Banking, Trade, and the Making of a Dominant Currency*

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Abstract

We explore the interplay between trade invoicing patterns and the pricing of safe assets in different currencies. Our theory highlights the following points: 1) a currency's role as a unit of account for invoicing decisions is complementary to its role as a safe store of value; 2) this complementarity can lead to the emergence of a single dominant currency in trade invoicing and global banking, even when multiple large candidate countries share similar economic fundamentals; 3) firms in emerging-market countries endogenously take on currency mismatches by borrowing in the dominant currency; 4) the expected return on dominant-currency safe assets is lower than that on similarly safe assets denominated in other currencies, thereby bestowing an "exorbitant privilege" on the dominant currency. The theory thus provides a unified explanation for why a dominant currency is so heavily used in both trade invoicing and in global finance.

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1 Introduction

The U.S. dollar is often described as a dominant global currency, much as the British pound sterling was in the 19th century and beginning of the 20th century. The notion of dominance in this context refers to a constellation of related facts, which can be summarized as follows:

- **Invoicing of International Trade:** An overwhelming fraction of international trade is invoiced and settled in dollars (Goldberg and Tille (2008), Gopinath (2015)). Importantly, the dollar's share in invoicing is far out of proportion to the U.S. economy's role as an exporter or importer of traded goods. For example, Gopinath (2015) notes that 60% of Turkey's imports are invoiced in dollars, while only 6% of its total imports come from the U.S. More generally, in a sample of 43 countries, Gopinath (2015) finds that the dollar's share as an invoicing currency for imported goods is approximately 4.7 times the share of U.S. goods in imports. This stands in sharp contrast to the euro, where in the same sample the euro invoicing share and the share of imports coming from countries using the euro are much closer to one another, so that the corresponding multiple is only 1.2.

- **Bank Funding:** Non-U.S. banks raise very large amounts of dollar-denominated deposits. Indeed, the dollar liabilities of non-U.S. banks, which are on the order of $10 trillion, are roughly comparable in magnitude to those of U.S. banks (Shin (2012), Ivashina, Scharfstein, and Stein (2015)). According to Bank for International Settlements (BIS) locational banking statistics, 62% of the foreign currency local liabilities of banks are denominated in dollars.

- **Corporate Borrowing:** Non-U.S. firms that borrow from banks and from the corporate bond market often do so by issuing dollar-denominated debt, more so than any other non-local "hard" currency, such as euros. According to the BIS locational banking statistics, 60% of foreign currency local claims of banks are denominated in dollars. BrÃ€uning and Ivashina (2017) document the dominance of dollar-denominated loans in the syndicated cross-border loan market. Importantly, this dollar borrowing is in many cases done by firms that do not have corresponding dollar revenues, so that these firms end up with a currency mismatch, and can be harmed by dollar appreciation (Aguiar (2005), Du and Schreger (2014), Kalemli-Ozcan, Kamil, and Villegas-Sanchez (2016)).

- **Central Bank Reserve Holdings:** The dollar is also the predominant reserve currency, accounting for 64% of worldwide official foreign exchange reserves. The euro is in second place at 20% and the yen is in third at 4% (ECB Staff (2017)).

- **Low Expected Returns and UIP Violation:** Gilmore and Hayashi (2011) and Hassan (2013), among others, document that U.S. dollar risk-free assets generally pay lower expected returns (net of exchange-rate movements) than the risk-free assets of most other currencies. That is, there is a violation of uncovered interest parity (UIP) that favors the dollar as a cheap funding currency. This violation is all the more striking–relative to other currencies that have historically also had low returns–in light of the fact that so much borrowing is done in dollars. Sometimes this phenomenon is referred to as the dollar benefiting from an "exorbitant privilege."[1]

The goal of this paper is to develop a model that can help to make sense of this multi-faceted notion of currency dominance. Our starting point is the connection between invoicing behavior and safe asset demand. Both of these topics have been the subject of much recent (and largely separate) work, but their joint implications have not been given as much attention.[2] Yet a fundamental observation is that in a multi-currency world, one cannot think about the structure of safe asset demands without taking into account invoicing patterns. Simply put, a financial claim is only meaningfully "safe" if it can be used to buy a known quantity of some specific goods at a future date, and this necessarily forces one to ask about how the goods will be priced.

Consider, for example, a representative household in an emerging market (EM). The household purchases some imported goods from abroad, both from the U.S. and from other emerging markets.[3] The household also holds a buffer stock of bank deposits that it can use to make these purchases over the next several periods. In what currency would it prefer to hold its deposits? If most of its imports are priced in dollars—and crucially, if these dollar prices are sticky—the household will tend to prefer deposits denominated in dollars, as these are effectively the safest claim in real terms from its perspective. In other words, while deposits in any currency may be free of default risk, in a world in which exchange rates are variable, only a dollar deposit held today can be used to purchase a certain quantity of dollar-invoiced goods tomorrow. Deposits in different currencies are therefore imperfect substitutes.

It follows that when more internationally-traded goods are invoiced in dollars, there will be a greater demand for dollar deposits—or more generally, for financial claims that pay off a guar-

[1]Our notion of "exorbitant privilege" is the empirical phenomenon whereby dollar safe assets pay a lower interest rate as compared to most other safe assets in the world. Gourinchas and Rey (2007) define the concept more broadly, to also capture the fact that the U.S. earns a higher return on its assets compared to what it pays on its liabilities.

[2]On the choice of invoicing currency, contributions include Friberg (1998), Engel (2006), Gopinath, Itskhoki, and Rigobon (2010), andGoldberg and Tille (2013). On safe asset determination in an international context, some recent papers are Hassan (2013), Gourinchas and Rey (2010), Maggiori (2017), He, Krishnamurthy, and Milbradt (2016) and Farhi and Maggiori (2018). We discuss these works in more detail below.

[3]This "representative household" label could also refer to firms that purchase imported inputs for production purposes.

anteed amount in dollar terms. Some of these may be provided by the U.S. government, in the form of Treasury securities, but to the extent that Treasury supply is inadequate to satiate global demand, private financial intermediaries will also have an important role to play. Specifically, banks operating in other countries will naturally seek to provide safe dollar claims to their customers who want them. However, in so doing, they must satisfy a collateral constraint: a bank that promises to repay a depositor one dollar tomorrow must have assets sufficient to back that promise. This collateral in turn, must ultimately come from the revenues on the projects that the bank lends to. And importantly, not all of these projects need be ones that produce revenues that are dollar-based. For example, a bank in an EM that is trying to accommodate a large demand for dollar deposits may seek to back these deposits by turning around and making a dollar-denominated loan to a local firm that produces non-tradeable, local-currency-denominated goods. Of course, this firm's revenues do not make particularly good collateral for dollar claims, because of exchange-rate risk: it would be more efficient to use the firm's revenues to back local-currency deposits, all else equal.

This inefficiency in collateral creation is at the heart of our results. If global demand for dollar deposits is strong enough, equilibrium inevitably involves having even those operating firms that generate revenues in other currencies serving as a marginal source of collateral for dollar deposits. Since these firms effectively have an inferior technology for producing dollar collateral relative to own-currency collateral, they can only be drawn into doing the latter if they are paid a premium for doing so, that is, if it is cheaper for them to borrow in dollars than in their home currency. The intuition is of walking up a supply curve: as worldwide demand for safe dollar claims expands, we exhaust the supply that can be provided by low-cost producers (the U.S. Treasury, and firms that naturally have dollar-denominated revenues) and therefore must turn to less efficient, higher cost producers, namely firms that have to take on currency risk in order to create the collateral that backs dollar claims. As a result, the safety premium on dollar claims deposits exceeds that on local-currency deposits. Or said differently, the expected return on dollar deposits is on average lower, in violation of uncovered interest parity (UIP). This is the exorbitant privilege associated with the dollar.

Note that this line of argument turns on its head much informal reasoning about why foreign firms borrow in dollars. In particular, if one takes the UIP violation as exogenous, it seems obvious why some firms might be willing to court exchange-rate risk by borrowing in dollars—it can be worth it to do so simply because dollar borrowing is on average cheaper. But this leaves open the question of where the UIP violation comes from in the first place. Our explanation is that dollar borrowing has to be cheaper because the worldwide demand for safe dollar claims is so large that even those firms that are not particularly well-suited to it must be recruited to help provide collateral for such claims; again, the intuition here is of walking up the supply curve. This recruiting can only happen in equilibrium if it is cheaper to borrow in dollars than in local currency. Thus the primitive in our story is the share of internationally-traded goods invoiced in dollars, which in turn drives the demand for safe dollar claims; the specific nature of the UIP violation, wherein the expected return on dollar deposits is on average lower, then emerges endogenously as the equilibrium "price" required to bring supply into line with demand.

Of course, this line of reasoning begs the question of where the dollar invoicing share comes from: what determines whether EM firms selling goods internationally price them in dollars, as opposed to their own currency or another potential dominant currency like the euro? Although a variety of factors likely come into play, we argue that there is an important feedback loop from UIP violations back to invoicing choices. Suppose for the moment that for an EM exporting firm dollar borrowing is cheaper in equilibrium than borrowing in either its own currency or in euros. All else equal, the EM firm then has an incentive to choose to invoice its exports in dollars, because doing so gives it more certainty about its next-period dollar revenues, which in turn allows it to safely borrow more in dollars, i.e., in the cheaper currency.

This then generates a link back to invoicing shares, safe asset demand and the UIP violation. To see this, consider two emerging markets i and j. An initially high dollar invoice share facing importer households in i leads to an increased demand on their part for safe dollar claims, which in turn drives down dollar borrowing costs. Responding to this financing advantage, exporting firms in j are induced to invoice more of their sales to importers in country i in dollars. So the dollar invoice share facing country-i importers goes up further. This same mechanism also increases the incentive for exporters in country i to price in dollars when selling to country j. In other words, a high dollar invoice share in country i tends to push up the dollar invoice share in country j, and vice-versa, through a safe asset demand-and-supply mechanism. As we show, this form of strategic complementarity can give rise to asymmetric equilibria in which a single currency becomes disproportionately dominant in both global trade and banking, even when two large candidate countries share similar economic fundamentals.

The model that we develop below formalizes this line of argument. For example, in a case where the U.S. and Europe are otherwise identical in all respects, we obtain asymmetric equilibrium outcomes where the majority of trade invoicing is done in dollars, and where most non-local-currency deposit-taking and lending by banks in other EM countries is dollar-denominated, rather than euro-denominated.

Finally, in such an asymmetric equilibrium, it seems natural to expect that the foreign-currency reserve holdings of a typical EM central bank would skew heavily towards dollars, as opposed to euros. Although we do not model this last link in the chain formally here, we do so in a companion paper (Gopinath and Stein (2018)). The logic is straightforward: given that an important role for the central bank is to act as a lender of last resort to its commercial banking system, the fact that the commercial banks' hard-currency deposits are primarily in dollars means that the central bank will want to have stockpile of dollars so as to be able to replace any sudden loss of bank funding that occurs during a liquidity crisis. Thus the central bank's asset mix is to some extent a mirror of the commercial banks' liability structure, and both are ultimately shaped by—and feed back on—the invoicing decisions made by exporters in other countries. This argument is consistent with the evidence in Obstfeld, Shambaugh, and Taylor (2010) who argue that the dramatic accumulation of reserves by central banks in emerging markets is driven in part by considerations of maintaining domestic financial stability.[4]

Our analysis is very much in the spirit of the historical narrative of Eichengreen (2010), which he summarizes by writing "...experience suggests that the logical sequencing of steps in internationalizing a currency is: first, encouraging its use in invoicing and settling trade; second, encouraging its use in private financial transactions; third encouraging its use by central banks and governments as a form in which to hold private reserves." As we discuss below, this logic may be helpful in thinking about the evolution of events in the early part of the 20th century, when the dollar first displaced the pound sterling as a dominant global currency. And it may also shed light on the strategy currently being undertaken by the Chinese government in their efforts to internationalize the renminbi, in particular, the fact that they are focusing at this early stage on creating incentives for the use of the renminbi in international trade transactions. The model also offers a lens through which to examine the creation of private currencies such as Facebook's Libra. Initially such a currency may be meant primarily for transactional purposes, but one can see how with sufficient scale this can lead to a demand for safe assets in this currency, which in turn triggers the production of further safe assets in the currency via borrowing, as well as other goods being invoiced in this currency.

Although our contribution is primarily theoretical, we also present some preliminary evidence which is consistent with our basic premise, namely that there is a close connection between the dollar's prominence in a country's import invoicing, and its role in that country's banking system. Specifically, we find a strong correlation at the country level between the dollar's share (relative to other non-local currencies) in the invoicing of its imports and the dollar's share (again relative

[4]Bocola and Lorenzoni (2017) also analyze central-bank reserve holdings from the perspective of a lender of last resort.

to other non-local currencies) in the liabilities of the domestic banking sector.

Related literature: This paper aims to connect two strands of research: one on trade invoicing, and the other on safe-asset determination in an international context. The former emphasizes the role of a dominant currency as a unit of account, while the latter focuses on its role as a store of value.[5] Our contribution is to highlight the strategic complementarity between these two roles, i.e., to show how they mutually reinforce each other. The only other work we are aware of that ties together trade invoicing and finance is contemporaneous work by Chahrour and Valchev (2017), who focus on the medium of exchange role of currencies. Farhi and Maggiori (2018) explore the unidirectional impact of trade invoicing on safe asset determination.

We also provide a novel perspective on both trade invoicing and safe-asset determination. The literature on trade invoicing sets aside financing considerations and instead focuses on factors that influence the optimal degree of exchange rate pass-through into prices, as in the contributions of Friberg (1998), Engel (2006), Gopinath, Itskhoki, and Rigobon (2010), Goldberg and Tille (2013). Doepke and Schneider (2017) rationalize the role of a dominant unit of account in payment contracts by the desire to avoid exchange rate risk and default risk. By contrast, we provide a complementary explanation that relates exporters' pricing decisions to their financing choices, and in particular to their desire to borrow in a cheap currency. In our model the only reason exporters choose to invoice in dollars is because by doing so they are able to more cheaply finance their projects.

On the safe asset role of the dollar and the lower expected return on the dollar relative to other currency assets, existing explanations are tied to the superior insurance properties of U.S. bonds that arise either from country size (Hassan (2013)); from the tendency of the dollar to appreciate in a crisis (Gourinchas and Rey (2010), Maggiori (2017)); from better fiscal fundamentals and liquidity of debt markets (He, Krishnamurthy, and Milbradt (2016)); or from the monopoly power of the U.S. as a safe asset provider (Farhi and Maggiori (2018)). We offer a distinct explanation that is tied to the invoicing role of the dollar in international trade. In our model, it is this invoicing behavior that generates the demand for dollar safe assets and importantly, that implies that the marginal supplier of dollar claims must have a mismatch of its assets and liabilities in equilibrium.

