Appendix: Model Details

As described in the text, in the simple textbook case, the IS curve is given by:

$$y_t = y^* - \gamma(r_t - r^*) + \epsilon_t \tag{A.1}$$

We assume that the central bank chooses r_t to minimize squared deviations of output from potential:

$$min \sum_{t=0}^{\infty} E\,(y_t - y^*)^2 \tag{A.2}$$

The solution is given by:

$$r_t = r^* + \frac{\epsilon_t}{\gamma} \tag{A.3}$$

Thus, as noted in the text, in this simple case, the central bank is able to perfectly stabilize output period-by-period, by raising rates to offset demand shocks.

Next, let us consider the case where the IS curve is given by the more elaborate equation (4) in the text, which we reproduce here for convenience:

$$y_t = y^* - \gamma((r_t + s_t) - (r^* + s^*)) - \beta(s_t - s_{t-1}) + \epsilon_t, \tag{A.4}$$

First, assume that $\beta = 0$, so there is no credit-bites-back effect. Now (A.4) simplifies to:

$$y_t = y^* - \gamma((r_t + s_t) - (r^* + s^*)) + \epsilon_t. \tag{A.5}$$

It is straightforward to show that output can again be perfectly stabilized in every period with a simple modification of the interest-rate rule:

$$r_t = r^* + \frac{\epsilon_t}{\gamma(1+\theta)} - \frac{v_t}{(1+\theta)}. \tag{A.6}$$

Relative to (A.3), the policy rate is less responsive to demand shocks, by a factor of $(1+\theta)$. Also, policy leans against exogenous movements in financial conditions, as given by v_t. When credit spreads are relatively low, the policy rate is higher, and vice-versa.

Finally, let us return to the case where $\beta > 0$. As noted in the text, we are interested in the optimal choice of policy rates r_1 and r_2 at times 1 and 2, and we assume that at time 0, $r_0 = r^*$, and $s_0 = s^*$. We further assume that $v_1 = v_2 = 0$, that the demand shock at times 1 and 2 are deterministic, with $\varepsilon_2 < \varepsilon_1 < 0$, and that this is all fully known as of time 1.

In order for the central bank to fully stabilize output at y^* at both time 1 and time 2, it is easy to show that it would have to set the following policy rates:

$$r_1^S = r^* + \frac{\varepsilon_1}{\gamma(1+\theta)+\beta\theta} \tag{A.7}$$

$$r_2^S = \frac{r^*\gamma(1+\theta)+r_1^S\beta\theta+\varepsilon_2}{\gamma(1+\theta)+\beta\theta} = r^* + \frac{\varepsilon_2}{\gamma(1+\theta)+\beta\theta} + \frac{\beta\theta(r_1^S-r^*)}{\gamma(1+\theta)+\beta\theta} \tag{A.8}$$

There are two important points to take away from (A.8). First, r_2^S now depends on r_1^S. In other words, there is hysteresis in the policy rate, and a lower rate at time 1 now requires the central bank to set a lower rate at time 2, all else equal, in order to stabilize output. Second, if ε_2 is sufficiently negative, the zero lower bound on the policy rate will bind at time 2, even if it was not binding at time 1. We assume that this is the case in what follows; in a more realistic stochastic setting it would suffice to simply assume that there is a non-zero probability of the zero lower bound binding at time 2.

When the zero lower bound binds at time 2 but not at time 1, we can write output at the two dates as:

$$y_1 = y^* - \gamma(1+\theta)(r_1 - r^*) - \beta\theta(r_1 - r^*) + \epsilon_1 \tag{A.9}$$

$$y_2(ZLB) = y^* + \gamma(1+\theta)r^* + \beta\theta r_1 + \epsilon_2 \tag{A.10}$$

Given these expressions, we can then ask what the central bank's optimal choice of r_1 is, given its quadratic loss function in (A.2). Note the intertemporal tradeoff: cutting r_1 is helpful in offsetting a negative demand shock at time 1, but when $\beta > 0$ and the zero lower bound binds at time 2, such a time-1 rate cut reduces output further below potential at time 2. We can solve for the optimal value of r_1 in this case as:

$$r_1(ZLB) = \frac{r^*[(\gamma(1+\theta)+\beta\theta)^2 - \beta\theta\gamma(1+\theta)] + \varepsilon_1(\gamma(1+\theta)+\beta\theta) - \varepsilon_2\beta\theta}{(\gamma(1+\theta)+\beta\theta)^2 + (\beta\theta)^2} \qquad \text{(A.11)}$$

To establish Proposition 1 in the text, note that by rearranging equation (A.8), it follows that a binding zero lower bound at time 2 (i.e. $r_2^s < 0$) implies that:

$$\varepsilon_2 < -r^*[\gamma(1+\theta) + \beta\theta] - \frac{\varepsilon_1\beta\theta}{\gamma(1+\theta)+\beta\theta} \qquad \text{(A.12)}$$

Next, we show that if (A.12) is satisfied, then $r_1(ZLB) > r_1^s$. To do so, let $D = r_1(ZLB) - r_1^s$. Using equations (A.7) and (A.11), a series of straightforward calculations yields:

$$D = \frac{r^*[(\gamma(1+\theta)+\beta\theta)^2 - \beta\theta\gamma(1+\theta)] + \epsilon_1[\gamma(1+\theta)+\beta\theta] - \epsilon_2\beta\theta}{[\gamma(1+\theta)+\beta\theta]^2 + (\beta\theta)^2} - \frac{r^*(\gamma(1+\theta)+\beta\theta) + \epsilon_1}{\gamma(1+\theta)+\beta\theta}$$

$$= -\beta\theta \frac{r^*[\gamma(1+\theta)+\beta\theta] + \epsilon_2 + \frac{\epsilon_1\beta\theta}{\gamma(1+\theta)+\beta\theta}}{[\gamma(1+\theta)+\beta\theta]^2 + (\beta\theta)^2} \qquad \text{(A.13)}$$

Therefore, since $\beta, \theta > 0$, we have that:

$$D > 0 \Leftrightarrow r^*[\gamma(1+\theta) + \beta\theta] + \epsilon_2 + \frac{\epsilon_1\beta\theta}{\gamma(1+\theta)+\beta\theta} < 0, \qquad \text{(A.14)}$$

that is, we will have $D > 0$ if $\varepsilon_2 < -r^*[\gamma(1+\theta) + \beta\theta] - \frac{\varepsilon_1\beta\theta}{\gamma(1+\theta)+\beta\theta}$. This is exactly the condition in (A.12) that holds if the zero lower bound binds at time 2.