Online Appendix for:

A Quantity-Driven Theory of Term Premia and Exchange Rates

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August 2022

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A Empirical analysis

A.1 Data

Our baseline sample includes monthly observations from 2001m1 and 2021m12 for the Australian dollar (AUD), the Canadian dollar (CAD), the Swiss franc (CHF), the euro (EUR), the British pound (GBP), and the Japanese yen (JPY).

We obtain data on nominal spot exchange rates from Bloomberg. The nominal foreign exchange rate for currency c, $Q_{c,t}$, is expressed as the number of US dollar per unit of foreign currency. Thus, a higher value of $Q_{c,t}$ means that currency c is stronger against the USD.

We obtain estimates of the nominal zero-coupon government yield curve for each currency from each country's central bank or finance ministry for $n = 1, 2, ..., 10$-year bonds. Many of these datasets lack estimates for 3-month government bill yields, so we obtain data on 3-month government bill yields from Global Financial Data. All bond yields are expressed as continuously compounded annual yields.

Our sources for these zero-coupon government yields are as follows:

Currency	**Website**
AUD	`https://www.rba.gov.au/statistics/tables/xls/f17hist.xls`
CAD	`https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/`
CHF	`https://data.snb.ch/en/topics/ziredev#!/cube/rendoblid`
EUR	`https://www.bundesbank.de/en/statistics/money-and-capital-markets` `/interest-rates-and-yields/term-structure-of-interest-rates`
GBP	`https://www.bankofengland.co.uk/statistics/yield-curves`
JPY	`https://www.mof.go.jp/english/jgbs/reference/interest_rate/index.htm`
USD	`https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`

Several notes are in order:

- As is standard in the literature, we use yields on German government bonds to proxy for the default-free term structure of interest rates in the Eurozone.
- The zero-coupon curves for AUD and CAD are estimated using the Merrill Lynch Exponential Splines model; see Finlay and Chambers (2008) and Bolder, Metzler, Johnson (2004), respectively. The CHF, EUR, and USD curves are estimated using the Svensson (1994) parametric model; see Müller (2005), Schich (1997), and Gurkaynak, Sack, Wright (2007), respectively. The GBP curve is estimated using the Waggoner (1997) splined-based approach; see Anderson and Sleath (2001). The JPY yields are yields on constant maturity coupon-bearing bonds. Following Cieslak and Pavlova (2015), we treat these as par-coupon yields and convert these to zero-coupon yields by bootstrapping the zero-coupon yield curve.
- Due to a paucity of short-dated government bonds in Australian, beginning in 2001 Finlay and Chambers (2008) supplement their data on short-maturity Australian government bonds with data on short-dated overnight index swaps (OIS).

Variable definitions:

- In Table 1, we measure the short-term interest rate as the 1-year government bond yield and the long-term interest rate as the 10-year zero-coupon government bond yield. Specifically, let $y_t^{(n)}$ denote the n-year zero-coupon yield in USD in month t and $y_{c,t}^{*(n)}$ denote the n-year zero-coupon yield in currency c in month t. Thus, we define $i_{c,t}^* \equiv y_{c,t}^{*(1)}$, $i_t \equiv y_t^{(1)}$, $y_{c,t}^* \equiv y_{c,t}^{*(10)}$, and $y_t \equiv y_t^{(10)}$.

- Table 2 presents regressions of the same form as in Table 1, but now using distant forward rates ($f^*_{c,t}$ and f_t) instead of long-term yields ($y^*_{c,t}$ and y_t) as our proxy for term premia. The distant forward we use is the 3-year 7-year forward government bond yield. Specifically, we have $f^*_{c,t} \equiv (10 \cdot y^{*(10)}_{c,t} - 7 \cdot y^{*(7)}_{c,t})/3$ and $f_t \equiv (10 \cdot y^{(10)}_t - 7 \cdot y^{(7)}_t)/3$.

- In Table 3, we work with the log excess returns on 10-year zero-coupon bonds. The 12-month log excess return on 10-year bonds in currency c from month t to $t+12$ is

$$rx^{y*}_{c,t\to t+12} = 10 \cdot y^{*(10)}_{c,t} - 9 \cdot y^{*(9)}_{c,t+12} - y^{*(1)}_{c,t}.$$

  The analogous 3-month excess return from t to $t+3$ is

$$rx^{y*}_{c,t\to t+3} = 10 \cdot y^{*(10)}_{c,t} - 9.75 \cdot y^{*(9.75)}_{c,t+3} - 0.25 \cdot y^{*(0.25)}_{c,t}.$$

  To compute 3-month returns, we estimate $y^{*(9.75)}_{c,t+3}$ by linearly interpolating between 9- and 10-year yields—i.e., we set $y^{*(9.75)}_{c,t+3} = 0.75 \cdot y^{*(10)}_{c,t+3} + 0.25 \cdot y^{*(9)}_{c,t+3}$. Since many central banks do not include estimates of the 3-month yield in the datasets, our estimate of $y^{*(0.25)}_{c,t}$ is the 3-month government bill yield from Global Financial Data. We compute the excess returns on U.S. dollar bonds in the analogous fashion.

- In Table 4, we work with the log excess return on foreign currency investments. Let $q_{c,t} = \log(Q_{c,t})$. We compute 12-month excess return on foreign currency c as

$$rx^{q}_{c,t\to t+12} = (q_{c,t+12} - q_{c,t}) + (y^{*(1)}_{c,t} - y^{(1)}_t).$$

  Analogously, the 3-month excess return on foreign currency c is

$$rx^{q}_{c,t\to t+3} = (q_{c,t+3} - q_{c,t}) + 0.25 \cdot (y^{*(0.25)}_{c,t} - y^{(0.25)}_t).$$

A.2 Sample choice: Taking our theory to the data

To test our theory, we need to choose a sample—i.e., we need to specify the set of currencies and the time period to examine. There are two sets of issues to confront when taking our theory to the data. The first set involves what we think of as the "boundary conditions" of our theory—e.g., the guidance our theory offers on how the empirical results should be expected to vary over currencies and over time. The second set involves issues about measurement and data availability.

Boundary conditions that emerge from our theory:

1. *"Convenience-free" and default-free rates*: As in other models in the Vayanos and Vila (2021) tradition, the interest rates in our model correspond to the "convenience-free" and default-free term structure of rates at which a permanently or "refreshed" AAA-rated private firm or financial institution can borrow. As a result, our model does not speak to the non-pecuniary "convenience" or "safety" premia than can push the yields on ultra-safe government bonds—e.g., U.S. Treasuries and German bunds—below those on default-free private yields (see e.g., Krishnamurthy and Vissing-Jorgensen (2012) and Du and Schreger (2016)). Today, we believe that the best empirical proxy for the short rate in our model is the secured overnight financing rate (SOFR) and that SOFR swap rates are the best proxy for the long-term rates in our model. Alternatively, one could associate the short rate in our model with the overnight Federal funds rate and the long-term rates in our model with Overnight Index Swap (OIS) swap rates. (OIS

and SOFR-linked rates have been extremely similar in recent years.) However, as we explain below, we lack long time series on OIS or SOFR-linked rates. As a result, we are forced to test our theory using data on government bond yields, which may embed convenience premia.

2. *Real versus nominal rates*: Our theoretical results hinge on the comovement between short-term interest rates on the one hand and exchange rates and long-term bonds on the other. As a result, it makes sense to think of the interest rates in our model as real interest rates and the exchange rate as the real exchange rate.

   To see why it makes sense to focus on real instead of nominal rates, note that if short-term nominal interest rates move one-for-one with changes in expected inflation, then news about *future* inflation will not impact real exchange rates. What is more, the arrival of pure news about future inflation will not lead to *unexpected* changes in *nominal* exchange rates: news about future inflation will simply lead to expected *future* movements in nominal exchange rates (see, e.g., Chapter 16 of Krugman, Obstfeld, and Melitz's International Economics textbook). Only news about future short-term real rates impacts both real and nominal exchange rates on arrival. Turning to long-term bonds, while both inflation-indexed (real) and non-indexed (nominal) long-term bonds are exposed to news about future short-term real rates, only long-term nominal bonds are directly exposed to news about future inflation.[1] All of this is spelled out in Section B.2 of the Appendix below where we extend our model to include both shocks to real interest rates and shocks to expected future inflation.

   The upshot is that the comovement patterns between long-term bonds and exchange rates that lie at the heart of our theory should be strongest when looking at inflation-indexed (real) bonds. This is because pure news about future inflation will impact long-term nominal bond yields, but should not impact nominal (or real) exchange rates on arrival. Similarly, the FX return predictability we emphasize should be strongest when looking at real rates: looking at nominal rates simply adds measurement error to the independent variables, biasing the results toward zero. Alternately, if one is forced to use data on nominal bonds to test our theory (e.g., due to a lack of historical data on inflation-indexed bonds), then we would expect our empirical predictions to emerge most strongly in periods where inflation expectations are stable and the resulting measurement error is small.

3. *Integration between FX and bond markets*: Our theoretical results hinge crucially on the idea that long-term bond markets and FX markets are at least partially integrated—i.e., that there are investors who are marginal in both markets—and vanish as these markets become highly segmented. This consideration argues strongly in favor of discarding many emerging market currencies where the presence of capital controls means that we are closer to this pure segmentation case. This consideration also argues in favor of discarding currencies that are pegged to, or are highly managed relative to, the base currency.[2] More generally, market integration considerations argue in favor of focusing on currencies that play an important role in international financial markets and, thus, are likely to play a non-trivial role in the portfolios of global fixed-income

[1] By contrast, shocks to the current nominal price level that do not change expected future inflation will impact nominal exchange rates, but not the yields on nominal bonds.

[2] In our framework, a central bank who is aggressively managing its currency can be thought of as an additional trader who is absorbing currency supply-and-demand shocks in order to prevent global rates investors from being required to absorb those shocks. If the central bank has significant foreign currency reserves and FX demand is relatively inelastic, such a peg may be effective in the short-run. By contrast, assuming the central bank has limited reserves over the longer run, a central bank who is trying to peg its currency may be forced to adjust its short rate to closely track movements in home short rates to avoid having to absorb large net FX flows.

investors. This consideration also argues in favor of looking at more modern data, when global bond and FX markets have arguably become more tightly integrated.

4. *Sufficiently symmetric currencies*: Our model has *qualitatively* different implications for the comovement between foreign exchange and bond returns when the short rate processes for the two countries are highly asymmetric. Specifically, if the foreign country's short rate tends to move *more than one-for-one* with the home country's short rate, then our framework will yield qualitatively different predictions. As a result, it makes sense to focus on "core" countries that pursue relatively independent monetary policies and to exclude "periphery" countries where monetary policy is largely dictated by that in the home country.

