm's cash flows. However, it seems quite possible that negative shocks to a firm's current cash flows might also make it more costly for the firm to raise money from outside investors. If this is the case, it may make sense for the firm to hedge more than it otherwise would. This will allow the firm to fund its investments while making *less* use of external finance in bad times than in good times.²⁰

²⁰ We thank Tim Luehrman for suggesting this case to us.

We can formalize this insight by generalizing the C function to be $C(e, \phi)$, where ϕ is given by $\delta(\epsilon - \bar{\epsilon}) + 1$. Such a generalization emerges naturally from the CSV model sketched in Section II.B. Suppose that instead of yielding x , the assets already in place yield ϕx . That is, the eventual proceeds from assets in place are correlated with the risk variable ϵ , and δ measures the strength of this correlation. As long as the distribution of x satisfies the increasing hazard rate property, then the $C(e, \phi)$ function, that emerges from the CSV setting has the feature that $C_{e\phi} < 0$ (for fixed first-period wealth). This simply means that marginal costs of external finance, C_e , are lower for higher realization of ϵ .

If we assume for the moment that α —which measures the correlation of investment opportunities with ϵ —is zero, we can derive an expression that gives us the pure effect of changing financing opportunities on the hedge ratio. The methodology is the same as before. In particular, the first-order condition in (19) still applies. But now the optimal hedge ratio is given by

$$h^* = 1 + \delta \frac{\bar{C}_{e\phi}}{w_0 \bar{P}_{ww}}. \quad (21)$$

Given that $C_{e\phi} < 0$, the optimal hedge ratio is greater than one, with the effect being greater the more sensitive are assets in place to the risk variable ϵ . Again, the intuition is that hedging must now allow the firm to fund its investments and yet conserve on borrowing at those times when external finance is most expensive.²¹

However, even with a nonstochastic production technology (i.e., $\alpha = 0$), it is no longer true that investment is completely insulated from shocks to ϵ . This is purely a consequence of the fact that we are restricting ourselves to linear hedging strategies. Nonstochastic investment would (by the firm's first-order conditions) require that, once the hedge is in place, C_e be independent of ϕ . This generally cannot be accomplished using futures alone. In Section V below, we argue that if options are available, the firm will indeed wish to construct a hedging strategy that leads to nonstochastic investment.

IV. Risk Management for Multinationals

Our framework also has implications for multinational companies' risk management strategies.²² Multinationals have sales and production opportunities in a number of different countries. In addition, the goods that they produce at any given location may either be targeted for local consumption

²¹ In this particular case, there is no default risk associated with the futures position that implements the desired hedge ratio. The futures position will only incur large losses in those states where assets in place are extremely valuable. In such states the funds that can be raised against assets in place ensure that the firm will make good on its future position.

²² Conversations with Don Lessard were especially helpful in motivating the work in this section. See Adler and Dumas (1983) for an overview of the traditional arguments for hedging exchange rate risk.

(i.e., nontradeable goods, such as McDonald's hamburgers) or for worldwide markets (i.e., tradeable goods, such as semiconductors). These factors complicate the hedging problem for multinational corporations.

We begin with a quite general framework which builds on that of the previous sections. Assume that the multinational can invest in two locations, "home" and "abroad," and that profits are given by

$$P(w) = f^H(I^H) + \theta f^A(I^A) - I^H - \gamma I^A - C(e) \quad (22)$$

where $\theta = \alpha(\epsilon - \bar{\epsilon}) + 1$, $\gamma = \beta(\epsilon - \bar{\epsilon}) + 1$, and the production functions, $f^i(I^i)$, $i = A, H$ are increasing and concave. In this expression, ϵ now represents the home currency price of the foreign currency, and α and β are parameters (between zero and one) which index the sensitivity of foreign revenues and foreign investment costs to the exchange rate.²³ Implicitly, equation (22) treats the domestic currency as the numeraire.²⁴

It is easiest to build an understanding of equation (22) by examining several special cases:

Case 1: Exchange rate exposure for both investment costs and revenues from foreign operations, $\alpha = \beta = 1$. This case might correspond to situations where both the outputs and the investment inputs are nontraded goods.²⁵ An example might be Euro-Disney in France, since local factors are required to begin operations.

Case 2: Exchange rate exposure for foreign investment costs but no exchange rate exposure for either foreign or domestic revenues, $\alpha = 0$ and $\beta = 1$. This case might correspond to a situation where the output from both plants is sold at the same price on the domestic market.²⁶ An example might be ball bearings, which can be produced using primarily local factors, but which are sold on a global market.