Outline of the paper: The full model that we consider below has two large countries, the U.S. and the Euro area, a continuum of emerging market economies, and endogenous invoicing and financing decisions. To provide a clear exposition of the mechanism we build up to the full model

[5] Matsuyama, Kiyotaki, and Matsui (1993), Rey (2001) and Devereux and Shi (2013) study the medium of exchange role of currencies and the emergence of a 'vehicle' currency. While we focus on the unit of account role of the currency, adding a medium of exchange role only strengthens our conclusions.

in steps. Section 2 starts with a simple case in which there is just the U.S. and one emerging market (EM), and in which invoice shares facing importers in the EM are exogenously specified. Here we highlight the fundamental source of the UIP violation. Section 3 endogenizes the invoicing decision of exporter firms in the EM and explains the financial incentive for invoicing in dollars. Section 4 brings in the continuum of EMs and demonstrates the strategic complementarity between their invoicing decisions and the safe asset demand that gives rise to multiple equilibria. Finally in Section 5 we add the euro as another candidate global currency and show that in spite of the symmetry in fundamentals, for some parameter values the equilibrium outcomes are asymmetric, with only one global currency being used extensively by emerging-market countries to invoice their exports and to finance their projects. Section 6 discusses several further implications of the model, and Section 7 concludes. All proofs not in the text can be found in the appendix.

2 Exogenous Import Invoice Shares and the UIP Violation

In the simplest version of the model, the world is comprised of just the U.S. and one emerging market. All of the focus is on decisions made by EM agents. The U.S. only plays two simple roles. First, an exogenous quantity M of goods are imported by EM households from the US and these are priced in dollars. And second, the U.S. supplies an exogenous net quantity $X_{\$}$ of safe dollar claims that are available to these same EM households. These safe claims could be, e.g. Treasury securities, or deposits in U.S.-based financial intermediaries such as banks or money-market funds.[6]

There are three groups of agents in the EM and there are two dates, denoted 0 and 1. There are also three types of assets: (i) risk-free home-currency deposits, D_h, (ii) risk-free dollar deposits, $D_{\$}$, and (iii) risky home-currency assets, A_R; that is assets with risky nominal payoffs such as bonds with credit risk, or equities. The first group of agents are risk neutral investors who can potentially invest in all three assets. As will become clear momentarily, their only role is to pin down the expected return on risky assets A_R. The second group are risk-averse importers, and they can only save in the two risk-free assets D_h and $D_{\$}$. This assumption captures a form of market segmentation, whereby importers are not informed enough to evaluate the underlying operations of the banks and firms in the economy that issue risky claims, and hence behave in an infinitely risk-averse fashion with respect to these risks. The importers also have finite risk

[6]To put a little more flesh on this assumption: imagine that U.S. households and firms have an inelastic demand for up to $Z_{\$}$ units of safe assets, and no more, and that the Treasury has issued $Y_{\$}$ units of safe Treasury securities. Then $X_{\$} = Y_{\$} - Z_{\$}$, and the empirically-relevant case for us to consider is when $X_{\$} \geqq 0$. The assumption of a perfectly inelastic supply of $X_{\$}$ is made for tractability and not essential to the analysis.

aversion over consumption risk, which, as we show, leads them to prefer a portfolio mix that is more tilted towards dollar deposits when they consume more dollar-invoiced goods at time 1. The third group, whom we call "banks", can be thought of as an agglomeration of the local banking sector with those firms—and by extension, those real projects—that the banks lend to. We describe each group in detail next. As a simplification, we assume all goods prices are normalized to one, and are assumed to be sticky over time in whatever currency they are set—so that those imports that are priced in dollars have sticky dollar prices.

2.1 Risk-Neutral Investors

Risk-neutral investors save at time 0, and consume only goods produced at home at both time 0 and time 1, in amounts given by C_0^n and C_1^n respectively. Their utility is therefore given by $C_0^n + \beta \mathbb{E}_0 C_1^n$, where β is a discount factor. If we define Q_R as the time-0 price of a risky asset that provides a payoff with an expected value of one at time 1—i.e., Q_R is the reciprocal of one plus the risky discount rate—then it follows immediately that $Q_R = \beta$. In principle, risk-neutral investors also have access to the safe local-currency and dollar-denominated claims, but given their risk-neutrality, they will choose not to invest in these safe claims if—as we establish below—they have a lower equilibrium rate of return than the risky asset. We also assume that these investors cannot short-sell the safe claims, presumably because they do not have the collateral that would be required to guarantee payment with absolute certainty. Hence the determination of the rate of return on safe claims will depend only on the preferences of the risk-averse importers.

2.2 Risk-Averse Importers

Importers consume quantities of home goods C_0 and C_1 at times 0 and 1 respectively, and of imported goods M at time 1 only. A key simplifying assumption is that importers have a completely inelastic demand for the bundle of imported goods, implying that M is a fixed constant, and consequently exchange-rate variation ends up impacting only the value of C_1.[7] Their initial endowment in home currency is given by W, and all imported-goods are priced in dollars. The time-t exchange rate is $\mathcal{E}_t$, and we adopt the normalization that $\mathbb{E}_0(\mathcal{E}_1) = \mathcal{E}_0 = 1$. We denote by Q_h the time-0 price of a deposit that pays off a certain one unit in the home currency at time 1. Similarly, $Q_\$$ is the time-0 price of a deposit that pays off a certain one unit in dollars at time 1. Finally, importers are risk-averse over their time-1 consumption of home goods C_1, and have a strictly greater discount factor than the risk-neutral investors, given by $\delta = \beta + \theta$.

[7] $M < \frac{W}{Q_\$}$ for feasibility.

Taken together, these assumptions allow us to write the importers' problem as:

$$\max_{D_h, D_\$} C_0 + \delta \mathbb{E}_0 U(C_1),$$

subject to:

$$C_0 = W - Q_h D_h - Q_\$ D_\$$$

$$C_1 = D_h + \mathcal{E}_1 D_\$ - \mathcal{E}_1 M,$$

$$C_0, C_1, D_h, D_\$ \geq 0$$

For concreteness, we assume a mean-variance utility function for $U(C_1)$ such that $\mathbb{E}_0 U(C_1) = \mathbb{E}_0 C_1 - \frac{\psi}{2} \mathbb{V}_0(C_1)$, where $\mathbb{V}$ stands for variance. It is easy to see that:

$$\mathbb{E}_0(C_1) = D_h + D_\$ - M$$

$$\mathbb{V}_0(C_1) = (D_\$ - M)^2 \sigma^2$$

where $\sigma^2 = \mathbb{V}_0[\mathcal{E}_1]$ is the variance of the exchange rate. Intuitively, importers face risk in their time-1 consumption of home goods to the extent that $D_\$ \neq M$, i.e., to the extent that their dollar deposit holdings do not match their time-1 consumption of imports that are dollar invoiced. Thus all else equal, there will be a greater hedging demand for dollar deposits when the quantity of dollar invoiced goods consumed, M, is larger. To see this explicitly, note that the first-order conditions for the importers' problem imply the following in an interior optimum where D_h and $D_\$$ are both positive:

$$D_\$ = M - \frac{1}{\psi \delta \sigma^2} (Q_\$ - Q_h)$$

$$Q_h = \delta$$

Thus the demand for dollar deposits increases with M, but decreases when $(Q_\$ - Q_h)$ goes up, i.e., when the interest rate on dollar deposits falls relative to that on home-currrency deposits. To close the model and solve for $Q_\$$, we therefore need to bring in the supply side, and pin down the equilibrium quantity of dollar safe claims provided by the banking sector. This is what we turn to next.

2.3 Banks

We model the representative EM bank as an entity that is endowed with N projects that collectively pay a risky return of γN in home currency at date 1, where γ is a random variable. Each project requires a unit of home currency investment at time 0 that the bank finances through borrowing with one of three types of liabilities: safe home currency claims B_h; safe dollar-denominated claims $B_\$$; and risky home currency bonds B_R. The bank is a price-taker in each of the three markets. Importantly, because the bank's projects are risky, there is an upper bound on how much it can promise in terms of safe claims. In other words, it faces a collateral constraint on its production of B_h and $B_\$$. Specifically, define γ_L to be the worst realization of the productivity shock γ, and $\bar{\mathcal{E}} > 1$ to be the most depreciated value of the home currency (recall that $\mathbb{E}_0(\mathcal{E}_1) = \mathcal{E}_0 = 1$).[8] Then the maximum quantities of safe claims B_h and $B_\$$ that the bank can issue are constrained by the condition: $\bar{\mathcal{E}} B_\$ + B_h \leq \gamma_L N$.

A central piece of intuition that emerges from this collateral constraint is that the bank has a comparative advantage in manufacturing home currency safe claims relative to dollar-denominated safe claims. This is because the bank's underlying collateral is a collection of projects that pay off in home currency. Given the risk of currency depreciation, an amount of home collateral that is sufficient to back one unit of safe home-currency claims is only enough to back $1/\bar{\mathcal{E}}$ units of safe dollar claims.

The bank's problem is therefore:

$$\max_{B_h, B_\$, B_R} \mathbb{E}_0 \left[\gamma N - B_h - \mathcal{E}_1 B_\$ - B_R \right]$$

subject to,

$$Q_h B_h + Q_\$ B_\$ + Q_R B_R \geq N \tag{1}$$

$$\bar{\mathcal{E}} B_\$ + B_h \leq \gamma_L N \tag{2}$$

Define λ and μ to be the Lagrange multipliers on the financing constraint eq. (1) and the collateral

[8] To be clear, $\bar{\mathcal{E}}$ is in units of home currency/dollar, so a higher value indicates a weaker home currency.

constraint eq. (2), respectively. The first-order conditions for the problem imply:

$$\begin{aligned} B_\$: \qquad & Q_\$ = \frac{\mu\bar{\mathcal{E}} + 1}{\lambda} \\ B_h : \qquad & Q_h = \frac{\mu + 1}{\lambda} \\ B_R : \qquad & Q_R = \frac{1}{\lambda} \end{aligned}$$

These conditions yield the following proposition.

Proposition 1 ***[Exorbitant Privilege]*** *In an interior equilibrium in which the bank issues all three forms of debt, we have that $Q_\$ > Q_h > Q_R$.*

$$\frac{Q_\$ - \beta}{Q_h - \beta} = \bar{\mathcal{E}}$$

In other words, in an interior equilibrium, UIP is violated, and dollar deposits benefit from an "exorbitant privilege" relative to local-currency deposits: they have a higher price and a lower expected return.

The proposition is a direct consequence of the bank's comparative disadvantage in creating dollar safe claims out of home-currency-denominated collateral. Because of this disadvantage, the bank will only be willing to fund these home projects with dollar borrowing if doing so is cheaper than funding with home currency deposits. However, it still remains to check, as we do just below, whether the bank does in fact fund its home-currency projects with dollar claims in equilibrium. Intuitively, it will do so only if the home demand for dollars is large relative to the exogenous supply of safe dollar claims $X_\$$ that are available from abroad.[9]

[9]One striking feature of the proposition is that the magnitude of the UIP violation is driven entirely by the worst-case realization of the domestic exchange rate, as captured by $\bar{\mathcal{E}}$; nothing else about the distribution of exchange-rate outcomes appears to matter. But this somewhat unnatural feature is easily tweaked. Consider a case where the exchange rate follows a known distribution, such as the normal. Suppose further that our collateral constraint implicitly reflects a form of capital regulation. In particular, the regulator allows the bank to issue short-term deposits (as opposed to equity) only up to the point where its risk of failure is some fixed probability, e.g. 0.1%, with the understanding that there will be a government bailout in the event of such a failure, so that deposits are alway riskless to households, just as they are in the model here. In this case, the relevant item will no longer be the worst-case realization of the exchange rate, but rather whatever moment of the distribution (e.g., the variance, in the normal case) it is that governs this tail probability. So if one wants to interpret $\bar{\mathcal{E}}$ more generally as a proxy for the variability of the exchange rate, rather than literally its worst-case value, this interpretation can be comfortably rationalized in a variant of our setting.

2.4 Market Clearing Conditions

In order to solve for the equilibrium of the model, we note that total safe dollar claims available to EM importers are the sum of those produced by the bank borrowing against home-currency collateral, and those exogenously supplied from abroad: $D_\$ = B_\$ + X_\$$. At the same time, safe home-currency claims can only be collateralized by home projects, meaning that $D_h = B_h$. The safe asset constraint always binds, which implies that $\bar{\mathcal{E}}(D_\$ - X_\$) + D_h = \gamma_L N$.[10]

The next proposition shows that, if the dollar invoice share is large enough relative to the supply $X_\$$ of safe dollar claims available from abroad, the bank will necessarily get drawn into the business of manufacturing dollar deposits backed by home-currency projects, which in turn requires the rate of return on these dollar deposits to be lower than that on home-currency deposits.

Proposition 2 *[**Import Invoice Shares and Exorbitant Privilege**] Define the cutoff* $\bar{M} = X_\$ + \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2}$*. The full solution to the model where the dollar invoice share* $\alpha_\$$ *is exogenously specified is given as follows (recall that* $\theta = \delta - \beta$*):*

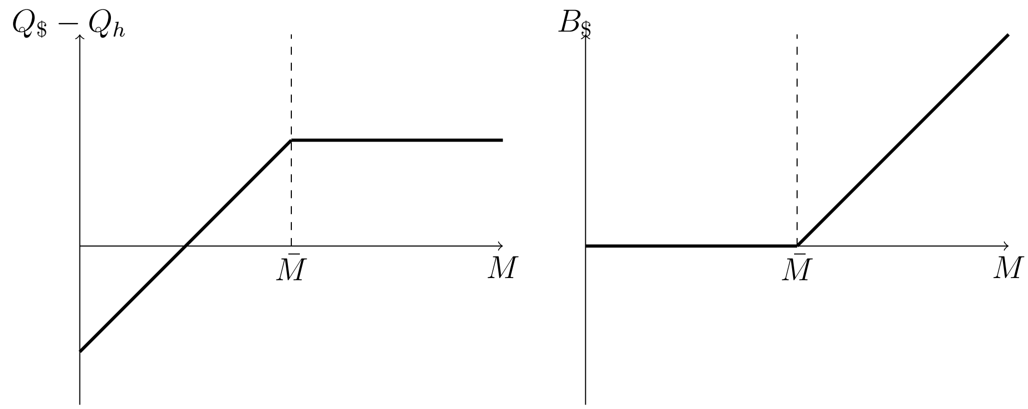
$$D_\$ = \begin{cases} X_\$ & \text{if } M < \bar{M} \\ M - \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2} & \text{if } M \geq \bar{M} \end{cases}$$

$$Q_\$ - Q_h = \begin{cases} (M - X_\$)\psi\delta\sigma^2 & \text{if } M < \bar{M} \\ \theta(\bar{\mathcal{E}} - 1) & \text{if } M \geq \bar{M} \end{cases}$$

$B_\$ = D_\$ - X_\$$ *and* $B_h = \gamma_L N - \bar{\mathcal{E}} B_\$$

Figure 1 illustrates, plotting the magnitude of the UIP deviation $(Q_\$ - Q_h)$ (in panel (a)) and the quantity of dollar funding by the banking system $B_\$$ (in panel (b)) versus the dollar invoice share in imports $\alpha_\$$. Note that $(Q_\$ - Q_h)$ has to become significantly positive—in particular, it has to reach a value of $\theta(\bar{\mathcal{E}} - 1)$— before the banks start using home-currency collateral to back dollar claims. This is because the cost of doing even the first unit of this kind of currency conversion is discretely positive, and is proportional to $(\bar{\mathcal{E}} - 1)$, which is effectively a proxy for the variability of the exchange rate.