   To see the idea, note that if we generalize our model to allow for asymmetric currencies, then the expected returns on long-term domestic bonds, long-term foreign bonds, and foreign exchange are given by

$$\begin{aligned} E_t\left[rx^y_{t+1}\right] &= \tau^{-1}\left(Var_t[rx^y_{t+1}]\cdot s^y_t + Cov_t[rx^y_{t+1}, rx^{y*}_{t+1}]\cdot s^{y*}_t + Cov_t[rx^y_{t+1}, rx^q_{t+1}]\cdot s^q_t\right), \\ E_t\left[rx^{y*}_{t+1}\right] &= \tau^{-1}\left[Cov_t[rx^y_{t+1}, rx^{y*}_{t+1}]\cdot s^y_t + Var_t[rx^{y*}_{t+1}]\cdot s^{y*}_t + Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]\cdot s^q_t\right], \\ E_t\left[rx^q_{t+1}\right] &= \tau^{-1}\left(Cov_t[rx^y_{t+1}, rx^q_{t+1}]\cdot s^y_t + Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]\cdot s^{y*}_t + Var_t[rx^q_{t+1}]\cdot s^q_t\right), \end{aligned}$$

   In our baseline model where the two currencies are perfectly symmetries, we have $Var_t[rx^y_{t+1}] = Var_t[rx^{y*}_{t+1}]$, $0 < Cov_t[rx^y_{t+1}, rx^q_{t+1}] = -Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]$, and $Cov_t[rx^y_{t+1}, rx^{y*}_{t+1}] > 0$. We first note that the qualitative predictions of our baseline model carry through so long as the two currencies are not highly asymmetric. Since the stable equilibrium in our model is continuous in the model's underlying parameters, Proposition 2 implies that $Cov_t[rx^y_{t+1}, rx^q_{t+1}] > 0$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] < 0$, whenever $\rho < 1$ and the short rates and bond supply follow sufficiently symmetric processes.[3] Furthermore, we have $Cov_t[rx^y_{t+1}, rx^{y*}_{t+1}] > 0$ so long as short rates and bond supply follow sufficiently symmetric processes, $\rho > 0$, and FX supply risk is sufficiently small relative to ρ.

   However, things grow more complicated if we allow for highly asymmetric currencies. Specifically, with highly asymmetric short rate processes, the sign of $Cov_t[rx^y_{t+1}, rx^q_{t+1}]$ is ambiguous and the sign of $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]$ need not be opposite that of $Cov_t[rx^y_{t+1}, rx^q_{t+1}]$. To see why, suppose $\sigma^2_{i^*} \equiv Var_t[\varepsilon_{i^*_{t+1}}] \neq Var_t[\varepsilon_{i_{t+1}}] \equiv \sigma^2_i$, but the two short rates share the same persistence ϕ_i. Focusing for simplicity on the limit where there is no supply risk, we have

$$\begin{aligned} Cov_t[rx^y_{t+1}, rx^{y*}_{t+1}] &= Cov\left[-\frac{\delta}{1-\delta\phi_i}\varepsilon_{i_{t+1}}, -\frac{\delta}{1-\delta\phi_i}\varepsilon_{i^*_{t+1}}\right] = \left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i\sigma_{i^*}, \\ Cov_t[rx^y_{t+1}, rx^q_{t+1}] &= Cov\left[-\frac{\delta}{1-\delta\phi_i}\varepsilon_{i_{t+1}}, \frac{1}{1-\phi_i}\left(\varepsilon_{i^*_{t+1}} - \varepsilon_{i_{t+1}}\right)\right] = \frac{1}{1-\phi_i}\frac{\delta}{1-\delta\phi_i}\sigma^2_i\left(1-\rho\frac{\sigma_{i^*}}{\sigma_i}\right) \\ Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] &= Cov\left[-\frac{\delta}{1-\delta\phi_i}\varepsilon_{i^*_{t+1}}, \frac{1}{1-\phi_i}\left(\varepsilon_{i^*_{t+1}} - \varepsilon_{i_{t+1}}\right)\right] = -\frac{1}{1-\phi_i}\frac{\delta}{1-\delta\phi_i}\sigma^2_{i^*}\left(1-\rho\frac{\sigma_i}{\sigma_{i^*}}\right). \end{aligned}$$

   While we still have $Cov_t[rx^y_{t+1}, rx^{y*}_{t+1}] > 0$ so long as $\rho > 0$, the behavior of $Cov_t[rx^y_{t+1}, rx^q_{t+1}]$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]$ is more complicated. Noting that $\rho\sigma_{i^*}/\sigma_i$ ($\rho\sigma_i/\sigma_{i^*}$) is the coefficient from a regression of i^*_t on i_t (i_t on i^*_t), there are three possible cases:

   - If $1 > \max\{\rho\sigma_{i^*}/\sigma_i, \rho\sigma_i/\sigma_{i^*}\}$—i.e., if the short rates are sufficiently symmetric, then $Cov_t[rx^y_{t+1}, rx^q_{t+1}] > 0$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] < 0$ as in our baseline model.

[3] If there are moderate asymmetries between the domestic and foreign short rates and bond supply processes, then $Cov_t[rx^y_{t+1}, rx^q_{t+1}] \neq -Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]$ but we still have $Cov_t[rx^y_{t+1}, rx^q_{t+1}] > 0$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] < 0$.

- If $\rho\sigma_{i^*}/\sigma_i > 1 > \rho\sigma_i/\sigma_{i^*}$—i.e., if foreign short rates move more than one-for-one with domestic short rates, then $Cov_t[rx^y_{t+1}, rx^q_{t+1}] < 0$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] < 0$.
- If $\rho\sigma_i/\sigma_{i^*} > 1 > \rho\sigma_{i^*}/\sigma_i$—i.e., if domestic short rates move more than one-for-one with foreign short rates, then $Cov_t[rx^y_{t+1}, rx^q_{t+1}] > 0$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] > 0$.

Suppose we take the base currency to be the US dollar as in our baseline empirical analysis. Then the primary concern is that there are countries in the second group whose short rates move more than one-for-one with USD short rates. Here news that USD short rates are set to rise going forward will typically lead such a currency to appreciate versus the dollar—this news suggests that foreign short rates will rise relative to USD short rates—leading to a negative correlation between the returns on this currency and those on long-term USD bonds. And, since these currencies typically appreciate when there is bad news about future USD short rates—i.e., news that USD short rates will rise—the expected returns on these currencies should decline, not rise, when the price of USD short rate risk increases.

The upshot is that this boundary condition argues against an empirical analysis that combines together large developed currencies who independently set their monetary policies with "periphery" currencies whose short rates may move more than one-for-one with U.S. monetary policy. Instead, it makes sense to focus on countries that pursue relatively independent monetary policies.

Data considerations:

1. *Using government bond data*: Our theory speaks to "convenience-free" and default-free rates. Today the best empirical proxies would be the rates of SOFR swaps or OIS swaps. Deep markets in OIS and SOFR swaps are a fairly recent innovation, so we are forced to choose between historical data on sovereign yields (which are closer to default free, but are impacted by convenience premia) and LIBOR swap rates (which are closer to convenience free, but reflect the time-varying credit risk component of LIBOR). Since convenience premia are generally thought to be slower moving than credit spreads, we judged that sovereign bond yields were the better empirical proxy for "convenience-free" and default-free rates. Also, we wanted to rely on off-the-shelf estimates of zero-coupon curves and we have been able to track down a wider cross-section of sovereign zero-coupon curves than LIBOR swap zero-coupon curves.

2. *Using nominal data*: Our theory is best thought of as characterizing real yields and real exchange rates. However, we test our theory using nominal bonds and nominal exchange rates due to two practical data considerations. First and foremost, inflation-indexed bonds are a fairly recent development—e.g., they were only introduced in Japan in 2004 and in Germany in 2007—and are still not issued in large quantities in currencies other than USD and GBP. Indeed, we have only been able to find off-the-shelf estimates of the real zero-coupon curve for the USD and GBP. Second, even in the US and UK, inflation-indexed bonds are far less liquid than nominal bonds and are thought to be impacted by localized "technical" factors—i.e., local price-pressure effects—that lie outside the scope of our model (Campbell, Shiller, and Viceira [2009]). For instance, forced sales of inflation-indexed bond in the U.S. and U.K. at the height of the 2008 Global Financial Crisis temporarily pushed up real yields relative to nominal yields, driving down breakeven inflation (Campbell, Shiller, and Viceira [2009]). From the standpoint of our theory, these localized technical factors that impact real yields are a source of measurement error.

3. *Sample of currencies*: As explained above, we do not expect our theory's predictions to apply to all currencies. Specifically, our theory is best viewed as a description of the exchange rates of

major developed countries that have floating or lightly-managed currencies, that pursue a semi-autonomous monetary policy, and that play an important role in international financial market. Thus, our baseline sample consists of six currencies: AUD, CAD, CHF, EUR, GBP, and JPY. However, we show that our results are robust to focusing solely on the EUR, GBP, and JPY, which arguably play the most significant role in international financial markets.

4. *Sample time period:* We focus our baseline analysis on the 2001–2021 time period. We do so for two main reasons. First, our theory makes predictions about real rates, but data considerations force us to test our theory using data on nominal rates. And, when working with nominal rates, we would expect the patterns predicted by our theory to emerge most strongly during periods when inflation is stable. Intuitively, shocks to nominal inflation simply add noise to our key dependent and independent variables, either reducing our statistical power or biasing our estimates towards zero. This is the first reason why we focus on the 2001-2021 period in our baseline analysis: this was a period when inflation expectations were firmly anchored and where movements in nominal interest rates largely corresponded to movements in real rates. Second, global bond and FX markets have arguably become more tightly integrated in recent dates (e.g., Schulz and Wolff [2008], Mylonidis and Kollias [2010], Ehrmann, Fratzscher, and Rigobon [2011], Pozzi and Wolswijk [2012]). Since our theory hinges on the idea that bond and FX markets are tightly integrated, this consideration also argues in favor of looking at more modern data.

   However, neither consider offers strong justification for beginning the analysis in precisely 2001. And, we do not think there was a structural break in either the stability of inflation expectations or the integration of markets in 2000. Instead, inflation expectations became more firmly moored throughout the 1990s. Similarly, FX and bond market integration likely increased over this same period. Comfortingly, we show that our conclusions are largely unchanged—although the significance is slightly attenuated—if we extend the analysis back to 1994m1, which is when zero-coupon data for all six currencies we examine first becomes available.

A.3 Bandwidth choice for Driscoll-Kraay standard errors

Here we discuss how we choose the bandwidth S in our Driscoll-Kraay (1998) variance estimator. Driscoll-Kraay (1998) is the panel data analog of the Newey-West (1987) heterostacity-and-autocorrelation robust variance estimator. Thus, we frame our econometric discussion of these issues here in a pure time-series setting for simplicity. However, all of the logic here carries through unchanged to the panel data case.

Suppose we have a time series of length T and let S denote the bandwidth used when computing Newey-West (1987) standard errors. The textbook Gaussian asymptotic theory for performing inference with Newey-West (1987) standard errors—a specific instance of heteroskedasticity-and-autocorrelation robust (HAR) standard errors—assumes that $S \to \infty$ and $S/T \to 0$ (Andrews (1991)). If the regression scores $z_t = x_{t-1}u_t$ follow an AR(1), $z_t = \rho_z z_{t-1} + \omega_t$, Andrews (1991) showed that the MSE-minimizing bandwidth choice for a Newey-West variance estimator is

$$S^* = 1.1447 \cdot \left(\frac{2\rho_z}{1-\rho_z^2} \right)^{\frac{2}{3}} \cdot T^{1/3},$$

which evaluates to $S^* = 0.75 \cdot T^{1/3}$ when $\rho_z = 0.25$. As Lazarus, Lewis, Stock, and Watson (LLSW, 2018) note, computing standard errors using this Andrews (1991) bandwidth choice and then using critical values based on a limiting standard normal distribution remains common in practice.