Case 3: No exchange rate exposure for investment costs but exchange rate exposure for foreign revenues, $\alpha = 1$ and $\beta = 0$. This case might correspond, as above, to a situation where the outputs are nontraded goods. However, now the investment inputs used in both locations are purchased on a single domestic market at the same price. An example might be a construction company, like Bechtel, which makes heavy use of construction equipment that is sold on a global market.

²³ Note that our earlier formulation in Section III can be interpreted as a degenerate case of equation (22), with $\beta = 0$ and I^H fixed at zero—i.e., no investment in one of the two countries.

²⁴ In this formulation, the external borrowing facility is also denominated in the home currency. In terms of CSV model developed in Section II.B, this amounts to assuming that the payoff x on the preexisting asset is home currency denominated. Thus, we are suppressing the issues relating to changing financing opportunities raised in Section III.B.

²⁵ Effectively, this assumes that the foreign currency price of nontradeable goods is not affected by exchange rate changes.

²⁶ This will be correct provided that this domestic currency price is constant.

In order to finance these different investments, the firm requires external finance of an amount

$$e = I^H + \gamma I^A - w. \quad (23)$$

Maintaining our focus on linear hedging strategies, w continues to be given by equation (14) above. In this formulation, a hedge ratio of one means that period zero wealth, w_0 , is held entirely in the domestic currency. In contrast, a hedge ratio of zero means that wealth is held entirely in the foreign currency.

Using arguments analogous to those developed above, we can solve for the optimal hedge ratio. (See the Appendix for a sketch of the derivation.)

$$h^* = 1 + \frac{E[(\alpha\gamma - \beta\theta)f_I^A P_{ww}/\theta f_{II}^A]}{w_0 \bar{P}_{ww}} - \beta \frac{E[I^A P_{ww}]}{w_0 \bar{P}_{ww}}, \quad (24)$$

where

$$P_{ww} = \frac{f_{II}^H \theta f_{II}^A C_{ee}}{C_{ee}(\gamma^2 f_{II}^H + \theta f_{II}^A) - \theta f_{II}^H f_{II}^A} < 0. \quad (25)$$

There are two basic components of the optimal hedge ratio in (24). First, there is a slightly more complex version of the “changing investment opportunity set” term,

$$\frac{E[(\alpha\gamma - \beta\theta)f_I^A P_{ww}/\theta f_{II}^A]}{w_0 \bar{P}_{ww}},$$

which effectively captures the *net* exchange rate exposure of foreign investment profitability. Second, there is a new “lock-in” term, $\beta(E[I^A P_{ww}]/w_0 \bar{P}_{ww})$, which is, loosely speaking, driven by the expected size of the foreign investment relative to internal wealth.

We can understand this lock-in term better by focusing on Case 1 above, where $\alpha = \beta = 1$. In this case (or in any case with $\alpha = \beta$), (24) can be simplified considerably—the changing investment opportunity set term disappears completely, and the lock-in term itself becomes easier to interpret. In particular, we demonstrate in the Appendix that:

PROPOSITION 1: *If $\alpha = \beta$, then the optimal hedging strategy is such that investment in both locations is independent of the exchange rate: $I^H(\epsilon) = \bar{I}^H$; and $I^A(\epsilon) = \bar{I}^A \forall \epsilon$. This hedging strategy is given by $h^* = 1 - \beta \bar{I}^A/w_0$.*

To understand the intuition behind the proposition, imagine that the company did not hedge at all but that the actual realization of the exchange rate coincided with its expectation, $\epsilon = \bar{\epsilon}$.²⁷ One could then solve for the optimal first-period levels of investment. What hedging does is to *assure* that

²⁷ Note that with $\bar{\epsilon} = 1$, the expected future spot rate is equal to the forward rate.

domestic and foreign investment will always be at exactly these levels, regardless of the actual realization of the exchange rate. In other words, hedging *locks in* the ability to carry out a predetermined (as of period zero) investment plan, where that plan is based on the expected future exchange rate.

In Case 2, with $\alpha = 0$ and $\beta = 1$, the lock-in term remains. However, it takes on a more complicated form, since I^A and P_{ww} are now random variables, and it is no longer generally true that $E[I^A P_{ww}] = \bar{I}^A \bar{P}_{ww}$. In addition, the hedge ratio is increased by the changing investment opportunity set term,

$$\frac{-E[f_I^A P_{ww}/f_{II}^A]}{w_0 \bar{P}_{ww}}.$$

This term implies that it is optimal to hold relatively *more* of the domestic currency than in Case 1. The logic is similar to that developed in Section III above. When the domestic currency depreciates, investments abroad become less attractive due to higher input costs. Thus, less foreign investment is warranted, and there is less need to hold foreign currency as a hedge against such an outcome.