[10]Given that importers discount the future less as compared to risk neutral investors, the interest rate on safe assets is always strictly below that of risky assets. Consequently, the bank/firm will always borrow first by issuing safe assets and only when this channel is exhausted will they switch to risky assets. Lastly, because the safe asset



(a) $Q_\$ - Q_h$ (b) $B_\$$

Figure 1: Invoicing Shares, Deposit Rates, and Dollar Borrowing

Proposition 2 and Figure 1 highlight our first key point: that in equilibrium, there is a fundamental link between the dollar's role as a global invoicing currency, and the low return on safe dollar claims, i.e., the exorbitant privilege. To the extent that the dollar enjoys a large invoicing share, this increases the demand on the part of importers for safe dollar deposits. Equilibrium then requires these claims to have a higher price, or equivalently, to offer a lower rate of return. This is true because when the demand is high enough, the marginal supply of safe dollar claims must be produced with a relatively inefficient technology—that is, it must be backed by the collateral coming from non-dollar-denominated projects.

Remark 1 *Exogenous Exchange Rates?*

We are taking exchange rates as exogenous, and also assuming that there is no expected appreciation or depreciation between time 0 and time 1. This is not important for our key conclusions. The logic of the model fundamentally pins down the net-of-exchange-rate expected returns on the different assets in the economy. With expected exchange-rate changes set to zero, this maps into own-currency rates of return; the analysis is therefore best thought of as suited to making on-average statements about the safe interest rate in different currencies. An alternative approach would be to add active monetary policy to the model, thereby allowing rates in each country to be displaced from their average values in response to aggregate demand shocks. In this case, there would still be the same violations of UIP described in Proposition 2. But now, if interest rates rose in the U.S. due to contractionary monetary policy, the dollar would have to be expected to weaken

constraint ensures that only $\gamma_L < 1$ can be raised for each unit of investment the bank/firm will necessarily exhaust this constraint and then turn to borrowing at the risky rate.

going forward so as to maintain the same relative expected return on dollar claims. This is how exchange rates might be endogenized in the richer version of the model. Note however, that we would still be making the same statements about on-average interest-rate differentials—i.e. rate differentials when monetary policy in both countries was at its neutral level.[11]

Remark 2 *Banks and Non-financial Firms*

The agents that we have been calling "EM banks" invest directly in real projects that yield returns in local-currency units. Thus they are more accurately thought of as an agglomeration of banks and the local non-financial firms that the banks lend to. To create a separation between these two types of entities, and a more well-defined account of the role of financial intermediation, assume that any individual non-financial firm can invest in a single project that pays a random amount γ/p if the project succeeds, which happens with probability p, and zero otherwise. This individual project-level success or failure draw is idiosyncratic, and uncorrelated across firms. Thus no single non-financial firm can issue any amount of safe claims, because there is always some chance that its project will yield zero. However, a bank that pools a large number N of these uncorrelated projects will be assured of a worst case payout of $\gamma_L N$, as we have been assuming.[12] Hence, as originally pointed out by Gorton and Pennacchi (1990), there is a specific pooling-and-tranching role for banks in creating safe claims.

However, this observation raises a further question of who bears the exchange rate risk. In the model, a bank that issues dollar deposits against its local-currency collateral bears some exchange-rate risk: if the dollar appreciates against the local currency, it will see its profits decline. But if the word "bank" is really a metaphor for the combined local banking and non-financial sectors, which of the two do we expect will actually wind up bearing the bulk of the currency risk? In other words, one possibility is that non-financial firms borrow from banks using local-currency debt, in which case the banks assume the currency mismatch. Alternatively, the non-financial firms could borrow using dollar-denominated debt, in which case they would be the ones bearing the currency risk, while the banks would be insulated. For the internal logic of the model, either interpretation works, since in either case the exchange-rate risk acts to limit the ultimate amount of safe dollar claims that can be produced from a given amount of local-currency collateral. As a matter of empirical reality, the existing evidence suggests that a significant amount of the exchange-rate risk is borne by the non-financial corporate sector in emerging

[11]Either version of our model is silent with respect to any higher frequency aspect of UIP violations such as the forward premium puzzle, according to which relative expected return to holding a given country's currency increases when the interest rate in that currency rises (Engel (2014)). Instead, we are interested in on-average cross-country rate of return differentials, of which we take the "exorbitant privilege" to be a leading example.

[12]This particular formulation follows Stein (2012).

markets (Galindo, Panizza, and Schiantarelli (2003), Du and Schreger (2014)). So when we develop propositions about the degree of exchange-rate mismatch in the "banking" sector in what follows, these propositions are best taken as statements that refer at least in part to mismatch among non-financial firms.[13]

3 Exporter Firms and Endogenous Invoicing

The next step is to allow exporter firms in the EM to choose how to invoice their sales to other countries, while temporarily maintaining the assumption that the invoice shares facing its importers are exogenously fixed. Bearing in mind the interpretation that the banks in the model are really agglomerations of banks and operating firms, we now assume that the EM banks have two types of projects. First, there are N_0 projects which, as before, necessarily produce home-currency revenues; these can be thought of as representing investments undertaken by firms that sell all of their output domestically. Second, there are N projects that can produce either dollar revenues or home-currency revenues. These latter projects are meant to capture the pricing decisions facing exporter firms in the EM: they have the choice of whether to invoice their sales in either dollars or their home currency. Moreover, if they do more of the former—and if prices are sticky—their dollar revenues will be more predictable, and hence will make better collateral for backing safe dollar claims.

We denote by η the fraction of the N export projects that are invoiced in dollars, with the remaining fraction $(1 - \eta)$ being invoiced in home currency. Our focus is on the financing motive behind trade invoicing. Accordingly, to shut down other motives—e.g., those having to do with imperfect goods-market competition—we assume the following: the sticky price of export goods is exogenous to the exporting firm, which acts as a price-taker. The export firm's invoicing decision thus amounts to only the choice between a given sticky home currency price or a dollar price.

To make the micro-foundations behind the bank/exporter's optimization problem more transparent, one can think of each export project as requiring a unit investment denominated in the home currency. Each project, if undertaken, produces γ units of a unique good and that can be sold either at the pre-fixed dollar price or the pre-fixed home price, each of which is normalized to one. The ultimate owners of the bank only consume local goods. So their consumption equals net profits denominated in home currency. If they invoice their exports in dollars, their revenue

[13] Niepmann and Schmidt-Eisenlohr (2017) provide evidence that distress caused by currency mismatch among non-financial firms spills over into credit risk for financial institutions.

in home currency is therefore $\gamma\mathcal{E}_1$ per project. Analogously, if they invoice their exports in home currency, their revenue in home currency is simply γ.

With these assumptions in place, the bank/exporter's profit-maximization problem can be stated as:

$$\max_{B_h, B_\$, B_R, \eta} \mathbb{E}_0 \left[\gamma N_0 + (1-\eta)\gamma N + \mathcal{E}_1 \eta \gamma N - B_h - \mathcal{E}_1 B_\$ - B_R\right]$$

subject to

$$Q_h B_h + Q_\$ B_\$ + Q_R B_R \geq N + N_0 \tag{3}$$

$$\bar{\mathcal{E}} B_\$ + B_h \leq \gamma_L N_0 + (1-\eta)\gamma_L N + \bar{\mathcal{E}} \eta \gamma_L N \tag{4}$$

$$B_h \leq \gamma_L N_0 + (1-\eta)\gamma_L N \tag{5}$$

There are a couple of points to note about this revised formulation. First, the collateral constraint (4) now reflects the fact that by invoicing in dollars, the bank-exporter coalition is able to increase the total quantity of safe dollar claims it can create. Again, this is because when it sets prices in dollars, and these prices are sticky, the lower bound on future dollar revenues is higher. Second, we have added an additional constraint in **??** which says that all local-currency safe claims must be backed by projects with local-currency revenues. This rules out a perverse outcome where exporters first bear a cost to invoice their projects in dollars, and then turn around and use these dollar revenues to back local-currency safe claims.[14]

Define λ, μ and κ to be the Lagrange multipliers on the three constraints in (3), (4) and (5) respectively. The first-order conditions for the bank's problem are given by:

$$B_\$: \qquad Q_\$ = \frac{\mu\bar{\mathcal{E}} + 1}{\lambda}$$

$$B_h : \qquad Q_h = \frac{\mu + 1 + \kappa}{\lambda}$$

$$B_R : \qquad Q_R = \frac{1}{\lambda}$$

[14]Such an outcome is endogenously ruled out as soon as one notes that the local currency can appreciate, as well as depreciate, against the dollar. For example, denoting the most appreciated value of the local currency by $\underline{\mathcal{E}} < 1$, one can never use a unit of dollar revenues to back more than $\underline{\mathcal{E}}$ units of local-currency safe claims. Incorporating this constraint explicitly into the optimization is formally identical to incorporating (5).

and η is now determined by the following condition

$$\eta = \begin{cases} 0 & \text{if} \quad Q_\$ - Q_h < 0 \\ \in [0,1] & \text{if} \quad Q_\$ - Q_h = 0 \\ 1 & \text{if} \quad Q_\$ - Q_h > 0 \end{cases} \tag{6}$$

Equation (6) captures the key wrinkle in this variant of the model: now when dollar financing is cheaper than home-currency financing, it must be that $\eta > 0$, i.e., there is dollar invoicing by EM exporters in equilibrium. Note that because of the linearity of the bank's problem—the bank is effectively a risk-neutral profit-maximizer in home currency—the solution for η is of a bang-bang nature: as soon as $(Q_\$ - Q_h)$ turns positive, η jumps from zero to one. The only way to maintain an interior value for η is in an equilibrium where $(Q_\$ - Q_h) = 0$, i.e., where dollar and local-currency interest rates are equalized.[15]

With this apparatus in hand, we can generalize Proposition 2. Now as M increases, we pass through four distinct regions of the parameter space, rather than just two. In the first, lowest-M region, $M < \underline{M}$, we have $(Q_\$ - Q_h) < 0$ and $B_\$ = 0$. That is, banks do not finance any of their projects with safe dollar claims, because the interest rate on dollar deposits is higher than that on local-currency deposits. In the second region, $\underline{M} < M < \hat{M}$ we have $(Q_\$ - Q_h) = 0$, $\eta > 0$, and $B_\$ = \eta\gamma_L N$. Here there is some amount of dollar invoicing by exporters, and dollar-invoiced projects are the only source of collateral that is used to back safe dollar claims—no dollar claims are backed by home currency projects. In the third region, $\hat{M} < M < \bar{M}$we have $(Q_\$ - Q_h) > 0$, $\eta = 1$, and again $B_\$ = \gamma_L N$. Now all invoicing by exporters is done in dollars, but this is still the only source of backing for safe dollar claims. Finally, in the highest-M region, $M > \bar{M}$, we have $(Q_\$ - Q_h) > 0$, $\eta = 1$, and $B_\$ > \gamma_L N$. Here, dollar deposits are backed *both* by dollar-invoiced projects, as well as by the remaining home-currency projects, as they were in the earlier setting. Or said differently, here the banks or the home-oriented firms they lend to take on some degree of currency mismatch, as they did in Proposition 2.

The full solution to this version of the model is characterized in Proposition 3, as follows:

Proposition 3 ***[Endogenous Invoicing]*** *Define the three cut-offs*

$$\underline{M} = X_\$, \qquad \hat{M} = X_\$ + \gamma_L N, \qquad \bar{M} = X_\$ + \gamma_L N + \frac{\theta(\bar{\mathcal{E}} - 1)}{\psi\delta\sigma^2}$$

[15] While this bang-bang type of solution for η emerges most naturally from a well-specified risk-neutral profit-maximization objective for the bank, it is not crucial for what follows. We have also derived qualitatively similar results in a version of the model which adds an ad hoc friction that makes η a more continuous function of $(Q_\$ - Q_h)$. See the NBER working paper version Gopinath and Stein (2018a) for this formulation of the model.

We can characterize the solution to the model as

$$\eta = \begin{cases} 0 & \textit{if } M < \underline{M} \\ \frac{M - X_\$}{\gamma_L N} & \textit{if } \underline{M} \leq M < \hat{M} \\ 1 & \textit{if } M \geq \hat{M} \end{cases}$$

$$D_\$ = \begin{cases} X_\$ & \textit{if } M < \underline{M} \\ M & \textit{if } \underline{M} \leq M < \hat{M} \\ X_\$ + \gamma_L N & \textit{if } \tilde{M} \leq M < \bar{M} \\ M - \frac{\theta(\bar{\mathcal{E}} - 1)}{\psi \delta \sigma^2} & \textit{if } M > \bar{M} \end{cases}$$

$$Q_\$ - Q_h = \begin{cases} (M - X_\$)\psi\delta\sigma^2 < 0 & \textit{if } M < \underline{M} \\ 0 & \textit{if } \underline{M} \leq M < \hat{M} \\ (M - X_\$ - \gamma_L N)\psi\delta\sigma^2 > 0 & \textit{if } \hat{M} \leq M < \bar{M} \\ \theta(\bar{\mathcal{E}} - 1) > 0 & \textit{if } M > \bar{M} \end{cases}$$

■

Note that as before, we have that $B_h = D_h$ and $B_\$ = D_\$ - X_\$$. Figure 3 illustrates Proposition 3, showing how the equilibrium values of the dollar export share η (in panel (a)), the dollar premium $(Q_\$ - Q_h)$ (in panel (b)) and dollar borrowing $B_\$$ (in panel (c)) all vary as the exogenous dollar import-invoice share $\alpha_\$$ increases.

This figure and the associated proposition summarize the second key message of the paper: we offer a novel argument for why EM firms choose to invoice their exports in dollars. The existing literature has no role for financing considerations and instead focuses on factors that influence the optimal degree of cost pass-through into prices, such as the contributions of Friberg (1998), Engel (2006), Gopinath, Itskhoki, and Rigobon (2010), Goldberg and Tille (2013). An alternate explanation, as developed in Rey (2001) and Devereux and Shi (2013), is that the dollar is used as a vehicle currency to minimize transaction costs of exchange.