However, as is well known, noise in these Newey-West variance estimator leads the resulting t-statistics to have "fat tails" in finite samples relative to this limiting Gaussian distribution. See, for example, Muller (2014) for a summary. As a result, tests based on critical values from the standard normal distribution tend to result in significant over-rejections of the null in moderately-sized samples—i.e., there are serious size distortions. This problem is analogous to the reason we teach undergraduates to use critical values from the Student's t-distribution instead of the standard normal when they have a small number of observations. And, these size distortions become more severe when the serial correlation in the underlying data is more pronounced and when the bandwidth parameter, S, is large relative to the sample size.

In the years since Andrews (1991) and Newey-West (1994) developed their approaches to bandwidth selection, several authors have noted that the HAR testing problem entails a tradeoff between bias and variance. This contrasts with MSE minimization which entails a tradeoff between *squared* bias and variance. As a result, the testing-optimal bandwidth choice with Newey-West standard errors should be $S^* = s \cdot T^{1/2}$ for some s, increasing faster with T than the MSE-optimal bandwidth which only grows with $T^{1/3}$ (Velasco and Robinson (2001) and Sun, Phillips, and Jin (2008)).

Relatedly, Kiefer and Vogelsang (2005) derived a modified asymptotic theory that has better finite-sample performance than the standard Gaussian theory by assuming that the bandwidth S is a *fixed fraction* b of the sample size: $S = bT$ for some $b \in (0, 1]$. Specifically, letting $Z(r)$ denote a standard Brownian motion (i.e., $Z(r) \sim N(0, r)$) and $\tilde{Z}(r) = Z(r) - rZ(1)$, Kiefer and Vogelsang (2005) show that limiting distribution for a Newey-West t-statistic under these so-called "fixed-b" asymptotics is $t \xrightarrow{d} Z(1) / \sqrt{Q(b)}$, where $Q(b) = \frac{2}{b} \int_0^1 \tilde{Z}(r)^2 \, dr - \frac{2}{b} \int_0^{1-b} \tilde{Z}(r+b) \tilde{Z}(r) \, dr$. Furthermore, they show that $Q(b) \xrightarrow{p} 1$ as $b \to 0$, so fixed-b asymptotics reduce to standard Gaussian asymptotics as $b \to 0$.[4]

Even though econometricians knew to use Kiefer and Vogelsang (2005) standard errors and that optimal bandwidth choice took the form $S^* = s \cdot T^{1/2}$ for some s, until LLSW (2018) there still was no formal guidance econometric on how to choose s. To choose the optimal value of s in $S^* = s \cdot T^{1/2}$, LLSW (2018) proposed minimizing a natural loss function, namely, $Loss = \kappa_z (\Delta_S)^2 + (1 - \kappa_z) (\Delta_P^{\max})^2$, where Δ_S is the size distortion from a test based on the critical values from Kiefer and Vogelsang's (2005) fixed-b distribution and $\Delta_P^{\max}$ is the worst-case power-loss against local alternatives. As LLSW (2018, 2021) explain, there is a size versus power tradeoff here: choosing a larger bandwidth S reduces size-distortions (a good thing) but increases worst-case power-loss (a bad thing).

Assume a desired test size of $\alpha = 5\%$ and that one is using Newey-West standard errors (a HAR variance estimator that uses the Bartlett or triangular kernel) and a weight κ_z on controlling size distortions. Then, LLSW (2018) show that the loss-function minimizing bandwidth S for testing hypotheses about the the mean of a Gaussian AR(1) process with persistence ρ_z is given by $S^* = s^*(\kappa_z, \rho_z) \cdot T^{1/2}$ where

$$s^*(\kappa_z, \rho_z) = 0.453 \cdot \left(\frac{\kappa_z}{1 - \kappa_z} \right)^{\frac{1}{4}} \left(\frac{2\rho_z}{1 - \rho_z^2} \right)^{\frac{1}{2}}.$$

Naturally, $s^*(\kappa_z, \rho_z)$ is increasing in both κ_z and ρ_z, equals zero if either $\kappa_z = 0$ or $\rho_z = 0$, and becomes infinite if either $\kappa_z \to 1$ or $\rho_z \to 1$.

[4] To better see the relationship to Student's t-distribution, LLSW (2018) note that $t \xrightarrow{d} Z(1) / \sqrt{\sum_{j=1}^{M/2} K_j \xi_j}$ where the K_j are a set of fixed weights and the ξ_j are a set of random variables that are drawn independently (of each other and $Z(1)$) from the χ-squared distibution with 2 degrees of freedom (divided by 2)—i.e., $\xi_j \overset{d}{\sim} \chi_2^2/2$ so $E[\xi_j] = 1$. Thus, the term under the radical is a weighted average of variables that are distributed $\chi_2^2/2$, so it resembles the Student's t-distribution. Indeed, as discussed in LLSW (2018, 2021), this weighted average can be approximated as $\sum_{j=1}^{M/2} K_j \xi_j \overset{d}{\approx} \chi_\nu^2/\nu$ where $v = 1/b$. Thus, letting ζ be random variable drawn from the χ_ν^2 distribution, we have $t \xrightarrow{d} Z(1) / \sqrt{\sum_{j=1}^{B/2} K_j \xi_j} \overset{d}{\approx} Z(1) / \sqrt{\zeta/\nu}$, implying that the limiting fixed-b distribution is *similar* to Student's t-distribution with $\nu = 1/b$ degrees of freedom. To be clear, this is only an approximation: the true limiting fixed-b distribution with Newey-West standard errors is non-standard and must be computed using the formulae in Kiefer and Vogelsang (2005).

In deference to classical hypothesis testing, LLSW (2018) argue in favor of placing a large weight on controlling size distortions. Specifically, they set $\kappa_z = 0.9$. Finally, to consider the case of a moderately persistent process, LLSW (2018) set $\rho_z = 0.7$. Plugging into the above formula, this yields $s^*(0.9, 0.7) = 1.3$ as shown in their Table 2. Assuming we want to fix $\kappa_z = 0.9$ but allow ρ_z to vary, the rule then becomes $S^* = s^*(\rho_z) \cdot T^{1/2}$ where

$$s^*(\rho_z) = 0.7846 \cdot \left(\frac{2\rho_z}{1-\rho_z^2}\right)^{\frac{1}{2}}.$$

How should we apply these insights to regressions involving overlapping returns? Consider forecasting H-month returns in overlapping data using a regression of the form

$$\textstyle\sum_{h=1}^{H} r_{t+h} = A + B \cdot x_t + \varepsilon_{t\to t+H}.$$

Under the null that returns are independent of x_t—i.e., that returns are not predicted by x_t, the regression scores for this H-period forecasting regression are given by $z_t = x_t \sum_{j=1}^{H} (r_{t+j} - E[r_t])$. Assuming that the monthly returns are *iid*, the first-order autocorrelation of these scores is

$$\rho_z^{(1)} \equiv \frac{Cov(z_t, z_{t+1})}{Var(z_t)} = \frac{H-1}{H}\rho_x,$$

where ρ_x is the first-order auto-correlation of the forecasting variable x_t. And, the j-order autocorrelation is

$$\rho_z^{(j)} \equiv \frac{Cov(z_t, z_{t+j})}{Var(z_t)} = \frac{\max\{H-j, 0\}}{H}(\rho_x)^j.$$

Indeed, one can show that approximating these scores as following an AR(1) with persistence $\rho_z^{(1)}$ is going to be conservative. Specifically, since $\rho_z^{(j)} < (\rho_z^{(1)})^j$, the autocorrelations will decay faster than would be expected if the scores followed an AR(1) with persistence $\rho_z^{(1)}$.[5]

For the monthly regressions we consider in our paper, we have $\rho_x \approx 0.9$, implying that $\rho_z = (2/3) \cdot 0.9 = 0.6$ for our 3-month forecasting regressions and $\rho_z = (11/12) \cdot 0.9 = 0.825$ for our 12-month forecasting regressions. The forecasting regressions in Tables 3 and 4 have $T = 249$ observations when forecasting 3-month returns and $T = 240$ when forecasting 12-month returns. Plugging these values of ρ_z and T into the LLSW (2018) formula gives

$$S = 0.7846 \cdot \left(\frac{2 \cdot 0.6}{1-0.6^2}\right)^{\frac{1}{2}} \cdot 249^{1/2} = 16.95$$

when forecasting 3-month returns and

$$S = 0.7846 \cdot \left(\frac{2 \cdot 0.825}{1-0.825^2}\right)^{\frac{1}{2}} \cdot 240^{1/2} = 27.63$$

when 12-month forecasting regressions. We round up in both cases and use 17 and 28 lags, respectively.

With $T = 240$ observations and $S = 28$, we have $b = S/T = 0.117$. The resulting Kiefer-Vogelsang

[5] The inequality is trivial for $j \geq H$. For $0 < j < H$, we show that $(H-j)/H < ((H-1)/H)^j$. This is equivalent to $[\ln(H-j) + (j-1)\ln H]/j < \ln(H-1)$ which holds by Jensen's inequality.

(2005) fixed-b critical values for a two-sided t-test are as follows:

p-value	$b = 0.117$	$b = 0$
90%	1.90	1.64
95%	2.30	1.96
99%	3.12	2.58

These fixed-b critical values can be compared to the familiar Gaussian critical values, which corresponds to $b = 0$.

The persistences of the scores ρ_z in Tables 1 and 2 (where we estimate monthly regressions with H-month overlapping returns) are similar to those in Tables 3 and 4 (where we forecast overlapping H-month returns). Thus, we use the same values for ρ_z throughout (0.6 for 3-month changes/returns and 0.825 for 12-month changes/returns). However, we select slightly larger values of S since the regressions in Tables 1 and 2 contain $T = 252$ monthly observations. As a result, we select $S = 18$ when examining 3-month changes and $S = 29$ when examining 12-month changes.

A.4 Robustness for baseline empirical results

In this section, we conduct a battery of robustness checks on our main empirical findings in Tables 1, 2, 3, and 4.

A.4.1 Estimates using non-overlapping data

One might be concerned about our use of overlapping changes and returns in our baseline regression specifications. Fortunately, Appendix Tables A1, A2, A3, and A4 show that our estimates and our inferences are quite similar if we use non-overlapping H-month changes or returns. Specifically, using non-overlapping returns—which sidesteps the inferential issues associated with overlapping returns—has minimal effect on our point estimates. Of course, using non-overlapping returns results in less precise estimates since we are deliberately throwing away some amount of statistically independent data. But, while we loose some power when we inefficiently discard data in this way, our results generally remain significant. Of course, the best strategy is to use all of the data—i.e., to use overlapping returns—and then to get the standard errors right. This is what we do in the main text.

Econometric background: To begin, note that, in the absence of strong seasonalities, the Frisch-Waugh-Lovell theorem suggests that the coefficients in our panel regressions that use overlapping returns should approximately equal the simple average of coefficients from the corresponding *set* of panel regression specifications using non-overlapping returns.

To see the idea, recall that we are estimating panel regressions of the form:

$$y_{c,t} = A_c + B \cdot x_{c,t} + \varepsilon_{c,t},$$

where $y_{c,t} \equiv \sum_{h=1}^{H} r_{c,t+h}$, c indexes countries, and t indexes time.[6] Our regressions include currency fixed effects, A_c, and thus only exploit within-currency time-series variation.