Finally, in Case 3, with $\alpha = 1$ and $\beta = 0$, there is no lock-in effect. Because the price of foreign investment is insensitive to the exchange rate, it is unnecessary to hold foreign currency to guarantee a given level of foreign investment. At the same time, it is still worthwhile to hold *some* wealth in the form of foreign currency. This is because the correlation of net investment opportunities with the value of the domestic currency is now *negative*—when the domestic currency depreciates, returns on foreign investment are now *high*.

V. Nonlinear Hedging Strategies

Thus far we have restricted our attention to hedges which employ only forward or future contracts. With these instruments, the sensitivity of internal wealth to changes in the risk variable to be hedged is constrained to be a constant. That is, $dw/de = (1 - h)w_0$, which is independent of the realization of ϵ . While such linear hedges can add value, they generally will not *maximize* value if other, nonlinear instruments, such as options, are available. Options effectively create the possibility for hedge ratios to be “customized” on a state-by-state basis.

To see why a firm might want its hedge ratios to be sensitive to the realization of ϵ , let us return to our oil company example. We argued that the oil company’s investment opportunities become less attractive when the price of oil falls, and that this militated in favor of leaving its cash flows somewhat exposed to these fluctuations. But suppose we use futures to pick a single, state-independent hedge ratio, and that this hedge ratio results in the oil company cutting capital investment expenditures by 2 percent for every 1

percent decline in the price of oil. This might make good sense for small fluctuations in oil prices—perhaps the company's level of investment *should* be cut by 20 percent when oil prices fall by 10 percent. But it may not make equally good sense for the company to completely eliminate its investment spending when oil prices fall by 50 percent.

If this is the case, the oil company may wish to do some of its hedging with options. For example, by adding out-of-the-money puts on oil to its futures-hedging position, the company can give itself relatively more protection against large decreases in the price of oil than against small decreases. (Similarly, the company might also write out-of-the-money calls on oil, if a linear hedging strategy results in “too much” cash for very large increases in the price of oil.)

We can develop the general logic for nonlinear hedging strategies using the same basic setup as in Section IV. We denote the frequency distribution of the random variable, ϵ , by $p(\epsilon)$. If we assume complete markets, the firm's hedging problem now becomes one of choosing a profile for wealth across states of nature, $w^* = w^*(\epsilon)$, to maximize expected profits:

$$\max_{w(\epsilon)} \int_{\epsilon} P(\epsilon, w(\epsilon)) p(\epsilon) d\epsilon, \quad (26)$$

subject to the “fair pricing” constraint that hedging cannot change the expected level of wealth,

$$\int_{\epsilon} w(\epsilon) p(\epsilon) d\epsilon = w_0, \quad (27)$$

and to the first-order conditions for domestic and foreign investment (which are given in equations (A1) and (A2) of the Appendix.²⁸

The first-order condition for the constrained optimization problem in (26) is given by

$$P_w = \lambda, \quad (28)$$

where λ is the Lagrange multiplier on the constraint (27). Equation (28) says that the optimal hedging policy equalizes the shadow value of internal wealth across states. By smoothing the impact of costly external finance in this way, the firm has optimally matched the cash demand of investment with the supply of internal funds.

Equation (28) implicitly defines an optimal level of internal wealth in every state. Note that because λ is constant, the implicit function theorem can be applied to (28), which after some algebra yields an expression for the optimal

²⁸ It is also important to check whether the candidate solution that emerges from (26) and (27) involves negative wealth in any states. If so, then an additional, nonnegativity constraint on internal wealth, $w \geq 0, \forall \epsilon$, might also be imposed in the maximization problem, in order to address the concerns about credit risk raised in footnote 19.

hedge ratio in each state:

$$\frac{dw^*(\epsilon)}{d\epsilon} = \frac{P_{w\epsilon}}{-P_{ww}} = -(\alpha\gamma - \beta\theta) \frac{f_I^A}{w_0 \theta f_{II}^A} + \frac{\beta I^A}{w_0}, \quad (29)$$

where $w^* = w^*(\epsilon)$ describes the optimal level of wealth for every value of ϵ . The expression on the right-hand side of (29) can be shown to be a function (denoted by $l = l(w(\epsilon), \epsilon)$, of both internal wealth and ϵ :

$$\frac{dw^*(\epsilon)}{d\epsilon} = -(\alpha\gamma - \beta\theta) \frac{f_I^A}{w_0 \theta f_{II}^A} + \frac{\beta I^A}{w_0} = l(w^*(\epsilon), \epsilon). \quad (30)$$

This expression defines the basic differential equation which the optimal level of wealth must satisfy. The constraint (27) provides the restriction that ties down the constant of integration.