By contrast, here we set aside all these factors and provide a complementary explanation that relates exporters' pricing decisions to their desire to borrow in a cheap currency. Indeed, in our model the *only* reason exporters choose to invoice in dollars is because by doing so they are able

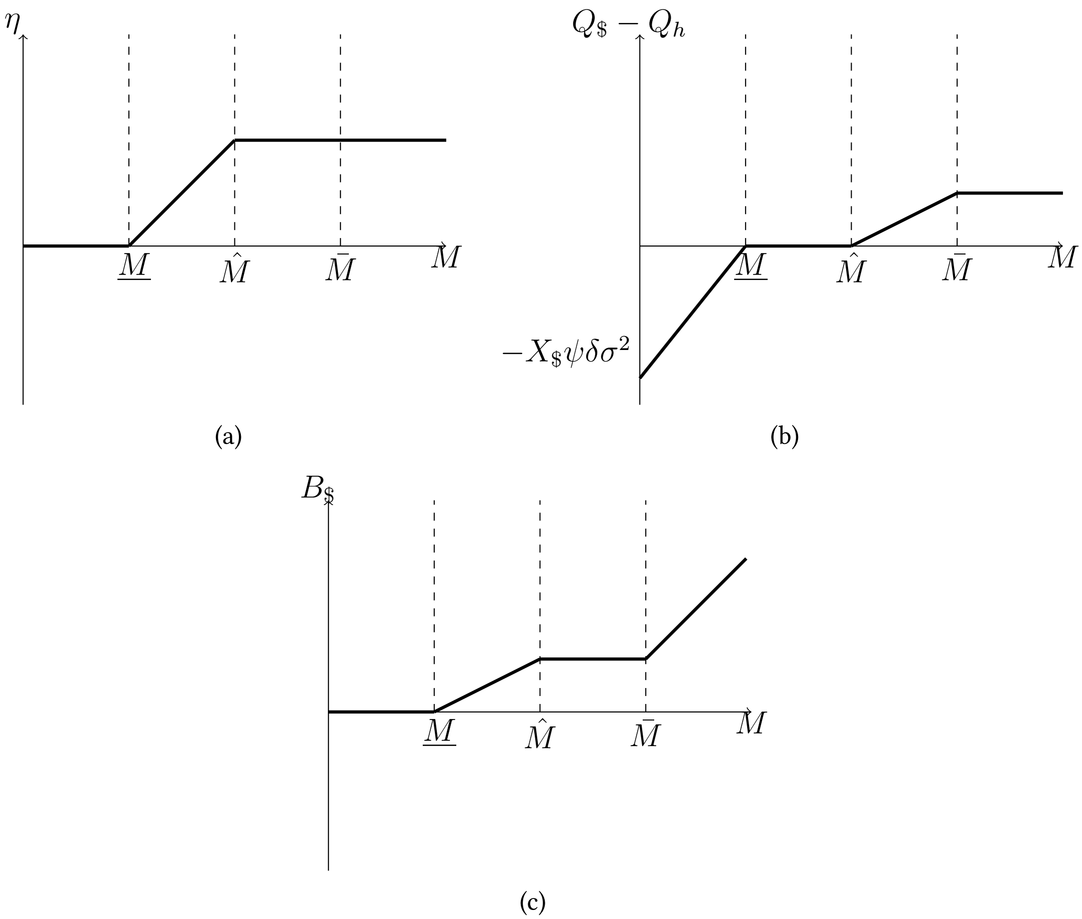


Figure 2: Equilibrium Values As Dollar Invoice Share Varies

to more cheaply finance their projects.[16]

Remark 3 *Why is the Export-Pricing Decision Relevant if Exporters Can Hedge?*

At first glance, one might think that there is no need for an exporter firm that wants to insulate its dollar revenues to invoice its sales in dollars; it could instead invoice in home currency and then overlay a foreign exchange swap to convert the proceeds from the sale into dollars. Or said a bit differently, invoicing in dollars bundles together a goods-pricing decision with a risk-management decision, and in principle these two decisions could be unbundled, in which case the model's predictions for invoicing behavior would be less clear cut.

A recent theoretical and empirical literature (Rampini and Viswanathan (2010), Rampini, Viswanathan, and Vuillemey (2017), among others) has argued that, due to financial contract-

[16]Baskaya, di Giovanni, Kalemli-Ozcan, and Ulu (2017) use micro data for Turkish firms and banks to to show that there is indeed a failure of UIP and bank loans denominated in dollars are cheaper than those in Turkish lira.

ing frictions, hedging of this sort by both operating firms and financial intermediaries tends to be quite constrained. The broad idea of this work is that when a firm wishes to enter (say) a forward contract to hedge its FX risk, it needs to post adequate collateral to ensure that it will be able to perform should the hedge move against it. In a world of financial frictions, posting such collateral is necessarily expensive, as it draws resources away from real investment activities.

To see why such frictions can make invoicing in dollars preferred to FX hedging in our setting, consider the following example. An exporter in Mexico plans to offer machines for sale in Brazil. It can either price these machines in Mexican pesos, and then enter into a forward contract with a derivatives dealer to convert the pesos into dollars; or it can price the machines in dollars. In the former case, it needs to be able to assure the derivatives dealer that the sale of the machines will actually happen and will generate the stipulated revenues, and that these revenues will not be diverted by the exporter before the dealer can get its hands on them. If this is difficult or expensive to do, the exporter will be required to post a significant amount of collateral in order to enter the hedging transaction. Moreover, if it is already liquidity-constrained, this posting of collateral will in turn compromise its ability to do real investment. In contrast, if the exporter invoices in dollars, these problems of assuring performance disappear. Effectively, by bundling the two decisions, it sources its hedge from somebody (the Brazilian importer) who is already fully protected from default on the part of the exporter, because the importer does not have to turn over any cash until it receives its machines, and is not promised anything other than the machines in any state of the world. Compare this with the derivatives dealer who makes a payment in one state (when the dollar depreciates against the peso) in the hopes of receiving a potentially default-prone payment in another state (when the dollar appreciates against the peso).

4 Endogenous Invoice Shares and Multiple Equilibria

In the previous section we endogenized the invoicing choices of exporter firms but did not link these decisions to the magnitude of dollar-invoiced imports that importers have to pay for, which in turn determines their preferences for dollar safe assets. In this section we close the loop. To do so we extend the model to include many emerging markets that trade with each other. Specifically, we now consider a world comprised of one large economy—namely the U.S.—and a continuum of small open economies (EMs) of measure one. The EM we described in the previous section is one of this continuum and therefore of measure zero. This extension of course introduces multiple exchange rates. To keep the analysis tractable we assume that households in each EM demand safe assets only in their own home currency and in dollars. The idea is that home-currency consumption and dollar-invoiced imports are always a non-negligible fraction of expenditures in each EM country; the latter because the U.S. is discretely large. By contrast, imports from any single other EM are only an infinitesimal share of the expenditure bundle. Therefore, if we think of there being a small fixed cost of setting up a deposit account in each currency, citizens of country i will only want to do so in dollars and in country-i currency, rather than having to set up an infinite number of such accounts to cover all the currencies of the world. Moreover, if we assume that exchange rates across the individual EM countries are uncorrelated, the law of large numbers implies that the failure to hedge non-dollar currency risk in this way has no effect on the aggregate cost of the overall import bundle, and hence imposes no undesirable variation in time-1 local currency consumption C_1.

Exporters in each of the EMs can choose to invoice their exports in either their own currency or in dollars.[17] With this, the budget constraint in period 1 is modified to,

$$C_{1i} = D_{hi} + \mathcal{E}_{1i} D_{\$i} - \mathcal{E}_{1i} M^U - \mathcal{E}_{1i} \int_{j \neq i} \eta_j M^{EM} dj - \int_{j \neq i} \mathcal{E}_{ij} (1 - \eta_j) M^{EM} dj$$

where η_j is the fraction of the N projects in country j that are priced to generate dollar revenues, as chosen by exporters in country j, M^U are total imports coming from the U.S., and M^{EM} are imports coming from EM countries and $\mathcal{E}_{ij}$ is the bilateral exchange rate between currency of i and of j expressed as the price of currency j in terms of currency i. The period 0 budget constraint is unchanged. Consistent with our description above, we assume $\int_j \mathcal{E}_{ij} dj = 1$, that is while bilateral EM exchange rates can move around, the average of currency i relative to all other EMs stays constant at 1. As before, all imports from the US, M^U are assumed to be invoiced in dollars. We can define the dollar invoice share of imports for emerging market i, as:

$$\alpha_{\$i} = \frac{M^U}{M} + \frac{M^{EM}}{M} \int_{j \neq i} \eta_j dj \equiv a + b \int_{j \neq i} \eta_j dj$$

where $M = M^U + M^{EM}$. Simply put, if exporters in the rest of the EM world price more of their exports in dollars, importers in i who import from these countries have a higher share of dollar-invoiced goods in their own expenditures.

The key exogenous parameters in the model are now $a \equiv \frac{M^U}{M}$ and $b \equiv \frac{M^{EM}}{M}$. These are the U.S. and EM shares in total (inelastic) import demand, respectively. Our assumption is that U.S. exporters always price in dollars, no matter what, so that if the U.S. share a is relatively large, $\alpha_{\$i}$ will necessarily be large as well. By contrast, EM exporters endogenously choose whether

[17] As will become clear there is never an incentive to invoice in the destination currency.

to price in dollars or in their own home currency, with this choice represented by η_j, so when their share b is large, the ultimate value of $\alpha_{\$i}$ is more up for grabs. In terms of the mechanics of the model, a acts as an exogenous anchor on import-invoice shares, while b serves as a feedback coefficient—meaning that the higher is b, the stronger is the feedback from the rest of the EM world's export-pricing decisions to import-invoice shares, and vice-versa, and hence the stronger are the strategic-complementarity effects that can give rise to multiple equilibria.

By keeping a and b constant across all EM countries, we are effectively assuming that all EMs are equally exposed to the U.S. as a trading partner. This makes for a convenient simplification, though it is straightforward to generalize. Finally, we also assume that the market for dollar deposits is integrated, meaning that country-i citizens can obtain safe dollar claims from anywhere in the world. This ensures that the interest rate on dollar deposits offered by banks is the same across countries. By contrast, home-currency markets are segmented across the countries. These assumptions imply that the market-clearing conditions are given by:

$$
\begin{aligned}
B_{hi} &= D_{hi} \\
B_{Ri} &= A_{Ri} \\
\int_i B_{\$i} di + X_\$ &= \int_i D_{\$i} di
\end{aligned}
$$

As just noted, for sufficiently large values of the invoicing-feedback coefficient b, we can obtain multiple equilibria, with differing degrees of dollar invoicing. Intuitively, if exporters in all countries $j \neq i$ price a lot of their sales in dollars, this raises the dollar invoice share $\alpha_{\$i}$ facing country-i importers—and more so if b is larger. Given this higher value of $\alpha_{\$i}$, country-$i$ importers demand more dollar-denominated deposits, which tends to push down dollar interest rates. These low dollar rates in turn validate the original decision on the part of country-j exporters to price in dollars; they do so precisely because it helps them to tap more of the cheap dollar funding. This line of reasoning explains how we can sustain an equilibrium where the dollar is used relatively intensively in both trade and banking. Conversely, a less dollar-intensive equilibrium can also be self-sustaining. In this case, there is less invoicing in dollars, which lowers the demand on the part of importers for safe dollar claims, and therefore leads to higher interest rates on safe dollar claims. These higher rates in turn validate the choice on the part of exporters to do less in the way of pricing their exports in dollars.

Proposition 4, which is illustrated in Figures 3 and 4, formalizes this intuition. The proposition divides the parameter space into three regions, but now the exogenous parameter that defines the regions is a, not $\alpha_\$$. Recall again that a can be interpreted as the U.S. share in imports of the representative EM country.

Proposition 4 *Define two cut-offs $\underline{a}$ and $\bar{a}$ as:*

$$\underline{a} \equiv \frac{X_\$ + \gamma_L N}{M} + \frac{\theta(\bar{\mathcal{E}} - 1)}{M\psi\delta\sigma^2} - b$$
$$\bar{a} \equiv \frac{X_\$}{M}$$

If the invoicing-feedback coefficient b is large enough—specifically, if

$$b > \frac{\gamma_L N}{M} + \frac{\theta(\bar{\mathcal{E}} - 1)}{M\psi\delta\sigma^2}$$

then $\underline{a} < \bar{a}$, and we can describe the solution of the model according to three regions. In the high-a region where $a > \bar{a}$, the only equilibrium is one in which $\eta = 1$, and in which there is mismatch, meaning that $B_\$ > \gamma_L N$—that is, dollar deposits are backed both by dollar-invoiced projects, as well as by local-currency projects. In the low-a region where $a < \underline{a}$, there is an equilibrium with $\eta = 0$, and the equilibrium with both $\eta = 1$ and mismatch ($B_\$ > \gamma_L N$) does not exist. And in the intermediate-a region where $\underline{a} < a < \bar{a}$, both types of equilibria co-exist.[18] *The values of all of the other endogenous variables in these two equilibria are the same as given by the corresponding expressions for the lower and upper ranges in Proposition 3. Specifically, when $\eta = 0$, $D_\$ = X_\$$, $Q_\$ - Q_h = -\delta\psi\sigma^2(X_\$ - aM)$, and when $\eta = 1$, $D_\$ = (a + b)M - \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2}$, $Q_\$ - Q_h = \theta(\bar{\mathcal{E}} - 1)$.*

There are two broad messages to take away from Proposition 4 and the accompanying figures. First, as the share of EM imports coming from the U.S.—proxied for by the parameter a—gradually increases from zero, we eventually must obtain a discrete jump in the global role of the dollar, by $\underline{a}$ at the earliest, or by $\bar{a}$ at the latest . This jump occurs when other countries besides the U.S. start pricing some of their exports in dollars as well. When they do so, the dollar premium jumps also, and the lower interest rate on dollar safe claims is precisely what helps to support the decision of non-U.S. exporters to price their sales in dollars. Second, because of these strategic complementarities, there can be some indeterminacy in the outcome when imports from the U.S. are in a middle range. This indeterminacy may leave the door open for historical factors to pin down what actually happens. We return to this point in more detail below.

[18]The statement of the proposition simplifies things somewhat, in the following sense. What we are calling the low-a region can in turn be divided into two sub-regions, with the addition of another cut-off $\underline{\tilde{a}} = \frac{X_\$ + \gamma_L N}{M} - b < \underline{a}$. When $a < \underline{\tilde{a}}$, the only possible equilibrium is one with $\eta = 0$. And when $\underline{\tilde{a}} \leq a \leq \underline{a}$, this zero-$\eta$ equilibrium co-exists with one in which with $0 < \eta < 1$, but where there is no mismatch: $B_\$ = \eta\gamma_L N$. We are downplaying the no-mismatch equilibrium in the presentation here in part just to streamline the exposition, and in part because it is less empirically relevant, given the body of evidence on mismatch among corporate borrowers in emerging markets.