Suppose we group the months in our sample into H buckets that contain non-overlapping data for the dependent variable $y_{c,t} = \sum_{h=1}^{H} r_{c,t+h}$. For instance with $H = 12$ returns, we could run regressions using returns from January to January, February to February, ..., or December to December. So, there are H different estimators that use non-overlapping returns, each corresponding to one of the possible starting points for H-month returns.

[6] For simplicity, we discuss these issues in the context of a univariate forecasting regression, but these same ideas extend naturally to multivariate regressions.

In the absence of strong seasonalities in the data, our panel regressions with currency fixed effects will be *nearly identical* to panel regressions with currency-by-starting-month fixed effects:

$$y_{c,t} = A_{c,h(t)} + B_{\text{mod}} \cdot x_{c,t} + \varepsilon_{c,t}.$$

(In other words, there are now H separate fixed effects for *each* currency, one for each set of possible starting months for H-month returns.) Applying the Frisch-Waugh-Lovell theorem to this modified estimator, we obtain

$$\begin{aligned}
\hat{B} &\approx \hat{B}_{\text{mod}} \\
&= \sum_{h=1}^{H} \overbrace{\frac{\sum_c \sum_{t \in h(t)} (x_{c,t} - \overline{x}_{c,h})^2}{\sum_{k=1}^{H} \sum_c \sum_{t \in k(t)} (x_{c,t} - \overline{x}_{c,k})^2}}^{w_h^{\textit{no-overlap}}} \overbrace{\frac{\sum_c \sum_{t \in h(t)} (x_{c,t} - \overline{x}_{c,h})(y_{c,t} - \overline{y}_{c,h})}{\sum_c \sum_{t \in h(t)} (x_{c,t} - \overline{x}_{c,h})^2}}^{\hat{B}_h^{\textit{no-overlap}}} \\
&= \sum_{h=1}^{H} w_h^{\textit{no-overlap}} \hat{B}_h^{\textit{no-overlap}}
\end{aligned}$$

where $\hat{B}_h^{\textit{no-overlap}}$ is the panel data estimator for $y_{c,t} = A_c + B \cdot x_{c,t} + \varepsilon_{c,t}$ that only makes use of data beginning in h-months.[7]

Again, absent strong seasonalities in the data, we would expect to have $w_h^{\textit{no-overlap}} = 1/H$ and thus

$$\hat{B} \approx \hat{B}_{\text{mod}} \approx H^{-1} \sum_{h=1}^{H} \hat{B}_h^{\textit{no-overlap}}.$$

In other words, our panel data estimator will approximately equal the simple average of the H different panel data estimators that use non-overlapping H-month returns. As we show below, this approximation is extremely accurate for our panel data estimator because seasonalities are negligible.

So, why then do we use overlapping returns? So, long as the estimators using non-overlapping returns are imperfectly correlated, there will be an efficiency gain from pooling them—i.e., from working with overlapping returns. And, this efficiency gain becomes larger when these correlations are smaller. Specifically, so long as $Corr(\hat{B}_h^{\textit{no-overlap}}, \hat{B}_k^{\textit{no-overlap}}) < 1$ for some pair of estimators, we will have

$$\begin{aligned}
\sqrt{Var[\hat{B}]} &\approx \sqrt{H^{-2} \sum_{h,k} Corr(\hat{B}_h^{\textit{no-overlap}}, \hat{B}_k^{\textit{no-overlap}}) \sqrt{Var[\hat{B}_h^{\textit{no-overlap}}] Var[\hat{B}_k^{\textit{no-overlap}}]}} \\
&< H^{-1} \sum_{h=1}^{H} \sqrt{Var[\hat{B}_h^{\textit{no-overlap}}]}.
\end{aligned}$$

Of course, pooling these non-overlapping estimators then puts the onus on us to properly estimate these correlations—i.e., $Corr(\widehat{\beta_h^{\textit{no-overlap}}}, \widehat{\beta_k^{\textit{no-overlap}}})$—but that is the whole point of using Driscoll-Kraay (1998) standard errors. This efficiency argument is precisely why, like us, researchers often work with over-lapping data and then strive to get the standard errors right.

Results: Here we show that our estimates and our inferences are quite similar if we simply use non-overlapping H-month changes and returns. There are four tables that use non-overlapping data, corresponding to Tables 1 to 4 in the paper:

Table A1	$\Delta_H q_{c,t} = A_c + B \cdot \Delta_H \left(i_{c,t}^* - i_t\right) + D \cdot \Delta_H \left(f_{c,t}^* - f_t^*\right) + \Delta_H \varepsilon_{c,t},$
Table A2	$\Delta_H q_{c,t} = A_c + B \cdot \Delta_H \left(i_{c,t}^* - i_t\right) + D \cdot \Delta_H \left(f_{c,t}^* - f_t^*\right) + \Delta_H \varepsilon_{c,t},$
Table A3	$rx_{c,t \to t+H}^{y*} - rx_{t \to t+H}^{y} = A_c + B \cdot \left(i_{c,t}^* - i_t\right) + D \cdot \left(f_{c,t}^* - f_t^*\right) + \varepsilon_{c,t \to t+H},$
Table A4	$rx_{c,t \to t+H}^{q} = A_c + B \cdot \left(i_{c,t}^* - i_t\right) + D \cdot \left(f_{c,t}^* - f_t\right) + \varepsilon_{c,t \to t+H}.$

[7] A quick note on the notation. For instance, for $h = \textit{December}$, $h(t)$ is the set of December observations in the sample. For $c =$ euro and $h = \textit{December}$, $\overline{x}_{c,h}$ is the average value of $x_{c,t}$ for the euro in December, $\overline{y}_{c,h}$ is the average value of $y_{c,t}$ for the euro in December, etc.

Panel A of each table shows the results for H = 3-month returns and Panel B shows the results for H = 12-month returns. In each panel, column (1) reports our estimates using overlapping H-month changes/returns. For these overlapping estimators, we report Driscoll-Kraay (1998) standard errors with Kiefer-Vogelsang (2005) p-values as in the main text. We report the slopes from the H underlying panel estimators that only use non-overlapping data. There are three such estimators for 3-month returns and 12 such estimators for 12-month returns. For these non-overlapping estimators, we compute standard errors that cluster by time to account for any contemporaneous correlation across currencies. Finally, in column (2) of each table, we report the (equal-weighted) average coefficient estimate across these H non-overlapping estimators as well as the average (estimated) standard error.

As can be seen by comparing columns (1) and (2) of each table, the average coefficient estimate across these H separate non-overlapping estimators is almost identical to our baseline estimator that uses overlapping returns. However, the average of the standard errors across these H estimators is larger than the Driscoll-Kraay standard error from our baseline estimates. This larger average standard error arises due to the efficiency loss from the fact that the non-overlapping estimators discard some amount of statistically independent data.

Even though they are each throwing away some informative data, many or most of the non-overlapping estimators are individually significant. And, our results generally remain significant if we use the average standard error from these H estimators to assess the significance of the average coefficient from the H estimators. Obviously, this procedure is extremely conservative and lacks any formal econometric justification.

In summary, the results in Tables A1, A2, A3, A4 give us considerable comfort that we are drawing appropriate inferences when using overlapping data.

A.4.2 Country-by-country results

In our baseline analysis in Tables 1, 2, 3, and 4, we obtain additional statistical power by pooling data across currencies in our panel—i.e., our panel data estimator is a weighted average of currency-level time-series estimators. While this is a standard econometric practice in empirical asset pricing and in the FX literature more specifically, it is natural to examine the results for each of the six currencies in our sample separately.

Tables A5, A6, A7, and A8 report the results for each country separately. Broadly speaking, our results are strong for AUD, CHF, GBP, EUR, and JPY when considered in isolation. However, our results for the CAD often have the opposite signs of those suggested by our model.

There are several potential explanations for the CAD results. First, the correlation between CAD and USD short rates is very high (e.g., the correlation between monthly changes in CAD and USD short rates is $\widehat{\rho}_{CAD} = 0.73$), which should attenuate the magnitude of the expected relationship between foreign exchange rates and term premium differentials in our theory. However, a high short-rate correlation alone should not lead the relevant coefficients to change sign. Second, if investors believe that the β of CAD short rates with respect to USD short rates exceeds 1, we would actually expect sign flipping of the sort we observe for the CAD. (See the discussion in Section A.5.1.) However, in our sample, this empirical β is high (we have $\widehat{\beta}_{CAD} \approx \widehat{\rho}_{CAD} = 0.73$), but still well below 1. Finally, the outlying results for the CAD could either stem from some unknown set of forces that lie outside of our model or could simply reflect chance sampling variation.

A.4.3 Panel results for major currencies only

Our theory is best viewed as a description of the exchange rates of major developed countries that have floating or lightly-managed currencies, that pursue a semi-autonomous monetary policy, and that play an important role in international financial market. Arguably, the EUR, GBP, and JPY are the three currencies that, alongside the USD, best fit the bill. Comfortingly, Tables A9, A10, A11, and A12 show that we obtain broadly similar results if we restrict our sample to the EUR, GBP, and JPY.

A.4.4 Varying the base currency

In light of the growing literature that emphasizes the special role of the U.S. dollar in international financial markets, it is natural to ask whether our results are driven by the decision to use the USD the base currency. The short answer is that our results are not driven by this choice. Tables A13, A14, A15, and A16 show that we obtain broadly similar results in Tables 1, 2, 3, and 4 if, instead of using the USD as the base currency, we use other currencies as the base. Specifically, the results are similar if we use AUD, CHF, EUR, GBP, or JPY as the base currency. However, the results are weaker if we use the CAD as the base currency. Overall, while we are convinced by the growing literature showing that the USD plays a special role in international financial markets, our main empirical results do not appear to derive from USD specialness.

A.4.5 Results for different time periods

As explained above, we focus on more recent data for two main reasons. First, our theory makes predictions about real rates, but data considerations force us to test our theory using data on nominal rates, which effectively adds measurement error. Thus, when working with nominal rates, we would expect the patterns predicted by our theory to emerge most strongly during periods when inflation is stable—i.e., when the resulting measurement error is small. Second, our theory hinges on the idea that bond and FX markets are tightly integrated and these markets have arguably become more integrated in recent decades. Motivated by these two considerations, the baseline sample for Tables 1, 2, 3, and 4 runs from 2001 to 2021.

However, neither consideration offers a strong justification for beginning the analysis in precisely 2001: we do not think there was a structural break in either the stability of inflation expectations or the integration of markets in 2000. Comfortingly, Tables A17, A18, A19, and A20 show that our results hold over the longer 1994 to 2021 sample. This is as far back as we have zero-coupon yields for all currencies in our sample. To be sure, if we were to extend our sample back to the 1980s or 1970s—which is possible for some of the currencies we consider—our results become weaker. However, are not overly concerned by this fact because our theory's boundary conditions suggest that its predictions should emerge less strongly during these earlier decades, especially when working with nominal data.

One might also wonder whether our results are somehow specific to the post-Global Financial Crisis period. The short answer is no. Specifically, Tables A21 and A22 show that our results generally hold in both the 2001-2007 period as well as the 2008-2021 period. If anything, the results are somewhat stronger during the 2001-2007 subsample. Of course, this needs to be taken with a healthy grain of salt since we are now working with fairly short subsamples. We simply lack the power to draw firms conclusions when slicing the data so thinly.