One can use (29) to see when the first-best hedge can be attained using only futures contracts. In such cases, it must be that $(dw^*/d\epsilon)$ is a constant. Thus, making use of the results of Proposition 1, we have:

PROPOSITION 2: *With $\alpha = \beta$, futures contracts alone can provide value-maximizing hedges. In all other cases, options may be required to obtain the value-maximizing hedge.*

Futures hedging alone is thus optimal: (i) in the simple models of Section II with fixed investment and financing opportunities (i.e., with α , δ , and β equal to zero); and (ii) in our multinational setup of Section IV whenever there is the complete lock-in described in Proposition 1. In contrast, options will be needed for implementing the optimal hedges when either $\alpha \neq \beta$ or when there are state-dependent financing opportunities ($\delta \neq 0$) as in Section III.B. In the latter case, the use of options allows investment to be completely insulated from shocks to financing opportunities.²⁹

For those cases in which options are required, equation (29) implicitly yields a recipe for the number of options to be purchased at different strike prices. While the first derivative of wealth, $(dw^*/d\epsilon)$, gives us the optimal exposure to ϵ , it is the *second* derivative, $(d^2w^*/d\epsilon^2)$, that describes the “density” of the options position at each strike price in the optimal hedge portfolio. Intuitively, an option at a strike price of $\hat{\epsilon}$ is indispensable for *changing* the degree of exposure at the point where $\epsilon = \hat{\epsilon}$. Thus, for example, if there are regions in which $(d^2w^*/d\epsilon^2)$ is large and positive, a substantial number of call options with strike prices in that region should be added. In contrast, for regions in which the hedge ratio is constant, $(d^2w^*/d\epsilon^2) = 0$, no additional options are required.

²⁹ To see this, note that with nonstochastic production technology, $F_I = P_w$, which by (28) is a constant.

Table I
Hypothetical Hedging Strategies and Investment Spending
(with Initial Wealth of 10)

State	Probability	Optimal Investment Spending	Net Funds Available for Investment			
			No Hedging (1)	Optimal Futures Hedge (2)	Payoffs to First-Best Options (3)	First-Best Hedge with Options (2) + (3) - cost
1	1/3	6	5	6	1	$6 + 1 - 1 = 6$
2	1/3	9	10	10	0	$10 + 0 - 1 = 9$
3	1/3	15	15	14	2	$14 + 2 - 1 = 15$
Total cost of options			$-1/3 - 2/3 = -.1$			

To see the role for options more concretely, consider the following numerical example. Suppose that there are three equally probable states of nature, 1, 2, and 3, and that a firm's first-best levels of investment (i.e., that for which $f_t = 1$) are 6, 9, and 15, in each state respectively. Suppose also that at any level of investment below 6, the firm will be unable to compete and will be forced into bankruptcy, and that the firm has no access to external finance. Finally, suppose that internal wealth is initially equal to 10, and that a no-hedging strategy yields 5, 10, and 15 of internal funds available for investment. (See Table I for a schematic.)

If the firm has only futures contracts available to it, it can increase state one internal wealth only through an equivalent reduction in state three wealth. Its optimal hedge will therefore be predicated on protecting revenues in the lowest state, and will lead to an internal wealth configuration of something like 6, 10, and 14. This is a better profile than without hedging, but it does not generate first-best levels of investment.

Now suppose that options become available. With its futures hedge in place, the firm has excess cash in state two and insufficient cash in state three. The value-maximizing hedging strategy therefore involves buying 1 state one "put" option (which pays 1 in state one and zero otherwise) and 2 state three "call" options (each of which pay 1 in state three or zero otherwise). Because each option costs $1/3$, their total cost is 1, which exactly eliminates the previously existing excess cash balance in state two. (See Table I.) Options are therefore valuable when value-maximizing hedge ratios are not constant.³⁰

VI. Further Extensions

Although we have explored a number of applications of our basic risk management paradigm, several interesting questions remain. In this section,

³⁰ By put-call parity, one can achieve an equivalent hedge by using only the put (or call) option together with a different quantity of futures, or by using options alone.

we briefly sketch some additional extensions, focusing on the basic intuition and leaving the formal development for future work.

A. Intertemporal Hedging Considerations

Since the model developed above is essentially a static one—there is only a single period during which investment takes place—we have not addressed any of the potentially important intertemporal issues associated with risk management.

To see how intertemporal considerations can complicate matters, suppose that at each of N dates, the firm has a random cash flow and a *nonstochastic* investment opportunity. (The simplest model in Section II.A is just a special case of this with $N = 1$.)