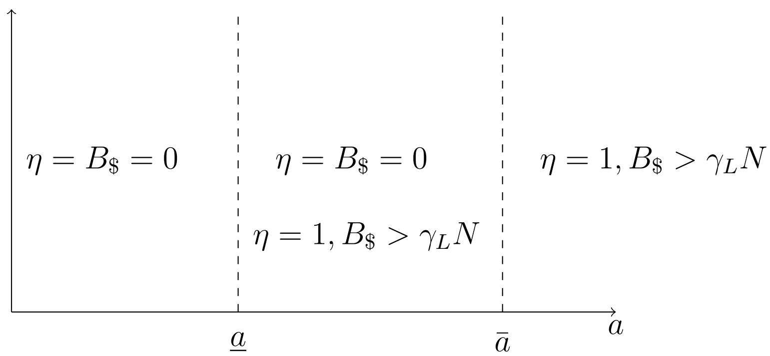
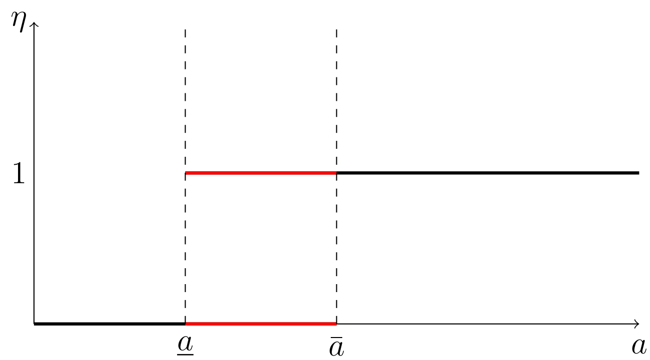
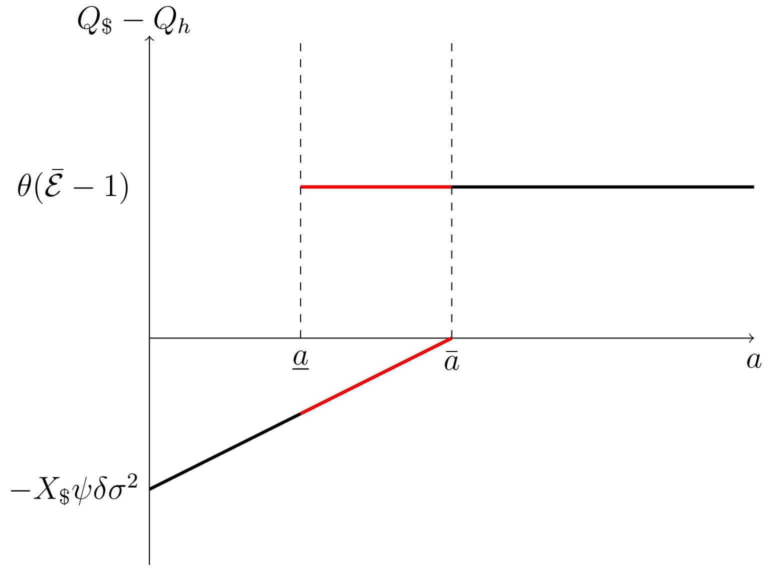


Figure 3: Dollar Equilibria as Share of Imports From U.S. Varies



(a)



(b)

Figure 4: Invoicing Decisions and Deposit Rates as Share of Imports From U.S. Varies

5 The Dollar vs. the Euro: Will One Currency Dominate?

In this section we explore the possibility of the emergence of a *single* globally dominant currency out of several possible alternatives. To do so we need to create a level playing field where we pit two candidate currencies against one another, and then ask what the potential outcomes are. This is what we do next. In particular, we now consider a symmetric setting where there are two possible global currencies, the dollar and the euro, with identical economic fundamentals. And the question we are going to be most interested in is this: are there circumstances where, in spite of the symmetry in fundamentals, the equilibrium outcomes are asymmetric, with one global currency being used extensively by emerging-market countries to invoice their exports and to finance projects, and the other global currency not being used at all in this way?

It turns out that such asymmetric outcomes arise naturally in our framework, and they are driven by the same invoicing-feedback mechanism that led to multiple equilibria in Proposition 4 above. Intuitively, once one currency—say the dollar—gets a bit of an edge in invoice share, this tends to feed on itself: as more global trade is invoiced in that currency, there is more demand for it as a safe store of value. This in turn makes it a cheaper currency to borrow in, which leads exporters in search of lower borrowing costs to invoice their sales in that currency. Such a virtuous circle can entrench the dollar as the dominant currency, and at the same time freeze out the euro, even if there is initially no fundamental difference between the two.[19]

5.1 Augmenting the Model

To capture this all in the model, we make several adaptations that allow us to incorporate the euro alongside the dollar. There is now an equal-sized exogenous external supply of dollar and euro safe assets available to emerging markets, that is $X_\$ = X_€ = X$. The goods purchased by importers in EM country i can now be invoiced in either dollars or euros. Imports from the US are priced in dollars, and those from the euro area are priced in euros. To maintain symmetry we assume $M^U = M^E = M^*$. The share of imports invoiced in dollars is now given by $\alpha_{\$i} = a + b\int_{j\neq i}\eta_{\$j}dj$, where as before $\eta_{\$j}$ is the fraction of the N export projects in country j that are invoiced in dollars. Similarly, the share of imports invoiced in euros is given by $\alpha_{€i} = a + b\int_{j\neq i}\eta_{€j}dj$.

[19]He et al. (2016) also study a model where strategic complementarities can strengthen the safe-asset position of one currency relative to another. In their setting, these complementarities run through the coordinated decisions of investors who must decide whether or not to roll over short-term sovereign debt claims. This contrasts with our model, where the complementarities run through the invoicing decisions of exporters. In other words, the role of goods trade, which is central in our story, is absent in theirs.

Note that the parameter a now not only proxies for the share of U.S. goods in total EM imports, it also proxies for the Eurozone share. This symmetry is designed to create a level-playing-field benchmark.

Importers in EM country i maximize the same utility function as before but now they have access to deposits denominated in euros as well as in dollars. The budget constraint of the importers therefore becomes:

$$\begin{aligned}
C_{0i} &= W - Q_{hi}D_{hi} - Q_{\$}D_{\$i} - Q_{€}D_{€i} \\
C_{1i} &= D_{hi} + \mathcal{E}_{\$1i}D_{\$i} + \mathcal{E}_{€1i}D_{€i} - \mathcal{E}_{\$1i}M^* - \mathcal{E}_{€1i}M^* \\
&\quad - \mathcal{E}_{\$1i}\int_{j\neq i}\eta_{\$j}M^{EM}dj - \mathcal{E}_{€1i}\int_{j\neq i}\eta_{€j}M^{EM}dj - \int_{j\neq i}\mathcal{E}_{ij}(1-\eta_{\$j}-\eta_{€j})M^{EM}dj
\end{aligned}$$

Note that, as before, $\mathbb{E}_0(\mathcal{E}_{\$1}) = \mathbb{E}_0(\mathcal{E}_{€1}) = 1$, $\int_j \mathcal{E}_{ij}dj = 1$, and we continue to assume that the dollar and euro deposit markets are globally integrated.

We also assume that the dollar and the euro are equally volatile with respect to the currencies of all EMs, and therefore that the maximally appreciated value of each is the same. That is, $\mathbb{V}_0[\mathcal{E}_{\$1}] = \mathbb{V}_0[\mathcal{E}_{€1}] = \sigma^2$ and $\bar{\mathcal{E}}_{€i} = \bar{\mathcal{E}}_{\$i} = \bar{\mathcal{E}}$. These assumptions have the effect of making it equally costly to use local-currency projects as collateral for either dollar or euro safe claims. Again, the goal here is to do everything we can to create a level playing field between the dollar and the euro based on fundamental considerations. Finally, to keep things simple, we assume that the correlation between the two exchange rates is zero. We have also derived the more general case where the correlation is given by some $\rho < 1$. All the qualitative results described below carry through to this case, albeit at the cost of some additional notational complexity.

Next, we turn to the supply side. We can restate the augmented version of the bank's problem as:

$$\max_{\substack{B_{hi},B_{\$i},B_{€i}B_{Ri},\\ \eta_{\$i},\eta_{€i}}} \mathbb{E}_0\left[\gamma N_0 + (1-\eta_{\$i}-\eta_{€i})\gamma N + \mathcal{E}_{\$1i}\eta_{\$i}\gamma N + \mathcal{E}_{€1i}\eta_{€i}\gamma N - B_{hi} - \mathcal{E}_{€1i}B_{\$i} - \mathcal{E}_{€1i}B_{€i} - B_{Ri}\right]$$

subject to

$$\begin{aligned}
Q_{hi}B_{hi} + Q_{\$}B_{\$i} + Q_{€}B_{€i} + Q_{Ri}B_{Ri} &\geq N + N_0 \\
\bar{\mathcal{E}}(B_{\$i} + B_{€i}) + B_{hi} &\leq \gamma_L N_0 + (1-\eta_{\$i}-\eta_{€i})\gamma_L N + \bar{\mathcal{E}}(\eta_{\$i}+\eta_{€i})\gamma_L N \\
B_{hi} &\leq \gamma_L N_0 + (1-\eta_{\$i}-\eta_{€i})\gamma_L N
\end{aligned}$$

The market-clearing conditions are now given by:

$$\begin{aligned}
D_{hi} =& B_{hi} \qquad \forall i \\
A_{Ri} =& B_{Ri} \qquad \forall i \\
\int_i D_{\$i} di =& \int_i B_{\$i} di + X \\
\int_i D_{€i} di =& \int_i B_{€i} di + X
\end{aligned}$$

Before formally stating the full solution to this version of the model, it is useful to preview the types of outcomes that one can expect. Broadly speaking, depending on the value of the exogenous parameter a, three kinds of equilibria can arise. The first is a symmetric zero-η equilibrium, where exporters do no pricing in either dollars or euros: $\eta_{\$i} = \eta_{€i} = 0$. The second is a dual-currency equilibrium, where exporters do some pricing in both dollars and euros: $\eta_{\$i} > 0, \eta_{€i} > 0$. And the third is an asymmetric dominant-currency equilibrium, where exporters exclusively use only one of the two currencies (in addition to the relevant local currency) to price their exports: $\eta_{\$i} > 0, \eta_{€i} = 0$ or $\eta_{€i} > 0, \eta_{\$i} = 0$.

If we focus for the moment on the dual-currency equilibria, it is important to note that these can be of two sub-types. In the first, there is no mismatch, meaning that the only source of collateral for dollar (euro) safe claims comes from exports invoiced in dollars (euros). In the second, there is mismatch, meaning that local-currency projects are also used to back dollar and euro safe claims. These two sub-types correspond to the intermediate-$\alpha_\$$ and high-$\alpha_\$$ regions shown in Figure 2 for the partial-equilibrium version of the model. A key insight for what follows—and one that captures a central piece of the intuition of our model—is that in the current general-equilibrium setting, only the latter mismatch types of equilibria are generally stable. That is, if we start in a hypothesized dual-currency equilibrium with no mismatch, a small deviation on the part of other EM countries will necessarily drive the invoicing choice in country i towards one of the extreme corner solutions: either $(\eta_\$, \eta_€) = (1, 0)$ or $(0, 1)$.

To see this explicitly, note that, given the bang-bang nature of the invoicing decision, it is only possible to sustain $\eta_{\$i} > 0, \eta_{€i} > 0$ if interest rates in dollars and in euros are exactly equalized, i.e., $Q_\$ = Q_€$. Now consider how these interest rates are determined in the no-mismatch region of the parameter space. From the first order conditions of the importers in country i, we have that:

$$
\begin{aligned}
Q_{\$} &= \delta + \delta\psi\sigma^2((a + b\eta_{\$,-i})M - D_{\$i}) && (7)\\
Q_{€} &= \delta + \delta\psi\sigma^2((a + b\eta_{€,-i})M - D_{€i}) && (8)\\
Q_h &= \delta
\end{aligned}
$$

Consider a potential no-mismatch equilibrium where $\eta_{\$} > 0$ and $\eta_{€} > 0$ for all countries. Now think about the situation facing a given country i, if all other countries deviate slightly from the proposed equilibrium, so that $\eta_{\$,-i}$ increases by a small amount, while $\eta_{€,-i}$ decreases by the same small amount. From the above first-order conditions, it follows that this deviation increases $Q_{\$}$ and reduces $Q_{€}$. In other words, the deviation towards more dollar invoicing increases the demand for dollar safe assets, and therefore drives dollar interest rates down relative to euro interest rates. But once we no longer have $Q_{\$} = Q_{€}$, it is no longer possible to sustain an equilibrium with positive amounts of invoicing in both dollars and euros, and indeed given the linearity of the problem they face, banks in country i will switch all the way towards an extreme invoicing choice of $\eta_{\$i} = 1, \eta_{€i} = 0$. This demonstrates that any potential dual-currency equilibrium with no mismatch is not robust to such small deviations.This is what we mean when we say that such equilibria are unstable.

By contrast, in the mismatch region of Figure 2—which occurs when $\alpha_{\$}$ is sufficiently high—it can be seen that $Q_{\$}$ is a constant, and therefore independent of $\eta_{\$}$. It follows that a deviation by other countries that increases $\eta_{\$-i}$ has no effect on the incentive for country i to invoice its exports in dollars; in other words, $d\eta_{\$i}/d\eta_{\$-i} = 0$, which implies that symmetric equilibria with mismatch are always stable.

If we restrict attention to such always-stable equilibria, it turns out that for any given value of a, it is possible that more than one type of equilibrium can be sustained. For example, for some values of a, it might be the case that we can have both a symmetric zero-η equilibrium, as well as an asymmetric dominant-currency equilibrium. Nevertheless, the symmetric zero-η equilibrium is more likely to arise when a is relatively low, while the dual-currency equilibrium with mismatch is more likely to arise when a is high. And the asymmetric dominant-currency equilibria are most prevalent for intermediate values of a. Intuitively, this is because the parameter a proxies for the exogenous component of non-home-currency invoicing, and hence the generalized demand for safe claims denominated in some non-home currency, be it the euro or the dollar. When this demand is very low, this tends to produce outcomes where neither the dollar nor the euro plays an important role in global trade. And when it is very high, we can get situations where both are prominently used. But in the intermediate region—and this is of particular interest to us—it can effectively be the case that while there is enough safe-asset demand to sustain one global currency, there is not enough to sustain two. This is what can lead to there being a single dominant currency.

Proposition 5 ***[Dominant Currency]*** *Consider a symmetric setting where the U.S. and the Eurozone have the same shares in global trade: $a_\$ = a_€ = a$. Define four cutoffs as:*

$$\bar{a}^n \equiv \frac{X}{M}, \qquad \underline{a}^s \equiv \frac{X}{M} + \frac{\gamma_L N}{M} + \frac{\theta(\bar{\mathcal{E}} - 1)}{M\delta\psi\sigma^2} - b$$
$$\bar{a}^s \equiv \frac{X}{M} + \frac{\theta(\bar{\mathcal{E}} - 1)}{M\delta\psi\sigma^2} \qquad \underline{a}^b \equiv \frac{X}{M} + \frac{\theta(\bar{\mathcal{E}} - 1)}{M\delta\psi\sigma^2} + \frac{\gamma_L N}{2M} - \frac{b}{2}$$

$\bar{a}^n$ is the cut-off for a below which a symmetric zero η equilibrium can be sustained and above which it cannot; $\underline{a}^b$ is the cut-off above which a symmetric positive η equilibrium is sustainable but below which it is not; $\underline{a}^s$ is the cut-off above which an asymmetric dominant currency equilibrium is sustainable, while it is not above $\bar{a}^s$.[20]

From simple inspection of the four cut-offs in Proposition 5 it follows that all the cut-offs shift leftward the higher the level of imports in consumption M and the lower is the supply of exogenous dollar/euro safe assets. This follows because a higher M weakly increases the demand for dollar/euro safe assets, and a lower X reduces the exogenous supply of such assets making it attractive for EM banks to produce these assets. The range of values of a for which an equilibrium with either an asymmetric dominant currency or a symmetric positive η can be sustained is greater the higher is the feedback coefficient b. Higher volatility has an ambiguous effect. On the one hand, raising volatility (σ^2) increases importers demand for foreign currency safe assets, while on the other hand a higher worse-case (depreciated) value $\bar{\mathcal{E}}$ reduces the attractiveness to EM banks of issuing safe foreign currency claims.