A.5 Additional empirical results

A.5.1 Supply shocks and short rate correlation

As discussed in the main text, our model implies bond supply shocks should have a larger impact on the bilateral exchange rate when the correlation between the two countries' short rates is lower. Specifically, we have

$$\frac{\partial E_t[rx^q_{t+1}]}{\partial s^y_t} = \tau^{-1} C_{y,q}$$

where $C_{y,q} \equiv Cov_t[rx^y_{t+1}, rx^q_{t+1}]$. And, we can show that

$$\frac{\partial^2 E_t[rx^q_{t+1}]}{\partial s^y_t \partial \rho} = \tau^{-1} \frac{\partial C_{y,q}}{\partial \rho} < 0,$$

where $\rho = Corr[\varepsilon_{i_{t+1}}, \varepsilon_{i^*_{t+1}}]$ is the correlation between domestic and foreign short rate shocks. For instance, in limiting cases where there are no supply shocks ($\sigma^2_{s^y} = \sigma^2_{s^q} = 0$), we have

$$C_{y,q} = (1-\rho) \frac{\delta}{1-\delta\phi_i} \frac{1}{1-\phi_i} \sigma^2_i > 0,$$

implying that[8]

$$\frac{\partial^2 E_t[rx^q_{t+1}]}{\partial s^y_t \partial \rho} = \tau^{-1} \frac{\partial C_{y,q}}{\partial \rho} = -\tau^{-1} \frac{\delta}{1-\delta\phi_i} \frac{1}{1-\phi_i} \sigma^2_i < 0.$$

Unfortunately, taking this direct prediction on the impact of bond supply shocks to the data is not straightforward because, other than on QE dates, we do not have a clean proxy for bond supply shocks. And, based on QE dates alone, we do not have enough power to detect this interaction effect.

That said, our theory suggests a less direct test of this same idea. Specifically, if there are bond supply shocks but no FX supply shocks—i.e., $\sigma^2_{s^y} > 0$ and $\sigma^2_{s^q} = 0$, our theory implies that

$$E_t\left[rx^q_{t+1}\right] = \overbrace{\left[-\frac{C_{y,q}}{V_y - C_{y,y^*}}\right]}^{<0} \cdot E_t\left[rx^{y*}_{t+1} - rx^y_{t+1}\right].$$

Moreover, we can show that the negative term in square brackets is increasing in ρ, implying that:

$$\frac{\partial^2 E_t\left[rx^q_{t+1}\right]}{\partial E_t\left[rx^{y*}_{t+1} - rx^y_{t+1}\right] \partial \rho} > 0.$$

To test this prediction, we add interaction terms involving the short-rate correlation to our specifications from Tables 2 and 4. Specifically, we estimate panel regressions of the form

$$\begin{aligned} \Delta_{12} q_{c,t} &= A_c + B_1 \cdot \Delta_{12}\left(i^*_{c,t} - i_{c,t}\right) + B_2 \cdot \hat{\rho}_c \times \Delta_{12}\left(i^*_{c,t} - i_{c,t}\right) \\ &\quad + D_1 \cdot \Delta_{12}\left(f^*_{c,t} - f_{c,t}\right) + D_2 \cdot \hat{\rho}_c \times \Delta_{12}\left(f^*_{c,t} - f_{c,t}\right) + \Delta_{12}\varepsilon_{c,t}, \end{aligned}$$

and

$$\begin{aligned} rx^q_{c,t\to t+12} &= A_c + B_1 \cdot \left(i^*_{c,t} - i_{c,t}\right) + B_2 \cdot \hat{\rho}_c \times \left(i^*_{c,t} - i_{c,t}\right) \\ &\quad + D_1 \cdot \left(f^*_{c,t} - f_{c,t}\right) + D_2 \cdot \hat{\rho}_c \times \left(f^*_{c,t} - f_{c,t}\right) + \varepsilon_{c,t\to t+12}. \end{aligned}$$

Our main interest is on D_2: the coefficient on the interaction between the short-rate correlation ($\hat{\rho}_c$) and the change/level of the difference in distant forwards, which is our proxy the change/level of $E_t\left[rx^{y*}_{t+1} - rx^y_{t+1}\right]$. In the first regression relating contemporaneous changes in FX rates to contemporaneous changes in interest rates, we expect $D_1 > 0$ and thus predict that $D_2 < 0$. In the FX return forecasting regression, we expect $D_1 < 0$ and thus that predict $D_2 > 0$. To operationalize this test, we estimate ρ_c using the sample correlation between monthly changes in USD short rates and monthly

[8]In this limit, a supply shock is a "MIT shock"—i.e., a one-off shock that investors think is impossible. However, it is still the case that $\partial C_{y,q}/\partial \rho < 0$ when $\sigma^2_{s^y} > 0$ and $\sigma^2_{s^q} > 0$.

changes in the short rate in currency c. Doing so, we obtain the following estimates of ρ_c:

Currency:	AUD	CAD	CHF	EUR	GBP	JPY
$\widehat{\rho}_c$	0.40	0.73	0.49	0.52	0.58	0.29

The rank ordering across currencies strikes us as intuitive.

The results of this exercise are shown below in Table A23. The results go in the direction predicted by our theory and are significant at the 10% level in both specifications. Intuitively, the coefficient D_2 on this interaction term is roughly equivalent to (1) first running a set of country-level time-series regressions and (2) then running a cross-sectional regression where the estimated currency-specific slopes are regressed on $\hat{\rho}_c$. This interaction term goes in the predicted direction in our panel because the results for the CAD, which has the highest short-rate correlation with the USD, look so different than those for the other currencies. That said, this result is not entirely robust—e.g., it vanishes if we drop the CAD from the sample—and is effectively being identified from a cross-section of just six observations.

B Additional theoretical results for the baseline model

B.1 Campbell-Shiller approximation of the return on a perpetuity

In this section, we derive the Campbell-Shiller (1988) log-linear approximation to the return on default-free long-term bonds that we use throughout the paper. We assume that long-term bonds are default-free, self-amortizing perpetuities with geometrically-declining payments. Specifically, consider a self-amortizing perpetuity that has a face value of 1 at time t and a coupon rate of $C > 0$. We assume that a holder of this perpetuity at time t receives (1) a coupon payment of C at $t+1$, (2) a principal repayment of $(1-\kappa)$ at $t+1$ for some $\kappa \in [0,1]$, and (3) and κ units of this self-amortizing perpetuity (which has a face value of 1) at $t+1$. Thus, κ controls the amortization rate of long-term bonds and hence their duration—i.e., the sensitivity of price to yield-to-maturity.

Let P_t^y denote the price and Y_t the yield-to-maturity of long-term bonds at time t. By the definition of yield-to-maturity—i.e., the constant return that equates bond price and the discounted value of promised cashflows, bond prices and yields are linked via the following formula:

$$P_t^y = \sum_{j=1}^{\infty} \frac{\kappa^{j-1}(1-\kappa+C)}{(1+Y_t)^j} = \frac{1+C-\kappa}{1+Y_t-\kappa}. \tag{3}$$

When $\kappa = 0$, this is the formula for the price of a 1-period bond. When $\kappa = 1$, this is the standard perpetuity formula. The Macaulay duration—i.e., the elasticity of price with respect to yield—of bonds at time t is

$$D_{\text{mac},t} \equiv -\frac{\partial P_t^y}{\partial Y_t}\frac{1+Y_t}{P_t^y} = \frac{1+Y_t}{1+Y_t-\kappa}. \tag{4}$$

Holding Y_t fixed, $D_{\text{mac},t}$ is increasing in the amortization rate κ. At one extreme, when $\kappa = 0$, $D_{\text{mac},t} = 1$ (the duration of a 1-period bond). At the other extreme, when $\kappa = 1$, $D_{\text{mac},t} = (1+Y_t)/Y_t > 1$ (the duration of a standard perpetuity).

The gross return on long-term bonds from time t to $t+1$ is

$$1 + R_{t+1}^y = \frac{C+1-\kappa+\kappa P_{t+1}^y}{P_t^y}. \tag{5}$$

To generate a tractable linear model, we use a Campbell-Shiller (1988) log-linear approximation to the return. Specifically, defining

$$\delta \equiv \frac{\kappa}{1+C} < 1, \tag{6}$$

the one-period log return on the long-term bond is approximately

$$r_{t+1}^{y} \equiv \ln\left(1+R_{t+1}^{y}\right) \approx \overbrace{\frac{1}{1-\delta}}^{D} y_t - \overbrace{\frac{\delta}{1-\delta}}^{D-1} y_{t+1} = y_t - \frac{\delta}{1-\delta}\left(y_{t+1}-y_t\right), \tag{7}$$

where $y_t \equiv \ln(1+Y_t)$ is the log yield-to-maturity on the long-term bond at time t and where

$$D \equiv \frac{1}{1-\delta} = \frac{1+C}{1+C-\kappa} \tag{8}$$

is the Macaulay duration when the bond is trading at par (i.e., when $Y_t = C$ and $P_t^y = 1$).[9]

To derive this approximation, we take a log-linear approximation of the log return about the point where the bond is trading at par at $t+1$ (i.e., where $P_{t+1}^y = 1$ and thus $p_{t+1}^y = \ln\left(P_{t+1}^y\right) = 0$) and obtain

$$r_{t+1}^{y} = \ln\left(1-\kappa+C+\kappa\exp\left(p_{t+1}^{y}\right)\right) - p_t^y \approx \theta + \delta p_{t+1}^y - p_t^y, \tag{9}$$

where $\theta \equiv \ln(1+C)$ and $\delta \equiv \kappa/(1+C)$. are parameters of the log-linearization. Iterating equation (9) forward, the log bond price is approximately

$$p_t^y \approx (1-\delta)^{-1}\theta - \textstyle\sum_{i=0}^{\infty}\delta^i E_t\left[r_{t+i+1}^y\right].$$

Applying this approximation to the yield-to-maturity (again, the constant return that equates price and the discounted value of promised cashflows), we obtain

$$p_t^y \approx (1-\delta)^{-1}\theta - (1-\delta)^{-1} y_t. \tag{10}$$

Equation (7) then follows by substituting the approximation for p_t^y in equation (10) into the Campbell-Shiller return approximation in equation (9).

Equation (10) also shows that the duration of this long-term bond is approximately $D = (1-\delta)^{-1} = (1+C)/(1+C-\kappa)$ which corresponds to Macaulay duration when the bond is trading at par. Specifically, when $P_t^y = 1$, we have $Y_t = C$ and thus $D_{\text{mac},t} = (1+C)/(1+C-\kappa)$.

Finally, note that, while κ is a free parameter that controls the bond's amortization rate and hence its duration, the coupon rate C cannot be regarded as a free parameter. Instead, based on the logic of the log-linearization, C must be chosen so that the perpetuity is priced at par in the steady state. Specifically, we need to set $\theta = \ln(1+C) = E\left[\ln\left(1+R_{t+1}^y\right)\right] = E\left[r_{t+1}^y\right]$, so that $E\left[p_t^y\right] = E\left[\ln\left(P_t^y\right)\right] = 0$.

B.2 Real versus nominal rates

In this section, we show how to generalize our model to allow for both inflation-indexed (real) bonds and non-indexed (nominal) bonds. To do so, we add two sets of pure nominal shocks: persistent shocks to expected future nominal inflation and transitory shocks to realized inflation.