Since investment opportunities are nonstochastic, a first guess might be—following the logic set out above—that the optimal strategy is to hedge *all* of the N random cash flows. For example, if the cash flows represent revenues from oil wells that will deliver 100 million barrels in each of the next ten years, it might seem that the best thing to do is to sell short 100 million barrels worth of futures with delivery one year hence, 100 million barrels worth with delivery two years hence, and so on, with contract maturities running out to ten years.

However, this raises a problem, at least if futures contracts are used in the hedge. If oil prices rise in the first year, the margin call on the aggregate futures position—representing ten years' worth of production—will be very large, and will much more than offset the positive impact of oil prices on first-year revenues. In other words, hedging the whole future stream of production leads to enormous margin fluctuations and hence to enormous variations in the year-by-year level of cash available for investment.

This suggests that if futures are indeed to be used, the aggregate size of the position will have to be lowered somewhat. The optimal hedge will have to trade off insulating the present value of all cash flows versus insulating the level of cash at each point in time.

An alternative possibility might be for the firm to structure its hedge using a series of forward contracts (or other “forward-like” instruments, such as swaps or indexed debt) rather than futures contracts. In an intertemporal setting, forwards might represent a more desirable instrument, since they do not have to be settled until maturity and hence do not entail interim margin calls. However, there are reasons to believe that forward contracts, while potentially useful, may not completely “solve” the problem sketched above. Precisely because they are not settled until maturity, forwards can involve substantially more credit risk than futures.³¹

³¹ If the oil production is literally certain to be 100 million barrels, then forward contracts do not involve credit risk, and would allow complete hedging. However, if, more realistically, production quantities are uncertain or subject to moral hazard problems, forward contracts *will* involve some credit risk, and therefore represent an imperfect hedging vehicle.

In effect, one can think of a forward contract as (loosely speaking) a combination of futures plus borrowing. In the context of our model, this means that a decision to use forwards may lower the firm's ability to raise external financing at any point in time. As a practical matter, it may simply be impossible for many firms to take very large positions in forwards because of the credit risks involved.

B. Capital Budgeting When Risks Are Not Marketable

We have assumed throughout that all risks impacting a firm's cash flows are marketable and thus can be hedged. However, this will not in general be true. For example, a firm's cash flows will be abnormally low if its new product introduction fails, but there may be no futures market in which this risk can be laid off.

If this is the case, such unmarketable idiosyncratic risks will (in a world with costly external finance) impose real costs on the firm. Capital-budgeting procedures should therefore take these costs into account. Consequently, the CAPM (or any other standard asset-pricing model) will no longer be universally valid as a capital-budgeting tool. In other words, when investment projects impose large idiosyncratic risks that cannot be directly sold off, a second-best risk management strategy will involve reducing the level of investment in these projects below that implied by a CAPM-type discounting procedure.

The *magnitude* of the deviation from traditional capital-budgeting principles should depend on the same sorts of factors that we identified above as determinants of the optimal hedging strategy. For example, if the unmarketable idiosyncratic risk on the investment currently being evaluated is closely correlated with the availability of future investment opportunities, then the logic developed in Section III.A suggests that there is relatively less reason to "hedge" by skimping on this investment. In contrast, if the investment in question is uncorrelated with the availability of future investment opportunities, it should be evaluated more harshly.

C. Hedging and Product-Market Competition

Our framework also has the implications for how companies' hedging strategies should depend on both (1) the nature of product market competition, and (2) their competitors' hedging strategies.³² To see this, suppose that there are two firms and they compete à la Cournot—they each choose production quantities, q_i , $i = 1, 2$, holding fixed the other's quantity decision. One can interpret the quantity decision as investment I_i , so that $I_i = cq_i$, where c is the marginal cost of a unit of capacity.

Assume that both firms have no access to external finance, so that investment can never exceed cash flow. Suppose further that cash flow is perfectly

³² Adler (1992) also considers the implications of product market competition for hedging policy.

correlated across firms and that its mean is equal to I^* , which we define as the investment level that would prevail in an unconstrained Cournot equilibrium.

The important feature of the Cournot model is that investment is less attractive the more a rival firm invests. In the terminology of Bulow, Geanakoplos, and Klemperer (1985), investment is a “strategic substitute.” This contrasts with other models in which the strategic variables are “strategic complements”—firms want to invest more when their rivals invest more. Such might be the case in a research and development (R & D) model in which there are informational spillovers across firms.

Suppose that neither firm hedges. When their cash flows exceed I^* , the unconstrained Cournot equilibrium prevails—both firms invest I^* . However, when cash flow is less than I^* , both firms invest what they have. Both would like to increase their investment in these states—since investment/output is relatively low and prices are high—but cannot due to liquidity constraints.