If the invoicing-feedback coefficient b falls within the following region

$$\frac{\theta(\bar{\mathcal{E}} - 1)}{M\delta\psi\sigma^2} + \frac{\gamma_L N}{M} < b < \frac{2\theta(\bar{\mathcal{E}} - 1)}{M\delta\psi\sigma^2} + \frac{\gamma_L N}{M}$$

the cutoffs satisfy $\underline{a}^s < \bar{a}^n < \underline{a}^b < \bar{a}^s$. We can then describe the stable equilibria of the model according to five regions, as depicted in Figure 5. In the first region, where $a < \underline{a}^s$, the only

[20]Note that because of the bang-bang nature of the solution for η, in the dual-currency equilibrium the specific values of $\eta_\$$ and $\eta_€$ are indeterminate. In this proposition we consider the case where $\eta_\$ = \eta_€$. However these conditions can be re-derived for any other split of η and the main insights are unchanged.

possible outcome is a symmetric zero-η equilibrium which there is no dominant currency. In the second region, where $\underline{a}^s < a < \bar{a}^n$, this equilibrium co-exists with an asymmetric dominant-currency equilibrium, where only one of the two $\eta's$ is positive. In the third region where $\bar{a}^n < a < \underline{a}^b$, the asymmetric dominant-currency equilibrium is the *unique* stable outcome. In the fourth region, where $\underline{a}^b < a < \bar{a}^s$, the asymmetric dominant-currency equilibrium co-exists with a symmetric positive-η equilibrium, where exporters do some pricing in both dollars and euros: $\eta_{\$i} = \eta_{€i} > 0$, and where there is mismatch in both currencies. And finally, in the fifth region where $a > \bar{a}^s$, the symmetric positive-η equilibrium is the unique outcome.

Again, what is particularly noteworthy about this ordering is that there is an intermediate range of values of a—namely, where $\bar{a}^n < a < \underline{a}^b$—where the only possible equilibrium is one with a single dominant currency.

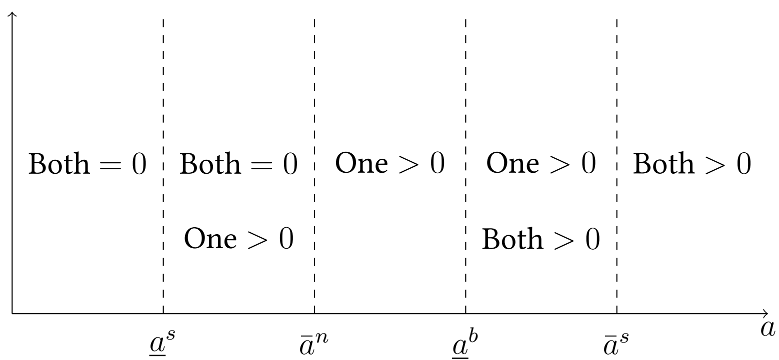


Figure 5: Possible equilibrium configurations for $\eta_{\$i}, \eta_{€i}$ as a function of a

5.2 Numerical Example

In this section we provide a detailed numerical example that generates the same ordering of cut-offs as in Figure 5. The parameters used are listed in Table 1.

Parameter	β	δ	ψ	N	N_0	X	M	b	γ_L	$\mathbb{E}[\gamma]$	σ	$\bar{\mathcal{E}}$	ρ	W
Value	0.75	0.8	0.8	2.6	10	0.3	4	0.4	0.55	1.4	0.55	3.8	0.05	10

Table 1: Parameter Values

As the figures show, in the no-dominant-currency case, which is the short-dashed (blue) line labeled "Both=0", we have that $\eta = 0$ and $B_\$ = B_€ = 0$. There is no incentive to invoice in a global currency, as $(Q_\$ - Q) = (Q_€ - Q) < 0$. In this range as a increases the negative gap between the dollar (euro) bond price and the EM bond declines as a consequence of the exogenous increase in demand for dollar and euro safe assets. The dollar invoicing share, $\hat{\alpha}_{\$}$, and the euro invoicing share, $\hat{\alpha}_{€}$, both increase by the same amount with the exogenous increase in a.[21]

In the case of a single dominant currency, depicted by the long-and-short dashed (yellow) line marked "One>0", there is positive invoicing in one of the two global currencies, whose η is plotted. For the purposes of discussion, we assign this dominant role to the dollar. The euro on the other hand is not used in trade invoicing, and EM banks do not create any safe euro claims. This difference in dollar and euro invoicing leads to a divergence between $\hat{\alpha}_{\$}$ and $\hat{\alpha}_{€}$, with the former jumping sharply relative to the no-dominant-currency case because of the endogenous increase in dollar invoicing, while the latter falls. Indeed, the dollar invoicing share exceeds the euro invoicing share for all values of a for which this asymmetric equilibrium is sustainable. Consistent with this, in this equilibrium, $(Q_{\$} - Q_h)$ is always positive and exceeds $(Q_{€} - Q_h)$. More subtly, $(Q_{€} - Q_h)$ is negative for lower values of a and then turns positive, but even at this point there is still no incentive to invoice in euros as long as $Q_{€} < Q_{\$}$. The figures also illustrate that in this equilibrium the bank-exporter coalition bears a dollar currency mismatch— in the sense that dollar deposits exceed dollar-denominated collateral— while there is no euro mismatch. In addition, the dollar's use in trade invoicing $\alpha_{\$}$ greatly exceeds the U.S. share in world trade a, while that same ratio equals one for the euro. This is very much in line with the empirical evidence on trade invoicing.

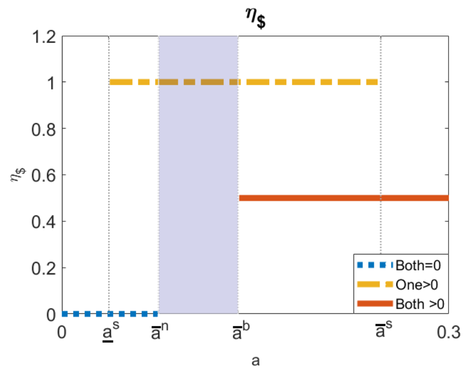
The case of dual dominant currencies is graphed as the solid (orange) line labeled "Both>0". Now η represents invoicing in both dollars and euros, and it is symmetric and constant over this range.[22] The size of the exorbitant privilege and the extent of currency mismatch are now also identical across dollars and euros. The vertical lines demarcate the regions that support the different equilibria.

5.3 Which Currency Dominates? The Role of History

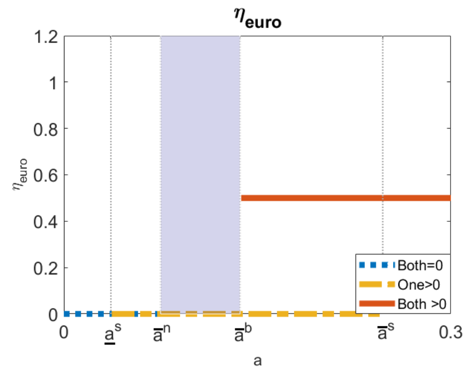
As discussed above, we are particularly interested in asymmetric dominant-currency equilibria, where exporters exclusively use only one of the two currencies (in addition to the relevant local currency) to price their exports: either $\eta_{€} = 0, \eta_{\$} > 0$, or $\eta_{€} > 0, \eta_{\$} = 0$. Our interest is motivated by the fact that the former configuration aligns very closely with what we observe in reality. In particular, although the U.S. and Eurozone economies are the two largest in the world,

[21] The two invoicing shares are defined as $\hat{\alpha}_{\$} \equiv \frac{a+b\eta_{\$}}{2a+b\eta_{\$}+b\eta_{€}+b(1-\eta_{\$}-\eta_{€})}$ and $\hat{\alpha}_{€} \equiv \frac{a+b\eta_{€}}{2a+b\eta_{\$}+b\eta_{€}+b(1-\eta_{\$}-\eta_{€})}$ respectively

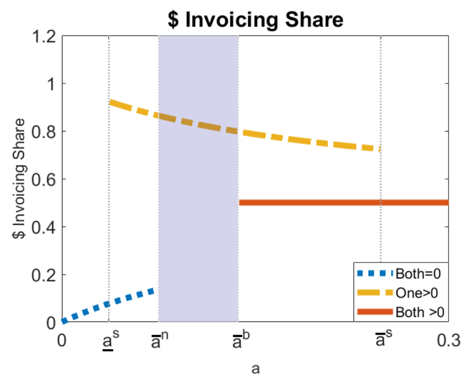
[22] For the parameter values in this example, we have verified numerically that any symmetric dual currency equilibrium without mismatch is indeed unstable, as per our discussion above. This is why such equilibria do not appear anywhere in Figure 6.



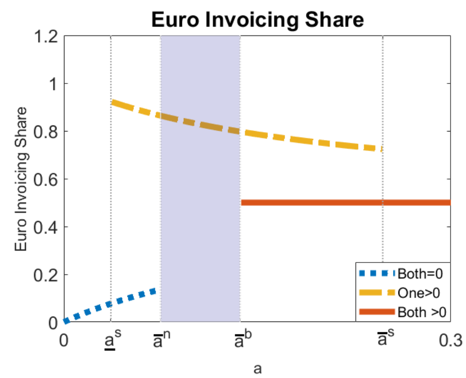
(a) \$ invoicing



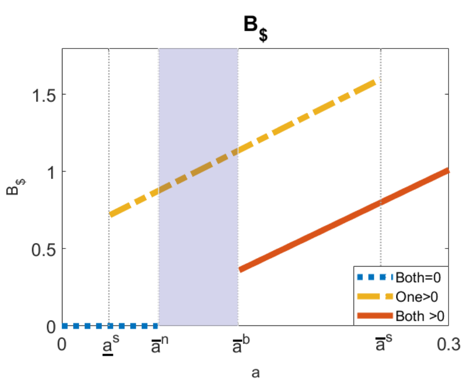
(b) € invoicing



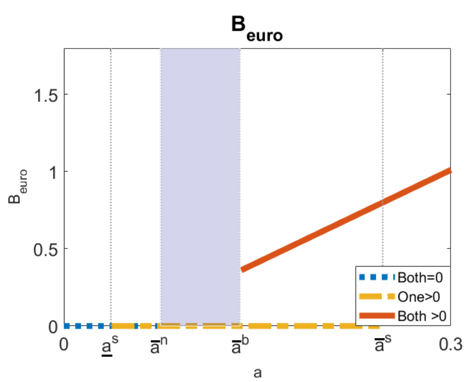
(c) \$ invoicing share



(d) € invoicing share

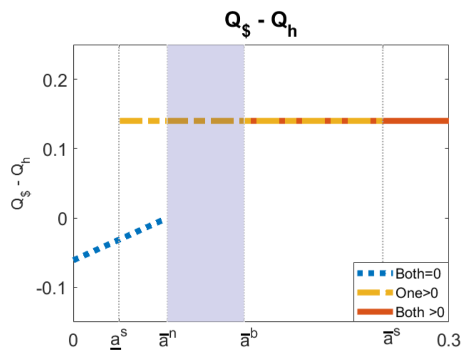


(e) Bank dollar deposits

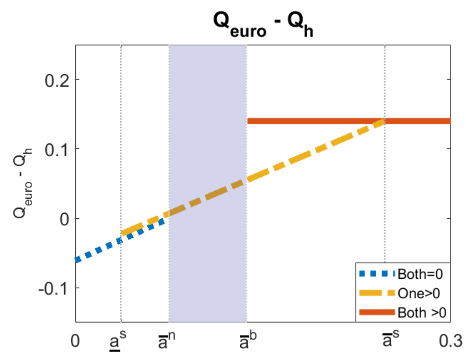


(f) Bank euro deposits

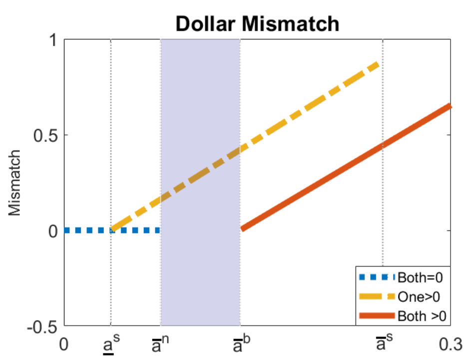
Figure 6: Numerical Example



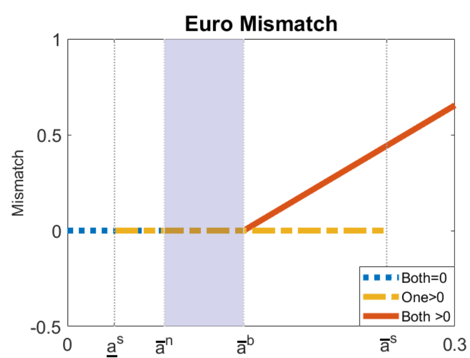
(g) Dollar Exorbitant Privilege



(h) Euro Exorbitant Privilege



(i) Dollar Mismatch



(j) Euro Mismatch

Figure 6: Numerical Example (continued)

trade invoicing by all countries other than these two skews almost entirely to the dollar: as noted in the introduction, the volume of international trade that is invoiced in dollars is several times that of imports coming from the U.S., while the volume of trade that is invoiced in euros is very similar to that of imports coming from the Eurozone.[23] Since the η's correspond precisely to export-pricing decisions made in countries other than the U.S. and Europe, it appears that we are in a situation that is strikingly similar to what the model envisions in an equilibrium with $\eta_{€} = 0$.

However, while the model suggests that we may well wind up in an asymmetric equilibrium where one currency dominates in this lopsided fashion, it is unable to speak to *which* currency that will be, given that it treats the U.S. and Europe as being identical on all fundamental dimensions. Taken literally, the model says that the outcome is indeterminate.

To break this indeterminacy, it may be useful to consider cases where the fundamentals differ across the U.S. and Europe, and to assign a role to history. Here is what we have in mind. If one steps away from the symmetric case where the U.S. and European shares in imports of other countries are the same—i.e., where $a_{\$} = a_{€}$—there can for a wide range of parameter values be just a single deterministic equilibrium outcome. We analyze this asymmetric-fundamentals case in the online appendix, and show, for example, that if $a_{\$}$ is much larger than $a_{€}$, it may well be that the only equilibrium is one in which $\eta_{€} = 0, \eta_{\$} > 0$. In the case of the U.S. and Europe, something like this might have been a good description of the situation that existed before the formation of Eurozone in 1999, when all the member countries had their own currencies, and the largest individual member, Germany, had a GDP only about a fifth that of the U.S. So applied to the pre-Eurozone period, our model might well have predicted that the only possible equilibrium outcome was one in which the dollar was the lone dominant currency.

Now suppose that after the Eurozone forms, it is large enough so that given current parameter values, the model admits two equilibrium outcomes: one where the dollar is dominant and one where the euro is dominant. Which of the two is likely to actually obtain? To the extent that there is any history-dependence, it would naturally seem to be the dollar-dominant equilibrium. In other words, any time we are faced with multiple possible equilibrium outcomes at some date t, a plausible selection mechanism would be to go back in time to the first date prior to t when one of those equilibria is uniquely pinned down by the model, and posit that it then remains as the focal equilibrium until the parameters change to the point where it is no longer viable.

If one accepts this line of reasoning, it suggests that even if the European economy grows to the point where it catches up with—or even somewhat surpasses—the U.S., this may not be

[23] The Eurozone refers to the 19 countries that use the euro as their common currency. Defined this way, the largest "countries" by GDP as of 2016 were, in descending order: the U.S., the Euro Area, China, Japan, and the U.K.

enough to dislodge the now-entrenched dollar from its dominant-currency perch. According to the dynamic equilibrium-selection process outlined above, this might require the European economy to get substantially bigger than the U.S., to the point where $\eta_{€} > 0, \eta_{\$} = 0$ becomes the unique equilibrium outcome. And of course, exactly the same observations apply if, instead of Europe, one asks about the prospects for the Chinese renminbi to become a globally-dominant currency: even as its fundamentals approach those of the U.S., it is likely to be handicapped by history, which we would argue can play an important role in selecting the equilibrium in a setting like that of our model.