The key insight here is that, as in any textbook theory of uncovered interest rate parity (see,

[9] This log-linear approximation for default-free coupon-bearing bonds appears in Campbell, Lo, and MacKinlay (1997) and Campbell (2018). As explained in Campbell (2018), equation (7) is an approximate generalization of the fact that the log-return on n-period zero-coupon bonds is $r_{t+1}^{(n)} = ny_t^{(n)} - (n-1)\,y_{t+1}^{(n-1)}$ where, for instance, $y_t^{(n)}$ is the log yield on n-period zero-coupon bonds at t.

e.g., Chapter 16 of Krugman, Obstfeld, and Melitz's International Economics textbook), the real foreign exchange rate only moves unexpectedly due to shocks to the expected path of future real short rate differentials. Shocks to expected future inflation differentials—which by construction have zero impact on the real exchange rate, but have a major impact on long-term nominal bond yields—impact the expected drift of nominal exchange rates but are not associated with *unexpected* movements in nominal exchange rates. By contrast, transitory shocks to realized inflation have no impact on long-term nominal bonds yields, but have a one-for-one impact on nominal exchange rates.

The takeaway is that our theory makes predictions about real rates, but data considerations force us to test our theory using data on nominal rates, which effectively adds measurement error. Thus, when working with nominal rates, we would expect the patterns predicted by our theory to emerge most strongly during periods when inflation is stable—i.e., when the resulting measurement error is small.

Setting: Let i_t and i_t^* denote the short-term nominal rates between t and $t+1$. Let $\widehat{r}_t$ and $\widehat{r}_t^*$ denote the short-term *ex ante* real interest rates between t and $t+1$. Expected nominal inflation between t and $t+1$—which is known at time t—is given by $\widehat{\pi}_t$ and $\widehat{\pi}_t^*$. Finally, we let P_t and P_t^* equal the nominal prices levels of goods in domestic and foreign currency, respectively. We assume that realized inflation between t and $t+1$ equals expected inflation plus a transitory price shock

$$\begin{aligned}\pi_{t+1} &\equiv \log\left(P_{t+1}/P_t\right) = p_{t+1} - p_t = \widehat{\pi}_t + \varepsilon_{p,t+1} \\ \pi_{t+1}^* &\equiv \log\left(P_{t+1}^*/P_t^*\right) = p_{t+1}^* - p_t^* = \widehat{\pi}_t^* + \varepsilon_{p^*,t+1},\end{aligned}$$

where $\varepsilon_{p,t+1}$ and $\varepsilon_{p^*,t+1}$ are the transitory price shocks at $t+1$.

We assume that short-term ex ante real rates are stationary, following symmetric AR(1) processes

$$\begin{aligned}\widehat{r}_{t+1} &= \overline{r} + \phi_{\widehat{r}}(\widehat{r}_t - \overline{r}) + \varepsilon_{\widehat{r}_{t+1}} \\ \widehat{r}_{t+1}^* &= \overline{r} + \phi_{\widehat{r}}(\widehat{r}_t^* - \overline{r}) + \varepsilon_{\widehat{r}_{t+1}^*}\end{aligned}$$

where $\phi_{\widehat{r}} \in (0,1)$. We assume that expected one-period ahead inflation follows

$$\begin{aligned}\widehat{\pi}_{t+1} &= \overline{\pi} + \phi_{\widehat{\pi}}(\widehat{\pi}_t - \overline{\pi}) + \varepsilon_{\widehat{\pi}_{t+1}} \\ \widehat{\pi}_{t+1}^* &= \overline{\pi}^* + \phi_{\widehat{\pi}}(\widehat{\pi}_t^* - \overline{\pi}^*) + \varepsilon_{\widehat{\pi}_{t+1}^*},\end{aligned}$$

where $\phi_{\widehat{\pi}} \in (0,1]$ and $\varepsilon_{\widehat{\pi}_{t+1}}$ and $\varepsilon_{\widehat{\pi}_{t+1}^*}$ are the shocks to expected 1-period ahead inflation at $t+1$. Thus, we can allow inflation to follow a random walk—i.e., we can consider the case where $\phi_{\widehat{\pi}} = 1$. And, we can allow the average long-run rate of domestic price inflation $\overline{\pi}$ to differ from the long-run rate of foreign inflation $\overline{\pi}^*$. Finally, we assume that

$$\begin{aligned}i_t &= \widehat{r}_t + \widehat{\pi}_t \\ i_t^* &= \widehat{r}_t^* + \widehat{\pi}_t^*.\end{aligned}$$

Thus, for simplicity, we assume we do not need to worry about pinning down an endogenous inflation risk premium on short-term nominal bonds and can take both short-term nominal and real rates as being exogenously given. It is easy to relax this assumption, but it adds little to the analysis other than complexity.[10]

[10] In principle, we should assume that $i_t = \widehat{r}_t + \widehat{\pi}_t + rp_t^p$ and $i_t^* = \widehat{r}_t^* + \widehat{\pi}_t^* + rp_t^{p*}$ where rp_t^i and rp_t^{i*} are endogenous risk premia that compensate investors for the fact that short-term nominal bonds are risky in real terms, earning lower real returns when inflation is unexpected high between t and $t+1$. For instance, the ex post real returns on domestic short-term non-indexed bonds between t and $t+1$ is $r_{t+1}^i \equiv i_t - \pi_{t+1} = (\widehat{r}_t + \widehat{\pi}_t + rp_t^p) - (\widehat{\pi}_t + \varepsilon_{p,t+1}) = \widehat{r}_t + rp_t^p - \varepsilon_{p,t+1}$.

Furthermore, we can allow the set of shocks $\boldsymbol{\varepsilon}_{t+1} \equiv (\varepsilon_{\widehat{r}_{t+1}}, \varepsilon_{\widehat{r}^*_{t+1}}, \varepsilon_{\widehat{\pi}_{t+1}}, \varepsilon_{\widehat{\pi}^*_{t+1}}, \varepsilon_{p,t+1}, \varepsilon_{p^*,t+1})'$ to be correlated with each other. For instance, if bad news about current inflation is associated with higher future inflation expectations, then $Cov(\varepsilon_{p_{t+1}}, \varepsilon_{\widehat{\pi}_{t+1}}) > 0$. If the domestic central bank follows a forward-looking Taylor rule with a coefficient exceeding one on expected inflation, then $Cov(\varepsilon_{\widehat{r}_{t+1}}, \varepsilon_{\widehat{\pi}_{t+1}}) > 0$. And, these real rate and inflation shocks in the domestic country can be correlated with those in the foreign country.

Inflation-indexed (real) and non-indexed (nominal) bonds: We first introduce inflation-indexed (real) and non-indexed (nominal) long-term and short-term bonds in both currencies. Short-term domestic nominal bonds earn a nominal return of i_t between t and $t+1$ and short-term domestic real bonds earn a real return of $\widehat{r}_t$ between t and $t+1$.[11] The corresponding rates on short-term foreign bonds are i^*_t and $\widehat{r}^*_t$.

Let y^r_t and y^{r*}_t denote the real yields on indexed bonds in domestic and foreign currency, respectively. Similarly, let y^i_t and y^{i*}_t the nominal yields on non-indexed bonds in domestic and foreign currency. We assume that all long-term bonds have the same duration of $1/(1-\delta)$ where $\delta \in (0,1)$. The excess nominal return on long-term non-indexed bonds over short-term non-indexed bonds is

$$rx^{y^i}_{t+1} = \left(y^i_t - i_t\right) - \frac{\delta}{1-\delta}\left(y^i_{t+1} - y^i_t\right),$$

and the excess real return on long-term indexed bonds over short-term indexed bonds is

$$rx^{y^r}_{t+1} = \left(y^r_t - \widehat{r}_t\right) - \frac{\delta}{1-\delta}\left(y^r_{t+1} - y^r_t\right).$$

Iterating these identities forward for bonds in both currencies, we have

$$\begin{aligned} y^i_t &= (1-\delta)\textstyle\sum_{j=0}^{\infty}\delta^j E_t[i_{t+j} + rx^{y^i}_{t+j+1}], \\ y^r_t &= (1-\delta)\textstyle\sum_{j=0}^{\infty}\delta^j E_t[\widehat{r}_{t+j} + rx^{y^r}_{t+j+1}], \\ y^{i*}_t &= (1-\delta)\textstyle\sum_{j=0}^{\infty}\delta^j E_t[i^*_{t+j} + rx^{y^{i*}}_{t+j+1}], \\ y^{r*}_t &= (1-\delta)\textstyle\sum_{j=0}^{\infty}\delta^j E_t[\widehat{r}^*_{t+j} + rx^{y^{r*}}_{t+j+1}] \end{aligned}$$

Thus, long-term indexed (real) bonds reflect the expected future path of future short-term real rates plus a real term premium. And, long-term non-indexed (nominal) bonds reflect the expected future path of future short-term nominal rates plus a nominal term premium.[12]

Thus, assuming that the central bank has perfect control over the ex ante short-term real rate and that it can be taken as exogenously given, we still need to clear the market for short-term nominal bonds: at the margin, investors must be indifferent between holding short-term real and nominal bonds. (An alternate and perhaps more realistic assumption would be that the central bank has perfect control over the short-term nominal rate, i_t. In this case, we would assume that nominal rates (i_t) and expected inflation ($\widehat{\pi}_t$) are both exogenously given. However, the equilibrium rate on short-term real bonds would have to be pinned down endogenously and would equal $\widehat{r}_t = i_t - (\widehat{\pi}_t + rp^p_t)$.) Assuming investors require a positive risk premium for bearing this inflation risk, this means that $rp^p_t = i_t - (\widehat{r}_t + \widehat{\pi}) > 0$ and $rp^{p*}_t = i^*_t - (\widehat{r}^*_t + \widehat{\pi}^*_t) > 0$. In practice, the inflation risk premia on short-term nominal bonds are thought to be de minimus, so we proceed under the simplifying assumption that $i_t = \widehat{r}_t + \widehat{\pi}_t$ where $\widehat{r}_t$ and $\widehat{\pi}_t$ are both endogenously given.

[11] The distinction between indexed and non-indexed short-term bonds arises from the fact that inflation is not perfectly known one-period in advance. If inflation is known perfectly one-period in advance—i.e., if $Var\left[\varepsilon_{p,t+1}\right] = 0$—then there is no distinction between indexed and non-index short term bonds. Both must pay the same nominal interest rate and will earn the same real return. However, with unexpected shocks to the price level, the nominal payoff of short-term domestic real bonds is $\widehat{r}_t + \widehat{\pi}_t + \varepsilon_{p,t+1} = i_t + \varepsilon_{p,t+1}$.