Now suppose that just Firm 1 hedges, locking in a cash flow of I^* . When Firm 2's cash flows exceed I^* , the unconstrained Cournot equilibrium is achieved—just as it would be without hedging. But, when Firm 2's cash flows are less than I^* , Firm 2 invests only what it has, while Firm 1 (which has hedged) gets to invest more. Because investment is a strategic substitute, the additional investment that hedging makes possible is particularly attractive to Firm 1 in these states: Firm 2 is not investing much; prices are high; and so are the marginal returns to the investment. Thus Firm 1 is clearly better off hedging. Indeed, Firm 1 would like to go even further—adopting a hedge ratio greater than one—because the returns to investments are now higher when cash flow is low than when it is high. In the context of our model with changing investment opportunities, this is analogous to the case of $\alpha < 0$.

One can also show that there are benefits to Firm 1 from hedging in this model if Firm 2 *does* hedge, but they are not as high as in the previous example. The reasoning is that if Firm 2 hedges—ensuring that it can invest I^* in all states—its generally stronger position makes investment less appealing to Firm 1. Thus, there is less reason for Firm 1 to use hedging to lock in a high level of investment.

There are two related implications that follow from this example. First, hedging policy inherits the strategic substitutability feature of the product-market game—a firm will want to hedge more when its rival hedges less. Second, the overall industry equilibrium will involve some hedging by both firms.

We conjecture that we might get very different results if investment were a strategic complement, such as in the R & D example mentioned above. In this framework, if Firm 2 does not hedge, the marginal returns to Firm 1 R & D are low when cash flow is low and high when cash flow is high. This is because when cash flow is low, Firm 2 is constrained and does little R & D. And when cash flow is high, just the opposite is true. This is analogous to the case of a positive α —a positive correlation between investment opportunities and cash flow—so that less than full hedging is optimal.

Thus it would seem that hedging is generally less attractive when investment is a strategic complement. One might also conjecture that, like in the previous model, hedging policy inherits the strategic character of the product-market game. In this case, that would imply that hedging policies are strategic complements: a firm will want to hedge *more* when its rival hedges more.

VII. Empirical Implications

In this section we discuss some of the model's empirical implications. However, before doing so we should note two points. First, it is not at all clear that our theory should be interpreted solely as a positive one, i.e., as an accurate description of the actual status of corporate hedging policy. Even if empirical work were to find that few firms currently hedge according to our theory, we nevertheless think that the theory has a number of useful *prescriptive* implications.

Second, empirical work in this area is made difficult by the fact that most hedging operations are off balance sheet (and thus are not included in databases such as COMPUSTAT). This lack of a well-developed database has led researchers to collect survey data on firms' hedging policies. We begin with a review of some of this evidence. Next, we propose a new type of test for optimal hedging, one which has the advantage of not requiring direct measurement of hedging positions.

A. Anecdotal and Survey Evidence

That the coordination of financing and investment is the basis for at least some managers' hedging strategies seems evident from what they say. For example, a Unocal executive, Matthew Burkhart, argues that "one possible added value of hedging is to continue on a capital program without funding and defunding."³³ And Lewent and Kearney (1990), in explaining Merck's philosophy of risk management, note that a key factor in deciding whether to hedge is the "potential effect of cash flow volatility on our ability to execute our strategic plan—particularly, to make the investments in R & D that furnish the basis for future growth."

It is, of course, far more difficult to say whether the considerations we outline are those that drive hedging strategies more broadly. A recent study by Nance, Smith, and Smithson (1993) uses survey data to compare the characteristics of firms that actively hedge with those that do not. Some of their findings are consistent with our framework, while others cut less clearly. One noteworthy result is that high R & D firms are more likely to hedge. There are a couple of reasons why this might be expected in the context of our model. First, it may be more difficult for R & D-intensive firms

³³ The quote is from "Shareholders Applaud Risk Management," *Corporate Finance*, June/July 1992.

to raise external finance either because their (principally intangible) assets are not good collateral (see Titman and Wessels (1988)) or because there is likely to be more asymmetric information about the quality of their new projects. Second, R & D “growth options” are likely to represent valuable investments whose appeal is *not correlated* with easily hedgeable risks, such as interest rates. Thus, the logic of Section III.A would imply more hedging for R & D firms.

Nance, Smith, and Smithson (1993), as well as Block and Gallagher (1986) and Wall and Pringle (1989), also find weak evidence that firms with more leveraged capital structures hedge more. To the extent that such firms have fewer unencumbered assets, and hence more difficulty raising large amounts of external finance, this finding also fits with our model.