We can also consider the case where the exogenous supply of safe assets differs across the two countries, i.e. $X_{\$} \neq X_{€}$. As shown in the online appendix, as the relative supply of safe assets increases for a country, all else equal, this reduces its ability to become a dominant currency. That is, an increase in the supply of euro safe assets, without any change in demand for euro safe assets, will lead to an increase in interest rates on euro safe assets and thereby reduce the incentive of EM banks to produce euro safe assets and of EM exporters to invoice in euros. It has often been argued that issuing a euro safe bond can help with internationalization of the euro. Interestingly, in the context of our model if such an issuance does not increase the demand for euro safe bonds it does not help with internationalization.

6 Further Implications

6.1 Historical Perspectives

As described in Eichengreen, Mehl, and Chitu (2017), the pound was the dominant global currency prior to World War I, with over 60% of world trade invoiced, financed and settled in pounds. This was despite the fact that the U.S. economy had already overtaken Britain as the world's largest economy (in the 1870s) and was almost as large as Britain in world trade. Things however changed quickly over the decade 1914-1924, when the dollar replaced the pound as the leading international currency. According to Eichengreen, Mehl, and Chitu (2017), this transition was triggered by two events: by the Federal Reserve Act of 1913 that allowed U.S. banks to deal in instruments of trade credit (also known as "bankers acceptances") and by World War I, which was relatively more disruptive to Britain, and to British trade.

Importantly, the Federal Reserve played an active role in fostering the market for trade credit in dollars, in the expectation that this would lead to the use of the dollar in the invoicing and settlement of international trade. Specifically, a policy decision was taken to make dollar trade credit available at concessionary rates, and as a result, between 1917 and 1930 the Federal Reserve held over half of all trade acceptances. In the language of our model, such a policy can be thought of as an increase in $a_{\$}$, the exogenous component in the dollar invoice share facing other countries. As we have shown, a small change in this parameter can lead to a large and abrupt change in the global role of the currency—both for invoicing decisions taken by exporters in other countries, as well as for the investment and financing decisions of firms, banks, and households in these countries. Thus our model offers one lens through which to interpret the rapid takeover of the dollar from the pound in the early part of the 20th century.[24]

The ability of a currency to dominate will of course depend on the strength of its financial and monetary institutions, the stability of its currency and the liquidity of its markets. However, historical efforts to globalize a currency have often started with an emphasis on the domain of international trade, as emphasized by Eichengreen (2010). This historical observation again highlights the key linkage between the role of a currency in trade invoicing and its role in banking and finance, which is the central focus of our paper.

With China now one of the largest economies in the world, and the biggest exporter, it appears that Chinese officials are taking these historical lessons to heart in their efforts to internationalize the renminbi. Similar to the U.S. interventions in the early 20th century, they have proceeded by encouraging the use of the renminbi in international trade transactions. Following this push, between 2010 and 2015 the renminbi's share as a settlement currency in China's trade has gone from 0% in 2010 to 25% in 2015.[25] In addition as documented in Eichengreen, Mehl, and Chitu (2017), the renminbi surpassed the euro in 2013 as the second most widely used currency in global trade finance, although it has since fallen back to third place.

Nevertheless, the renminbi currently remains far behind other major currencies in international financial transactions unrelated to trade. What might the future hold? In the medium term, the self-reinforcing mechanisms in our model might lead one to predict that the dollar's dominance would continue largely undisturbed, and that the renminbi would have a hard time gaining much traction in international banking and finance. However, in the longer run, if the gap between Chinese and U.S. shares in world exports widens far enough, we could eventually get to a point where a renminbi-dominant equilibrium becomes inevitable. At this point, the dollar's share in global trade and finance could potentially decline quite sharply.

[24]The ranking of the dollar and pound again switched in the 1930s as the Great Depression was more severe for the U.S., and trade collapsed more for the U.S. compared to Britain. In the 1950s the dollar once again took over as the world's leading currency, and has remained in that position since.

[25]Less is known about the extent of invoicing in RMB.

6.2 Cross-Country Empirical Evidence

A fundamental premise of the paper is that a currency's role as a unit of account is complementary to its role as a store of value. In this sub-section, we test a simple cross-sectional version of this hypothesis.

To get started, note that in any equilibrium of our model, if we generalize to allow the invoice shares $\alpha_{\$i}$ and $\alpha_{€i}$ to vary across countries, the first-order conditions for importers in country i imply that:

$$\frac{D_{\$i}}{D_{€i}} = \frac{\alpha_{\$i}M - \frac{1}{\psi\delta\sigma^2}\left(Q_{\$} - Q_{hi}\right)}{\alpha_{€i}M - \frac{1}{\psi\delta\sigma^2}\left(Q_{€i} - Q_{hi}\right)} \tag{9}$$

In other words, if importers in country i have a greater share of their imports invoiced in dollars—relative to euros—than importers in country j, they will hold a greater share of their deposits in dollars as well, all else equal. This is intuitive, since dollar deposits are effectively held to hedge the exchange-rate risk associated with dollar-priced imports. To operationalize a test of this prediction, we need to make two assumptions. First, country-i importers keep their deposits in country-i banks. Second, recalling that $D_{\$i} = B_{\$i} + X_{\$i}$, where $X_{\$i}$ denotes holdings by country-i importers of U.S. Treasury securities and the like, we need to assume that investments by importers in these securities are intermediated through the banking system, rather than being directly held. This implies that the asset side of bank balance sheets in country i includes not only their real projects, but also the $X_{\$i}$.

With these assumptions in place, the mapping from the theory to the data is straightforward. From Gopinath (2015) we have data at the country level on import invoice shares in different currencies. From the BIS Locational Banking Statistics, we can compute for any country in the dataset the fraction of foreign-currency local banking liabilities that are denominated in dollars. To be clear, for a given country i, these numbers are based on all banks domiciled in i, be they locally-headquartered banks or subsidiaries of foreign bank holding companies.

In the top panel of Figure 7 we plot this measure against the share of a country's foreign-currency-invoiced imports that are denominated in dollars. Our sample includes all countries for which both of these data series are available, with the following exceptions. First, we exclude Eurozone countries and the U.S., to ensure that we are not simply capturing the fact that countries use their own currency in trade and in finance. Second, we also exclude Brazil and India because they place strong restrictions on foreign currency deposits by private agents. These restrictions leave us with a sample of only 10 countries. However, if we drop the restrictions, and include all countries for which the data is available, the correlation depicted in the figure becomes even more pronounced. As it is, the 10-country sample already yields a very strong correlation, with the regression having an R-squared of 0.72. We also observe that it is those countries that are geographically closer to the Eurozone, and that trade heavily with Eurozone countries—namely, Denmark, Norway, Sweden, and Switzerland—that, unsurprisingly, have both lower dollar invoicing of their imports and fewer dollar-denominated banking sector liabilities.

In the lower panel of Figure 7 we attempt to create an alternative measure of banking activity that is somewhat cleaner relative to the theory we have in mind; we do so by restricting attention to banking liabilities in the sub-category "loans and deposits", and where the counterparty (i.e., the party making the loan to the bank in question) is itself a non-bank institution. The aim here is to eliminate inter-bank lending transactions, and other wholesale sources of funding that are less likely to come from local depositors. This refinement cuts our sample down from ten to eight countries, but leads to a very similar picture: the R-squared of the regression is now 0.82. Thus either way, there seems to be a strong empirical association at the country level between dollar-invoiced imports and dollar-denominated bank deposits.

6.3 Central Bank Reserve Holdings

In Gopinath and Stein (2018b), we extend the model of this paper to incorporate an explicit role for central-bank reserve holdings. To do so, we relax the assumption that banks always maintain enough collateral to render their deposits completely riskless in all states of the world. Instead there is a rare banking-crisis state in which the local currency depreciates against the dollar, and in which a fraction of banks fail and need to have their deposits bailed out by the government. The government can finance this bailout in one of two ways: it can impose distortionary taxes ex post on its citizens, or it can draw on a previously-accumulated stockpile of foreign-exchange reserves. The key proposition is that the larger is the share of dollar (as opposed to local-currency) deposits in bank liabilities, the more the government—i.e., the central bank—will choose to rely on dollar reserves to finance bailouts, as opposed to ex post taxation. This is because if a banking crisis tends to occur when the local currency is depreciating against the dollar, bailing out dollar deposits can require imposing very large ex post taxes, whereas holding dollar reserves effectively hedges against the currency risk associated with being a lender of last resort to a dollarized banking system.

Consistent with this proposition, we show that in a sample of 15 countries for which the data is available, there is a strong correlation between the dollar's share in import invoicing, and its share in central-bank foreign exchange reserves. Combined with the evidence in the previous sub-section, this suggests that a country's import invoice share not only influences the liability structure of its commercial banks, but in turn also affects the reserve-holding decisions of its

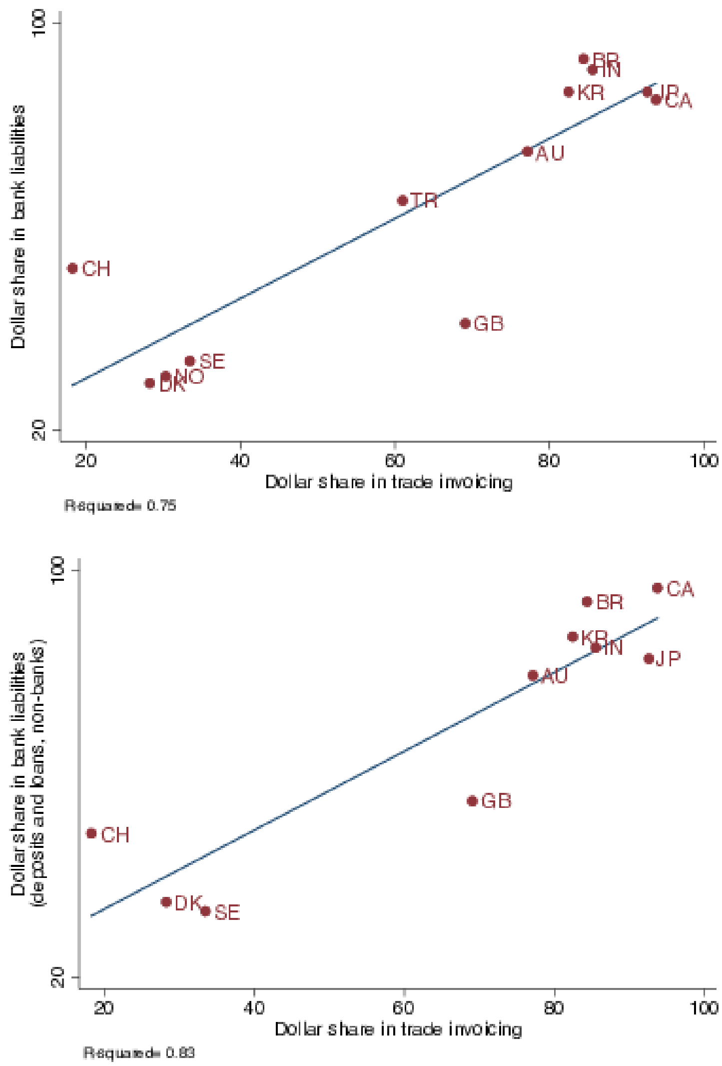


Figure 7: Dollar Share in Trade Invoicing and Banks Local Foreign Currency Liabilities

central bank.

7 Conclusion

The central theme of this paper is that there is a fundamental connection between the dollar's role as the currency in which non-U.S. exporters predominantly invoice their sales, and its prominence in global banking and finance. Moreover, these two roles feed back on and reinforce each other. Going in one direction, a large volume of dollar invoicing in international trade creates an increased demand for safe dollar deposits, thereby conferring an exorbitant privilege on the dollar in terms of reduced borrowing costs. Going in the other direction, these low dollar-denominated borrowing costs make it attractive for non-U.S. exporters to invoice their sales in dollars, so that they can more easily tap the cheap dollar funding. The end result of this two-way feedback can be an asymmetric entrenchment of the dollar as the global currency of choice, even when other countries are roughly similar to the U.S. in terms of economic fundamentals such as their share of overall world-wide imports.

Looking to the future, the self-reinforcing asymmetric equilibrium outcomes that we have highlighted carry a double-edged message about the dollar's potential prospects in a changing world. As noted above, the model suggests that the dollar's dominance is likely to be quite resilient in the medium run, even in the face of rapid growth in global exports from other leading economies like those of Europe and China. However, in the longer run, if the gap between the U.S. and one of these other economies widens far enough, the dollar may potentially fall off the world stage to a very substantial extent, much as the British pound sterling did in the early part of the 20th century. In other words, change may be slow to come, but when it finally does, the forces in our model suggest that the change may well be quite dramatic in magnitude.

Our model can also provide a lens through which to analyze recent developments in digital currencies. Potential new currencies like Facebook's Libra are designed for use in retail payments. With large enough scale (in this case Facebook's 2.4 billion user base), one could envisage a scenario where prices are invoiced in Libra and where this in turn generates a demand for Libra safe assets, which then triggers further invoicing in Libra and so on. Of course, the emergence of a dominant Libra-like currency would require that it be sufficiently liquid and stable in value, issues that we have taken as given in our analysis but that cannot be assumed away for novel would-be currencies.

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A Online Appendix (not for publication)

A.1 Proof of Proposition 5

We can rewrite the objective function of importers as

$$
\begin{aligned}
C_{0i} + \delta\left(\mathbb{E}_0[C_{1i}] - \frac{\psi}{2}\mathbb{V}_0[C_1]\right) =& W - Q_{hi}D_{hi} - Q_{\$}D_{\$i} - Q_{€}D_{€i} \\
&+ \delta(D_{hi} + D_{\$i} + D_{€i} - M) \\
&- \frac{\delta\psi\sigma^2}{2}[(D_{\$} - \alpha_{\$}M)^2 + (D_{\$} - \alpha_{\$}M)^2 + 2\rho(D_{€i} - \alpha_{€}M)(D_{\$} - \alpha_{\$}M)]
\end{aligned}
$$

The first order conditions yield

$$
\begin{aligned}
0 =& -Q_{\$} + \delta - \delta\psi\sigma^2\left((D_{\$i} - \alpha_{\$}M) + \rho(D_{€i} - \alpha_{€}M)\right) \\
0 =& -Q_{€} + \delta - \delta\psi\sigma^2\left((D_{€i} - \alpha_{€}M) + \rho(D_{\$i} - \alpha_{\$}M)\right) \\
0 =& -Q_h + \delta
\end{aligned}
$$

Solving this system of equations, it follows that

$$
\begin{aligned}
D_{\$i} =& \alpha_{\$}M - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{\$} - Q_{hi}) - \rho(Q_{€} - Q_{hi})\right) \\
D_{€i} =& \alpha_{€}M - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{€} - Q_{hi}) - \rho(Q_{\$} - Q_{hi})\right) \\
Q_h =& \delta
\end{aligned}
$$

From this point onwards, we consider three types of equilibria and provide conditions under which these equilibria exist.