[12] The real excess return on long-term indexed bonds over short-term non-indexed bonds equals $rx^{y^r}_{t+1} + \varepsilon_{p,t+1}$ since long-term indexed bonds are protected against unexpected increased in the nominal price level ($\varepsilon_{p,t+1}$) but short-term

For simplicity, we first consider a steady state where long-term bond supply is constant and where is there is no supply of foreign exchange. In this case simple case, we have

$$
\begin{aligned}
y_t^i &= (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[i_{t+j}] + E[rx_{t+1}^{y^i}] = \overline{r} + \overline{\pi} + E[rx_{t+1}^{y^i}] + \frac{1-\delta}{1-\delta\phi_{\widehat{r}}}(\widehat{r}_t - \overline{r}) + \frac{1-\delta}{1-\delta\phi_{\widehat{\pi}}}(\widehat{\pi}_t - \overline{\pi}), \\
y_t^r &= (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[\widehat{r}_{t+j}] + E[rx_{t+1}^{y^{ir}}] = \overline{r} + E[rx_{t+1}^{y^r}] + \frac{1-\delta}{1-\delta\phi_{\widehat{r}}}(\widehat{r}_t - \overline{r}), \\
y_t^{i*} &= (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[i_{t+j}^*] + E[rx_{t+1}^{y^{i*}}] = \overline{r} + \overline{\pi}^* + E[rx_{t+1}^{y^{i*}}] + \frac{1-\delta}{1-\delta\phi_{\widehat{r}}}(\widehat{r}_t^* - \overline{r}) + \frac{1-\delta}{1-\delta\phi_{\widehat{\pi}}}(\widehat{\pi}_t^* - \overline{\pi}^*), \\
y_t^{r*} &= (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[\widehat{r}_{t+j}^*] + E[rx_{t+1}^{y^{r*}}] = \overline{r} + E[rx_{t+1}^{y^{r*}}] + \frac{1-\delta}{1-\delta\phi_{\widehat{r}}}(\widehat{r}_t^* - \overline{r}).
\end{aligned}
$$

As a result, the unexpected returns on domestic nominal long-term bonds, domestic real long-term bonds, foreign nominal long-term bonds, and foreign real long-term bonds are given by

$$
\begin{aligned}
rx_{t+1}^{y^i} - E[rx_{t+1}^{y^i}] &= -\frac{\delta}{1-\delta\phi_{\widehat{r}}}\varepsilon_{\widehat{r},t+1} - \frac{\delta}{1-\delta\phi_{\widehat{\pi}}}\varepsilon_{\widehat{\pi},t+1} \\
rx_{t+1}^{y^r} - E[rx_{t+1}^{y^r}] &= -\frac{\delta}{1-\delta\phi_{\widehat{r}}}\varepsilon_{\widehat{r},t+1} \\
rx_{t+1}^{y^{i*}} - E[rx_{t+1}^{y^{i*}}] &= -\frac{\delta}{1-\delta\phi_{\widehat{r}}}\varepsilon_{\widehat{r}*,t+1} - \frac{\delta}{1-\delta\phi_{\widehat{\pi}}}\varepsilon_{\widehat{\pi}*,t+1} \\
rx_{t+1}^{y^{r*}} - E[rx_{t+1}^{y^{r*}}] &= -\frac{\delta}{1-\delta\phi_{\widehat{r}}}\varepsilon_{\widehat{r}*,t+1}.
\end{aligned}
$$

In words, the unexpected returns on long-term nominal bonds reflect shocks to the expected path of future nominal rates and, hence, are exposed to news about future inflation and future short-term real rates. By contrast, the unexpected returns on long-term real bonds only reflect shocks to the expected path of future real rates and, hence, are not exposed to news about future inflation.

It is easy to extend all of this logic to allow for time-varying supply and hence time-varying risk-premia—i.e., discount rate news. Once we allow for supply shocks unexpected returns are given by

$$
\begin{aligned}
rx_{t+1}^{y^i} - E_t[rx_{t+1}^{y^i}] &= -\frac{\delta}{1-\delta\phi_r}\varepsilon_{\widehat{r},t+1} - \frac{\delta}{1-\delta\phi_\pi}\varepsilon_{\widehat{\pi},t+1} + N\text{-}DR_{t+1}^{y^i} \\
rx_{t+1}^{y^r} - E_t[rx_{t+1}^{y^r}] &= -\frac{\delta}{1-\delta\phi_r}\varepsilon_{\widehat{r},t+1} + N\text{-}DR_{t+1}^{y^r} \\
rx_{t+1}^{y^{i*}} - E_t[rx_{t+1}^{y^{i*}}] &= -\frac{\delta}{1-\delta\phi_r}\varepsilon_{\widehat{r}*,t+1} - \frac{\delta}{1-\delta\phi_\pi}\varepsilon_{\widehat{\pi}*,t+1} + N\text{-}DR_{t+1}^{y^{i*}} \\
rx_{t+1}^{y^{r*}} - E_t[rx_{t+1}^{y^{r*}}] &= -\frac{\delta}{1-\delta\phi_r}\varepsilon_{\widehat{r}*,t+1} + N\text{-}DR_{t+1}^{y^{r*}}.
\end{aligned}
$$

Now, the conditional expected returns can vary over time and the $N\text{-}DR_{t+1}^Z$ for $Z \in \{y^i, y^r, y^{i*}, y^{r*}\}$ terms reflect the discount rate news associated with supply shocks. Specifically, we have

$$
N\text{-}DR_{t+1}^Z = -\frac{\delta}{1-\delta}(1-\delta)\sum_{j=0}^{\infty}\delta^j (E_{t+1} - E_t)[rx_{t+j+2}^Z] = -\frac{\delta}{1-\delta}(\boldsymbol{\alpha}_{DR}^Z)'\boldsymbol{\varepsilon}_{t+1}
$$

non-indexed bonds are not. Similarly, the real excess return on long-term nominal bonds over short-term real bonds is $rx_{t+1}^{y^i} - \varepsilon_{p,t+1}$ since short-term indexed bonds are protected against unexpected increased in the nominal price level ($\varepsilon_{p,t+1}$) but long-term nominal bonds are not.

In this model, the exact form of these discount rate news shocks (i.e., the equilibrium coefficients $\boldsymbol{\alpha}_{DR}^{Z}$ that map state variables into bond term premia) will depend on whether global bond investors must hold real long-term bonds—which are only exposed to news about future real rates—or nominal bonds—which are also exposed to news about future inflation.

Nominal and real exchange rates: Next, we introduce nominal and real exchange rates. We let Q_t^n denote the nominal exchange rate. The nominal log excess return on foreign currency is

$$rx_{t+1}^{q^n} = \left(q_{t+1}^n - q_t^n\right) + \left(i_t^* - i_t\right).$$

This is the familiar nominal return that is obtained by making an investment in short-term nominal foreign bonds. Let $Q_t^r = Q_t^n P_t^*/P_t$ denote the real exchange rate—i.e., the price of goods in foreign currency ($Q_t^n P_t^*$) divided by price of goods in domestic currency (P_t). The real log excess return on foreign currency between t and $t+1$ is

$$rx_{t+1}^{q^r} = \left(q_{t+1}^r - q_t^r\right) + \left(\widehat{r}_t^* - \widehat{r}_t\right).$$

The real excess return can be obtained by making an investment in short-term real (inflation-indexed) foreign bonds.[13]

Assuming that the real exchange rate is stationary and equals to zero in the long-run steady state, we have

$$q_t^r = \sum_{j=0}^{\infty} E_t[(\widehat{r}_{t+j}^* - \widehat{r}_{t+j}) - rx_{t+j+1}^{q^r}] = \overbrace{\left\{\frac{1}{1-\phi_{\widehat{r}}}\left(\widehat{r}_t^* - \widehat{r}_t\right)\right\}}^{\text{Uncovered interest rate parity}} - \overbrace{\sum_{j=0}^{\infty} E_t[rx_{t+j+1}^{q^r}]}^{\text{FX risk premium}}.$$

By definition, the log nominal exchange rate is given by

$$q_t^n = q_t^r + (p_t - p_t^*)$$

where p_t and p_t^* are the log price levels of domestic and foreign goods. Thus, the nominal exchange rate need not be stationary: it can trend deterministically if $\overline{\pi} \neq \overline{\pi}^*$ or it can trend stochastically if $\phi_\pi = 1$.[14]

---

[13]Specifically, since short-term foreign real bonds pay $(\widehat{r}_t^* + \widehat{\pi}_t^* + \varepsilon_{p*,t+1})$ at $t+1$, short-term domestic real bonds pay $(\widehat{r}_t + \widehat{\pi}_t + \varepsilon_{p,t+1})$, and $\left(q_{t+1}^n - q_t^n\right) = \left(q_{t+1}^r - q_t^r\right) + \left(\pi_{t+1} - \pi_{t+1}^*\right)$, we have

$$rx_{t+1}^{q^r} = \left(q_{t+1}^n - q_t^n\right) + \left((\widehat{r}_t^* + \widehat{\pi}_t^* + \varepsilon_{p*,t+1}) - (\widehat{r}_t + \widehat{\pi}_t + \varepsilon_{p,t+1})\right) = \left(q_{t+1}^r - q_t^r\right) + \left(\widehat{r}_t^* - \widehat{r}_t\right).$$

[14]If $\phi_\pi = 1$, then

$$E_t\left[p_{t+T} - p_{t+T}^*\right] = (p_t - p_t^*) + \sum_{j=0}^{T-1} E_t[\widehat{\pi}_{t+j} - \widehat{\pi}_{t+j}^*] = (p_t - p_t^*) + T\left(\widehat{\pi}_t - \widehat{\pi}_t^*\right)$$

and $\lim_{T\to\infty} E_t\left[p_{t+T} - p_{t+T}^*\right]$ will either be $+\infty$ or $-\infty$. By contrast, if $\phi_\pi < 1$, then

$$E_t\left[p_{t+T} - p_{t+T}^*\right] = (p_t - p_t^*) + \sum_{j=0}^{T-1} E_t[\widehat{\pi}_{t+j} - \widehat{\pi}_{t+j}^*] = (p_t - p_t^*) + T\left(\overline{\pi} - \overline{\pi}^*\right) + \frac{1-\phi_{\widehat{\pi}}^T}{1-\phi_{\widehat{\pi}}}[(\widehat{\pi}_t - \overline{\pi}) - (\widehat{\pi}_t^* - \overline{\pi}^*)].$$

If $\overline{\pi} \neq \overline{\pi}^*$, $p_{t+T} - p_{t+T}^*$ will contain a deterministic trend. Otherwise if $\overline{\pi} = \overline{\pi}^*$, then

$$\lim_{T\to\infty} E_t\left[p_{t+T} - p_{t+T}^*\right] = (p_t - p_t^*) + \frac{1}{1-\phi_{\widehat{\pi}}}\left(\widehat{\pi}_t - \widehat{\pi}_t^*\right).$$

Let $\epsilon_t X_{t+j} \equiv (E_t - E_{t-1}) X_{t+j}$ denote time t news about some random variable X_{t+j} for $j \geq 0$. Consider the following thought experiment. At time t, the only news is about *future inflation* between t and $t+T$. If future nominal rates move one-for-one with inflation so that real rates are unchanged, we have

$$\overbrace{\epsilon_t q_t^r}^{=0} = \overbrace{\sum_{j=0}^{T-1} \epsilon_t [(\widehat{r}_{t+j}^* - \widehat{r}_{t+j}) - rx_{t+1+j}^{q^r}]}^{=0} + \overbrace{\epsilon_t q_{t+T}^r}^{=0},$$

and

$$\overbrace{\epsilon_t q_t^n}^{=0} = \overbrace{\sum_{j=0}^{T-1} \epsilon_t [(i_{t+j}^* - i_{t+j}) - rx_{t+1+j}^{q^n}]}^{\sum_{j=0}^{T-1} \epsilon_t [\widehat{\pi}_{t+j}^{*n} - \widehat{\pi}_{t+j}^n]} + \overbrace{\epsilon_t q_{t+T}^n}^{-\sum_{j=0}^{T-1} \epsilon_t [\widehat{\pi}_{t+j}^* - \widehat{\pi}_{t+j}]}.$$

Thus, in this benchmark case, *pure* inflation news is not associated with unexpected movements in either real or nominal foreign exchange rates: inflation news just changes the expected path of future nominal foreign exchange rates. Since this change in the expected future drift perfectly offsets the news about future nominal interest rate differentials, the nominal exchange rate does not jump in response to the news.