Finally, Nance, Smith, and Smithson (1993) also find that high-dividend-paying firms are more likely to hedge. It is not obvious how this fact squares with our model. One interpretation—which is inconsistent with our model—is that high-dividend payers are not likely to be liquidity constrained since they have chosen to pay out cash rather than use it for investment.³⁴ However, a second interpretation would be that high-dividend firms need to hedge more if they are to maintain both their dividends *and* their investment. This interpretation is more consistent with our model.³⁵

B. A New Test for Optimal Hedging

The broadest implication of our model is that firms use hedging to lower the variability of the shadow value of internal funds. In the model of Section III.A, this was accomplished by choosing the hedge ratio, h , such that $\text{cov}(P_w, \epsilon) = 0$ (equation 19); in the model of Section V, it was done by setting P_w equal to a constant (equation (28)). Either way, the first-order condition of our model generates a clear testable restriction: that the shadow value of internal funds and ϵ ought to be uncorrelated.

Consider then the model of Section III.A, in which firm value is a function $P = (w(\epsilon), \epsilon)$. This means that the risk variable, ϵ , may affect P directly through its impact on investment opportunities *given* internal funds, w , and indirectly through its effect on w *given* investment opportunities. In addition, there is a third possible effect on P : changes in w that are unrelated to ϵ . This suggests a simple empirical specification of the form

$$P_{t,i} = \alpha + w_{t,i}(\alpha_1 + \alpha_2 \epsilon_t) + \alpha_3 \epsilon_t + \nu_{t,i}, \quad (31)$$

³⁴ This reasoning is certainly consistent with Fazzari, Hubbard, and Petersen (1988) who found that investment was least sensitive to cash flow for high-dividend firms.

³⁵ Nance, Smith, and Smithson also find that smaller firms are less likely to hedge. This fact is generally inconsistent with our model if one believes that smaller firms are more likely to be liquidity constrained due to greater informational asymmetries. However, the tendency toward greater information asymmetries may be offset by relationships with certain capital providers, such as banks. Also, if there are fixed costs of setting up a hedging program, the gains from hedging for small firms may not be enough to justify the cost.

where t denotes time and i denotes firm i . The error term, $\nu_{t,i}$, is interpreted as all other exogenous shocks to firm value. To get unbiased estimates of the coefficients involving ϵ , we would require that any unobserved shocks to P are independent of ϵ .

To implement this regression, we need to consider the choice of actual data. Take for example, a gold-mining firm. In this case, we would interpret: P as the market value of the firm; w as the amount of contemporaneous cash flow; ϵ as the price of gold. One also might want to scale value and cash flow by the book value of assets, or some other indicator of size, in order to facilitate cross-firm comparisons.

Equation (31) says that the marginal value of internal funds, P_w , is given by $\alpha_1 + \alpha_2 \epsilon_t$. The cross term thus allows ϵ_t to have an effect on the *marginal* value of internal funds. As discussed above, optimal hedging should eliminate this effect. Thus, according to the model's first-order condition, the null hypothesis that the firm is hedging optimally is given by $\alpha_2 = 0$.

To understand the intuition behind the test, imagine that we estimated α_2 to be significantly negative. This would mean that firm value is more sensitive to cash flow in low ϵ states, or, said differently, that liquidity constraints are more costly when ϵ is low. In this case, the firm could be made better off by shorting the source of ϵ risk.

Note that the model does *not* predict that α_3 should be zero. This is the point we made earlier: firm value should generally *not* be completely insulated from ϵ .

One possible problem with using firm value as a dependent variable in a regression of this sort is that firm value may respond to cash flow for reasons outside our model. For example, even if there are no liquidity constraints, α_1 is likely to be positive simply because cash flow is serially correlated and the dependent variable is forward looking. This will not create a problem in the estimation of α_2 , however, unless the degree of serial correlation is a function of ϵ . For example, if current cash flows are a better predictor of future cash flows when ϵ is low, we will estimate a negative α_2 even when the firm is hedging optimally. Thus, a key identifying assumption of our methodology is that other exogenous variables which simultaneously drive w and P are independent of ϵ .

If this identifying assumption is not appropriate, a second-best alternative might be to use investment, rather than firm value, as the dependent variable and to add Tobin's Q as another explanatory variable. Here too, the test would involve checking to see whether α_2 is equal to zero. The benefit of such an approach is that it would be harder to argue here that a nonzero α_2 was spurious. The drawback, however, is that investment is not quite the right variable to be measuring. Such a specification implies that the impact of liquidity constraints on the *quantity* of investment should not vary with ϵ . In contrast, the theory implies that the impact of liquidity constraints on the *value* of investment should not vary with ϵ .