[Case 1: $\eta_{\$i} = \eta_{\$i} = 0$] Suppose that all imported products are invoiced in local currency; we later verify that this is indeed an optimal behavior of firms since foreign currency interest rates are higher than local currency interest rate i.e. $Q_{\$} - Q_{hi} < 0$ and $Q_{€} - Q_{hi} < 0$ hold. Given this invoicing decision, importers face $\alpha_{\$} = \alpha_{€} = a$. The demands for foreign currency deposits are now characterized by

$$
\begin{aligned}
D_{\$i} =& aM - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{\$} - Q_{hi}) - \rho(Q_{€} - Q_{hi})\right) \\
D_{€i} =& aM - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{€} - Q_{hi}) - \rho(Q_{\$} - Q_{hi})\right) \\
Q_h =& \delta
\end{aligned}
$$

In view of the market clearing conditions, these equations can be converted to

$$\begin{aligned} X =& aM - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_\$ - \delta) - \rho(Q_€ - \delta)\right) \\ X =& aM - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_€ - \delta) - \rho(Q_\$ - \delta)\right) \end{aligned}$$

So we have

$$Q_\$ = Q_€ = \delta - \delta\psi\sigma^2(1+\rho)\,(X - aM)$$

A sufficient and neccesary condition to sustain this equilibrium is therefore $X \geq aM$. Let $\bar{a}^n = \frac{X}{M}$ denote the cutoff.

[Case 2 $\eta_{\$i}, \eta_€ = (1,0)$ or $(0,1)$] Suppose now that exporters invoice all of their foreign revenues in dollars. That is, $\alpha_\$ = a + b$ and $\alpha_€ = a$. It follows from the demand functions that

$$Q_\$ = \delta - \delta\psi\sigma^2(D_{\$i} - (a+b)M) + \rho\,(X - aM)) \tag{10}$$

$$Q_€ = \delta - \delta\psi\sigma^2((X - aM) + \rho\,(D_{\$i} - (a+b)M)) \tag{11}$$

On the other hand, the first order conditions for the bank's problem yield

$$Q_\$ - Q_{hi} = \theta(\bar{\mathcal{E}} - 1) \tag{12}$$

at an interior solution where $B_{\$i} > 0$ and $B_{hi} > 0$. Plugging (12) into (10) and (11), we can pin down the equilibrium value of dollar deposits:

$$D_{\$i} = (a+b)M - \rho(X - aM) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2}$$

We can then compute interest rates on foreign currency deposits by plugging this back to (10) and (11). That is,

$$\begin{aligned} Q_\$ &= \theta(\bar{\mathcal{E}} - 1) + \delta \\ Q_€ &= \rho\theta(\bar{\mathcal{E}} - 1) + \delta - \delta\psi\sigma^2(1-\rho^2)(X - aM) \end{aligned}$$

To verify that this is indeed an equilibrium, we need to ensure that there is currency mismatch in the balance sheet and that the interest rate on dollar deposits is lower than the interest rate on euro deposits.

Put differently,

$$D_{\$i} \geq \gamma_L N + X \tag{13}$$

$$Q_{\$} \geq Q_{€} \tag{14}$$

Let $\underline{a}^s$ denote the threshold such that eq. (13) holds with equality and $\bar{a}^s$ denote the threshold such that eq. (14) holds with equality.

$$\underline{a}^s = \frac{X}{M} + \frac{\gamma_L N}{(1+\rho)M} + \frac{\theta(\bar{\mathcal{E}}-1)}{(1+\rho)M\delta\psi\sigma^2} - b$$

$$\bar{a}^s = \frac{X}{M} + \frac{\theta(\bar{\mathcal{E}}-1)}{(1+\rho)M\delta\psi\sigma^2}$$

Therefore, it follows that a single dominant-currency equilibrium is sustainable if and only if $a \in [\underline{a}^s, \bar{a}^s]$

[Case 3: $\eta_{\$i} = \eta_{€} = 0.5$] Next, we turn to the case where both euros and dollars are used for international transactions. Consider a potential equilibrium where $\eta_{\$i} = 0.5$ and $\eta_{€i} = 0.5$ hold for all country i. From the bank's first order conditions, we have

$$Q_{\$} - Q_{hi} = Q_{€} - Q_{hi} = \theta(\bar{\mathcal{E}} - 1) \tag{15}$$

The demand functions, on the other hand, lead to

$$Q_{\$} = \delta - \delta\psi\sigma^2(D_{\$i} - (a+0.5b)M) + \rho\,(D_{€i} - (a+0.5b)M)) \tag{16}$$

$$Q_{€} = \delta - \delta\psi\sigma^2\,(D_{€i} - (a+0.5b)M) + \rho(D_{\$i} - (a+0.5b)M)) \tag{17}$$

Since $Q_{\$} - \delta = Q_{€} - \delta$ and $\rho < 1$, it is easy to show

$$D_{\$i} - (a+0.5b)M = D_{€i} - (a+0.5b)M$$

Let $D \equiv D_{\$i} = D_{€i}$. Combining (15), (16) and (17), we have

$$D - (a + b\eta_{\$})M = -\frac{\theta(\mathcal{E}-1)}{\delta\psi\sigma^2(1+\rho)}$$

Let $\bar{a}^b$ denote the cutoff such that $D = X + 0.5\gamma_L N$. That is,

$$\bar{a}^b = \frac{X}{M} + \frac{\theta(\mathcal{E}-1)}{\delta\psi\sigma^2 M(1+\rho)} + \frac{\gamma_L N}{2M} - \frac{b}{2}$$

Therefore, the proposed equilibrium is sustainable if and only if $a \geq \bar{a}^b$.

A.2 Asymmetric Case: Euro versus Dollar

In this section, we elaborate on computational details when the two currencies, the dollar and the euro, have different technological characteristics. We shed light on two factors: (i) the share of imports and (ii) the outside supply of foreign currency deposits. That is, the share of emerging markets' imports from the U.S. and European countries are different, $a_\$ \neq a_€$, which constitutes $\alpha_\$ = a_\$ + b\int_i \eta_{\$i} di$ and $\alpha_€ = a_€ + b\int_i \eta_{€i} di$. Also, the exogenous supply of foreign currency assets, $X_\$$ and $X_€$, can have different values as opposed to the symmetric case in the main draft. Given this setup, this section aims to characterize equilibrium conditions under which the dollar or the euro emerges as a global dominant currency. We then present numerical examples that underscore the asymmetry. Path dependency, which we briefly discussed in Section 5, begins to play a key role.

[Case 1: $\eta_{\$i} = \eta_{\$i} = 0$] In this case, the demand fucntions for dollar- and euro-denominated deposits can be written as

$$
\begin{aligned}
Q_\$ =& \delta - \delta\psi\sigma^2\left((X_\$ - a_\$ M) + \rho(X_€ - a_€ M)\right) \\
Q_€ =& \delta - \delta\psi\sigma^2\left((X_€ - \alpha_€ M) + \rho(X_\$ - a_\$ M)\right) \\
Q_h =& \delta
\end{aligned}
$$

To make this equilibrium sustainable, we should ensure that the parameter space, $(a_\$, a_€, X_\$, X_€)$, satisfies $Q_\$ < Q_{hi}$ and $Q_€ < Q_{hi}$.

[Case 2 $\eta_{\$i}, \eta_€ = (1,0)$ or $(0,1)$] Suppose now that exporters invoice all of their foreign revenues in dollars. As before, it follows from the demand functions that

$$Q_\$ = \delta - \delta\psi\sigma^2(D_{\$i} - (a_\$ + b)M) + \rho\left(X_€ - a_€ M\right)) \tag{18}$$

$$Q_€ = \delta - \delta\psi\sigma^2(X_€ - a_€ M) + \rho\left(D_{\$i} - (a_\$ + b)M\right)) \tag{19}$$

The first order conditions for the bank's problem yield

$$Q_\$ - Q_{hi} = \theta(\bar{\mathcal{E}} - 1)$$

at an interior solution. In view of this condition, we can write the equilibrium value of dollar deposits as

$$D_{\$i} = (a_\$ + b)M - \rho(X_€ - a_€ M) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2}$$

Substituting this equation into (18) and (19), we have

$$
\begin{aligned}
Q_\$ &= \theta(\bar{\mathcal{E}} - 1) + \delta \\
Q_€ &= \rho\theta(\bar{\mathcal{E}} - 1) + \delta - \delta\psi\sigma^2(1 - \rho^2)(X_€ - a_€)
\end{aligned}
$$

To make this equilibrium sustainable, we need to ensure that

$$\begin{aligned}
D_{\$i} &\geq \gamma_L N + X_\$ \\
Q_\$ &\geq Q_€
\end{aligned}$$

These conditions are used to draw a region of parameter space where a single dominant-currency equilibrium is sustainable.

[Case 3: $\eta_{\$i} = \eta_€ = 0.5$] Next, we turn to the case where both euros and dollars act as dominant currencies. Among possible equilibria, we focus on the symmetric invoicing case where $\eta_{\$i} = \eta_€ = 0.5$. It follows from the bank's first order conditions that

$$Q_\$ - Q_{hi} = Q_€ - Q_{hi} = \theta(\mathcal{E} - 1)$$

We can then write the demand functions as

$$\begin{bmatrix} Q_\$ - \delta \\ Q_€ - \delta \end{bmatrix} = \begin{bmatrix} -\delta\psi\sigma^2 & -\rho\delta\psi\sigma^2 \\ -\rho\delta\psi\sigma^2 & -\delta\psi\sigma^2 \end{bmatrix} \begin{bmatrix} D_{\$i} - (a_\$ + 0.5b)M \\ D_{€i} - (a_€ + 0.5b)M \end{bmatrix}$$

The above system of equations can be converted into

$$\begin{aligned}
D_{\$i} &= (a_\$ + 0.5b)M - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)} \\
D_{€i} &= (a_€ + 0.5b)M - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)}
\end{aligned}$$

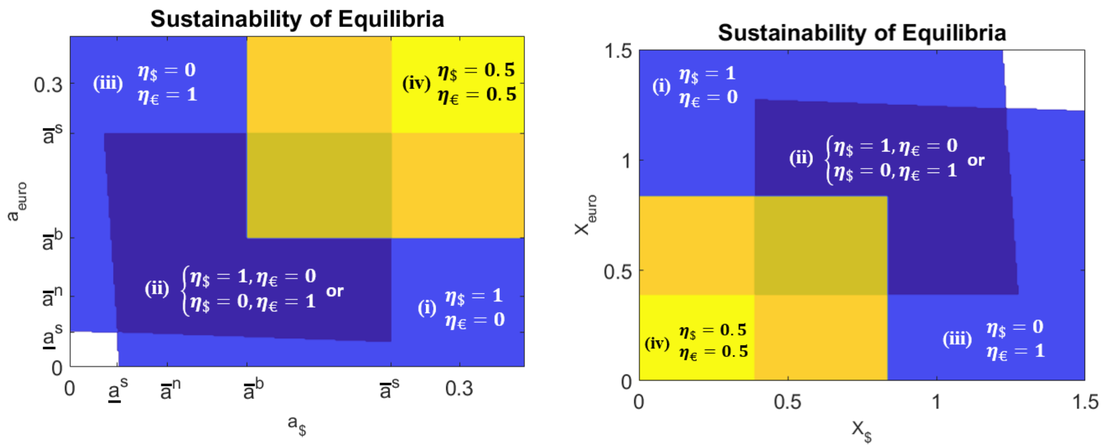
To make this equilibrium sustainable, we need to ensure that

$$\begin{aligned}
(a_\$ + 0.5b)M - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)} &\geq X_\$ + 0.5\gamma_L N \\
(a_€ + 0.5b)M - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)} &\geq X_€ + 0.5\gamma_L N
\end{aligned}$$

We now turn to a numerical example.

A.2.1 Example 1: Asymmetric $a_\$$ and $a_€$

The equilibrium conditions we have derived in the preceding subsection can be used to illustrate various scenarios associated with the competition between the euro and the dollar to become a global dominant currency. As the first exercise, we revisit the formation of Eurozone in Section 5. Conceptually, the formation of the currency union can be thought of as a structural change which raises $a_{€i}$ relative to $a_{\$i}$ in emerging markets. We have claimed that the global economy prior to this change might as well be described by an equilibrium in which the dollar was the lone dominant currency. The diagram in Figure 8 illustrates this point when we allow for $a_{€} \neq a_{\$}$. Assume, for the moment, that $X_{€} = X_{\$}$. The set of no-dominant currency equilibria is ruled out here because this is not our current interest. One can easily compute the corresponding region by using conditions in Appendix A.2. In any case, the light blue area at the bottom of the figure displays the region of parametric space where the dollar is the only dominant currency. As $a_{€}$ rises, we can reach the space where the equilibrium selection is indeterminate. Multiple equilibria can survive in darker areas. We can consider two equilibrium pathways in association with the Eurozone example.



(a) Exogenous Trade Shares (b) Exogenous Safe Asset Supply

Figure 8: Sustainable Equilibria Regions

Path 1: (i) → (ii) One thought experiment we can conduct in this context is the rising importance of Euro as a medium of exchange in international trade. After the integration of European currencies, emerging markets have to bear a larger amount of euro-denominated revenue irrespective of their invoicing decisions. To describe such a trajectory, we investigate an equilibrium path starting from $(a_{\$}, a_{€}) = (0.125, 0)$ towards $(a_{\$}, a_{€}) = (0.125, 0.125)$. The diagram in Figure 8 shows that this equilibrium path passes through three different regions colored by light blue, dark blue and dark yellow respectively. We then computed how $\eta_{\$}$ and $\eta_{€}$ respond as the global economy moves along this trajectory. Our numerical example indeed illustrates the intuition that the dollar could still retain its sole dominance despite the fact that, in theory, multiple equilibria can arise. One caveat here is that we fixed $X_{\$} = X_{€}$ in this example, while $a_{€}$ is increased. In the context of the Eurozone example, we can think of this assumption as implying that the supply of euro denominated assets has barely been changed relative to the trade invoicing.

Path 1: (i) → (iv) An alternative scenario we can consider is the rising importance of both central countries, U.S. and Eurozone, even with no catch-up of the later relative to the former. In other words, the two countries are so important as a share of global trade — that the only possible equilibrium is one where both the dollar and the euro are used by other countries to invoice their exports. To conduct this experiment, we investigate the equilibrium path starting from $(a_{\$}, a_{€}) = (0.125, 0)$ towards $(a_{\$}, a_{€}) = (0.3, 0.3)$. The diagram confirms our proposition that the global economy may end up in a situation where the only possible outcome is a dual-dominant equilibrium.

A.2.2 Example 2: Asymmetric $X_{\$}$ and $X_{€}$

In addition to $a_{\$}$ and $a_{€}$, our model can flexibly admit a difference between the foreign safe asset supplies $X_{\$}$ and $X_{€}$. Essentially, this setting reflects the fact that the market of the European bonds are more fragmented than that of U.S. treasury bonds. One can think of a situation in which the supply of dollar-denominated assets outside emerging markets is greater than the supply of Euro denominated assets. How much does this gap between $X_{\$}$ and $X_{€}$ matter in sustaining a single dominant currency equilibrium? To address this question, we revisit the numerical example and draw a region of the parameter space for sustaining each type of equilibria. We fix $a_{\$} = a_{€} = 0.27$ and, again, rule out no-dominant currency equilibria to focus on the main question. The diagram in Figure 8(b) shows that a dollar dominant equilibrium is more likely to emerge when $X_{\$}$ is low relative to $X_{€}$. Yet, path dependency still matters as we have seen in the previous subsection.