In fact, regressions very much like the latter set that we propose have been implemented in the literature. Gertler and Hubbard (1988), Hoshi,

Scharfstein, and Singleton (1993) and Kashyap, Lamont, and Stein (1993) all find that investment spending is more sensitive to liquidity during episodes of tight monetary policy, i.e., that liquidity constraints are more binding at these times. Subject to the above caveats, these regressions would seem to suggest that the firms in these samples could have benefitted by hedging more actively against the risk of tight monetary policy, say by using interest rate futures.

VIII. Conclusion

When external finance is more costly than internally generated sources of funds, it can make sense for firms to hedge. While this basic point seems to have already been recognized in the literature, its implications for optimal hedging strategy have not been fully developed. In this paper, we have argued that there is a rich set of such implications:

1. Optimal hedging strategy does not generally involve complete insulation of firm value from marketable sources of risk.
2. Firms will want to hedge less, the more closely correlated are their cash flows with future investment opportunities.
3. Firms will want to hedge more, the more closely correlated are their cash flows with collateral values (and hence with their ability to raise external finance).
4. In general, multinational firms' hedging strategies will depend on a number of additional considerations, including the exchange rate exposure of both investment expenditures and revenues. In some special cases, multinationals will want to hedge so as to "lock in" a fixed quantity of investment in each country in which they operate.
5. Nonlinear hedging instruments, such as options, will typically allow firms to coordinate investment and financing plans more precisely than linear instruments, such as futures and forwards.
6. In an intertemporal setting, there is a meaningful distinction between futures and forwards as hedging tools. In particular, the use of futures will involve a difficult tradeoff between insulating the present value of all cash flows versus insulating the level of cash at each point in time.
7. Optimal hedging strategy for a given firm will depend on both the nature of product market competition and on the hedging strategies adopted by its competitors.

Appendix

Derivation of Equation (24)

First, note that at the moment when the investments are made, ϵ has already been realized. It follows that the first-order condition of (22) with

respect to domestic investment is

$$f_I^H = \frac{\theta}{\gamma} f_I^A, \quad (\text{A1})$$

which says that the firm equalizes the marginal revenue product of an additional unit of domestic currency across investments. Second, note that the marginal return on domestic investment will always be set equal to the marginal cost of an additional unit (in domestic currency terms) of external finance,

$$f_I^H = C_e + 1. \quad (\text{A2})$$

Together these equations, along with the budget constraint in (23), tie down the optimal choices for domestic and foreign investment, *for given wealth* of w . By applying the implicit function theorem to them, one can determine the sensitivity of optimal investment plans to changes in ϵ , $(dI^H/d\epsilon)$ and $(dI^A/d\epsilon)$.

Moving back to the initial period when the hedging decision is made, equation (22) must be maximized with respect to h . The first-order condition for this problem is identical to that given in equations (17) through (19). Applying the formula for covariance given in footnote 18, equation (19) can be rewritten

$$E \left[C_{ee} \left(\frac{dI^H}{d\epsilon} + \gamma \frac{dI^A}{d\epsilon} + \beta I^A - (1-h)w_0 \right) \right] = 0. \quad (\text{A3})$$

Substituting in the expressions for $(dI^H/d\epsilon)$ and $(dI^A/d\epsilon)$ derived above and simplifying yields equation (24).

Proof of Proposition 1: We start by hypothesizing that I^H and I^A are nonstochastic, and $h = 1 - \beta \bar{I}^A/w_0$. We then verify that this is optimal, i.e., that the first-order conditions for both hedging (equation (24)) and investment (equations (A1) and (A2) above) are satisfied.

First, note that I^H and I^A constant and $h = 1 - \beta \bar{I}^A/w_0$ together imply, from the budget constraint in (23), that $(de/d\epsilon) = 0$ – external financing is independent of the exchange rate. This implies that C_e is independent of ϵ . But, given the first-order condition in (A2), this in turn implies that it is optimal for I^H to be independent of ϵ . Similarly, when $\alpha = \beta$, the first-order condition in (A1) reduces to $f_I^H = f_I^A$. So if it is optimal for I^H to be constant, then it is optimal for I^A to be constant also.

This establishes that a constant I^H and I^A are optimal, given

$$h = \frac{1 - \beta \bar{I}^A}{w_0}.$$

We now must check that this hypothesized hedge ratio is itself optimal. This now follows immediately from (24), once we note that $E[I^A P_{ww}]$ can be simplified to $\bar{I}^A \bar{P}_{ww}$ when I^A is nonstochastic.

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