Of course, to the extent that news about inflation coincides with news about future short-term real rates—i.e., because the short-term nominal rate is pinned at some lower bound or because the central bank's policy rule for the short-term nominal rate reacts more than one-for-one to news about future inflation as with a standard forward-looking Taylor rule—then it will be associated news about future short-term real rates. In this case, inflation news will be generally be associated with unexpected movements in both real and nominal exchange rates. But it is the induced news about real rates that is doing the work; not the pure inflation news.

More generally, we have

$$q_{t+1}^n - q_t^n = (q_{t+1}^r - q_t^r) - (\pi_{t+1}^* - \pi_{t+1}) = (q_{t+1}^r - q_t^r) - (\widehat{\pi}_t^* - \widehat{\pi}_t) - (\varepsilon_{p^*,t+1} - \varepsilon_{p,t+1}).$$

The nominal appreciation of foreign currency $(q_{t+1} - q_t)$ equals the real appreciation $(q_{t+1}^r - q_t^r)$ minus realized foreign less domestic inflation between t and $t+1$, $(\pi_{t+1}^* - \pi_{t+1})$. As a result, we have

$$q_{t+1}^n - E_t[q_{t+1}^n] = q_{t+1}^r - E_t[q_{t+1}^r] - (\varepsilon_{p^*,t+1} - \varepsilon_{p,t+1}).$$

In other words, the unexpected appreciation of the nominal exchange rate, $q_{t+1}^n - E_t[q_{t+1}^n]$, equals the unexpected appreciation of the real exchange rate, $q_{t+1}^r - E_t[q_{t+1}^r]$ minus the difference in contemporaneous surprise inflation, $\varepsilon_{p^*,t+1} - \varepsilon_{p,t+1}$. This implies that we have

$$\begin{aligned} rx_{t+1}^{q^r} - E_t[rx_{t+1}^{q^r}] &= \frac{1}{1-\phi_{\widehat{r}}}(\varepsilon_{\widehat{r}_{t+1}^*} - \varepsilon_{\widehat{r}_{t+1}}) + N\text{-}DR_{t+1}^{q^r} \\ rx_{t+1}^{q^n} - E_t[rx_{t+1}^{q^n}] &= \frac{1}{1-\phi_{\widehat{r}}}(\varepsilon_{\widehat{r}_{t+1}^*} - \varepsilon_{\widehat{r}_{t+1}}) - (\varepsilon_{p^*,t+1} - \varepsilon_{p,t+1}) + N\text{-}DR_{t+1}^{q^n} \end{aligned}$$

where

$$N\text{-}DR_{t+1}^Z = \sum_{j=0}^{\infty} (E_{t+1} - E_t) [rx_{t+j+2}^Z] = (\boldsymbol{\alpha}_{RP}^Z)' \boldsymbol{\varepsilon}_{t+1}$$

for $Z \in \{q^r, q^n\}$.

Thus, FX investments are exposed to news about the future path of short-term real rate differentials. However, they are not exposed to news about future inflation differentials. But, when implemented in nominal terms, FX investments are exposed to contemporaneous shocks to the levels of domestic and foreign goods prices. By contrast, the nominal returns on bonds are exposed to news about future inflation. But, nominal bond returns are not exposed to contemporaneous shocks to the levels of domestic and foreign goods prices.

Implications: The upshot of this discussion is that the co-movement patterns between bonds and foreign exchange rates that we emphasize in our paper should be most pronounced for real (inflation-indexed) bonds and real exchange rates.

The existence of nominal shocks will attenuate these co-movement patterns if one looks at nominal bonds and nominal currency returns. Specifically, the unexpected nominal appreciation of the exchange rate depends on transitory price shocks, but not news about future inflation. By contrast, the returns on nominal bonds are exposed to news about future inflation, but not to one-off transitory changes in the nominal price level.

Alternately, if one is looking the co-movement between nominal (non-indexed) bonds and nominal currencies, these co-movement patterns should be strongest in environments like that observed in recent decades where inflation was highly stable and movements in long-term nominal interest rates primarily reflected movements in long-term real rates.

B.3 Allowing for asymmetries between the two countries

This subsection discusses how the results of our baseline model in Section 3 generalize if we allow the two countries to have different short rate and bond supply processes.

First, since the stable equilibrium is continuous in the model's underlying parameters, Proposition 2 implies that $C_{y,q} > 0$ whenever $\rho < 1$ and the short rates and bond supply follow sufficiently symmetric processes. For example, while $Cov_t[rx^y_{t+1}, rx^q_{t+1}] \neq -Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}]$, we still have $Cov_t[rx^y_{t+1}, rx^q_{t+1}] > 0$ and $Cov_t[rx^{y*}_{t+1}, rx^q_{t+1}] < 0$ if there are moderate asymmetries between the domestic and foreign short rate and bond supply processes—e.g., there can be moderate differences in either the volatilities or persistences. Furthermore, $C_{y,y^*} > 0$ whenever $\rho > 0$, the short rates and bond supply follow sufficiently symmetric processes, and when FX supply risk is sufficiently small relative to ρ.

However, things grow more complicated if we allow for highly asymmetric short rate and bond supply processes. For instance, with highly asymmetric short rate processes, the sign of $C_{y,q}$ is ambiguous and the sign of $C_{y*,q}$ need not be opposite that of $C_{y,q}$. For instance, suppose that $\sigma^2_{i^*} \equiv Var_t[\varepsilon_{i^*_{t+1}}] \neq Var_t[\varepsilon_{i_{t+1}}] \equiv \sigma^2_i$, but the two short rates share the same persistence ϕ_i. Then, focusing on the limit where there is no supply risk for simplicity, we have

$$C_{y,y^*} = Cov\left[-\frac{\delta}{1-\delta\phi_i}\varepsilon_{i_{t+1}}, -\frac{\delta}{1-\delta\phi_i}\varepsilon_{i^*_{t+1}}\right] = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i\sigma_{i^*}, \tag{13a}$$

$$C_{y,q} = Cov\left[-\frac{\delta}{1-\delta\phi_i}\varepsilon_{i_{t+1}}, \frac{1}{1-\phi_i}\left(\varepsilon_{i^*_{t+1}} - \varepsilon_{i_{t+1}}\right)\right] = \frac{1}{1-\phi_i}\frac{\delta}{1-\delta\phi_i}\sigma^2_i\left(1-\rho\frac{\sigma_{i^*}}{\sigma_i}\right) \tag{13b}$$

$$C_{y^*,q} = Cov\left[-\frac{\delta}{1-\delta\phi_i}\varepsilon_{i^*_{t+1}}, \frac{1}{1-\phi_i}\left(\varepsilon_{i^*_{t+1}} - \varepsilon_{i_{t+1}}\right)\right] = -\frac{1}{1-\phi_i}\frac{\delta}{1-\delta\phi_i}\sigma^2_{i^*}\left(1-\rho\frac{\sigma_i}{\sigma_{i^*}}\right). \tag{13c}$$

While we still have $C_{y,y^*} > 0$ so long as $\rho > 0$, the behavior of $C_{y,q}$ and $C_{y^*,q}$ is more complicated. Noting that $\rho\sigma_{i^*}/\sigma_i$ ($\rho\sigma_i/\sigma_{i^*}$) is the coefficient from a regression of i^*_t on i_t (i_t on i^*_t), there are now three possible case:[15]

1. If $1 > \max\{\rho\sigma_{i^*}/\sigma_i, \rho\sigma_i/\sigma_{i^*}\}$—i.e., if the short rates are sufficiently symmetric, $C_{y,q} > 0$ and $C_{y^*,q} < 0$.

2. If $\rho\sigma_{i^*}/\sigma_i > 1 > \rho\sigma_i/\sigma_{i^*}$—i.e., if foreign short rates move more than one-for-one with domestic short rates, then $C_{y,q} < 0$ and $C_{y^*,q} < 0$.

[15] However, since $\min\{\rho\sigma_{i^*}/\sigma_i, \rho\sigma_i/\sigma_{i^*}\} < 1$, we can never have $C_{y,q} < 0$ and $C_{y^*,q} > 0$.

3. If $\rho\sigma_i/\sigma_{i^*} > 1 > \rho\sigma_{i^*}/\sigma_i$—i.e., if domestic short rates move more than one-for-one with foreign short rates, $C_{y,q} > 0$ and $C_{y^*,q} > 0$.

Thus, in the event of a positive shock to the supply of long-term dollar bonds, foreign currencies with $\rho\sigma_{i^*}/\sigma_i < 1$ would be expected to *depreciate* against the dollar on impact and then appreciate going forward: this is the case emphasized in the main text. By contrast, foreign currencies with $\rho\sigma_{i^*}/\sigma_i > 1$ would be expected to *appreciate* versus the dollar on impact and then depreciate going forward. To see the intuition, suppose that $\rho\sigma_{i^*}/\sigma_i > 1 > \rho\sigma_i/\sigma_{i^*}$, so foreign short rates move more than one-for-one with domestic short rates. Here an increase in the supply of long-term domestic bonds leads to a larger increase in the price of foreign short rate risk than in the price of domestic foreign short rate risk. Since foreign exchange has a positive exposure to domestic short rates and a negative—and opposite—exposure to foreign short rates, the increase in domestic bond supply actually reduces the expected future return on foreign exchange, leading foreign currency to appreciate today. And, since an increase in foreign bond supply also has a larger impact on the price of foreign short rate risk, such a shock also leads foreign currency to appreciate.

B.4 A unified approach to carry trade returns

In this subsection, we show that our model can deliver a unified explanation that links return predictability in foreign exchange and long-term bond markets to the *levels* of domestic and foreign short-term interest rates. For foreign exchange, Fama (1984) showed that the expected return on the borrow-domestic to lend-foreign FX trade is increasing in the foreign-minus-domestic short rate differential, $i_t^* - i_t$, a well-known and empirically robust failure of UIP. For long-term bonds, Fama and Bliss (1987) and Campbell and Shiller (1991) showed that the expected return on the borrow-short to lend-long yield curve trade is increasing in the slope of the yield curve, $y_t - i_t$, a well-known and empirically robust failure of the expectations hypothesis of the term structure.

The baseline model we developed above does not generate either predictability result. In our baseline model, shocks to short-term interest rates make foreign exchange and long-term bonds risky investments for global bond investors. As a result, supply shocks impact the expected returns on foreign exchange and long-term bonds. However, the levels of domestic and foreign short-term interest rates do not affect the expected excess returns on FX and long-term bonds.

However, a simple extension of our model can simultaneously match these two facts if we follow Gabaix and Maggiori (2015) and, appealing to balance-of-trade flows, assume that global bond investors' exposure to foreign currency is increasing in the strength of the foreign currency. Put simply, our model makes it possible to "kill two birds with one stone." Specifically, the assumption that Gabaix and Maggiori (2015) need to make to match the Fama (1984) pattern in their model, immediately delivers the Campbell-Shiller (1991) result for both the domestic and foreign yield-curve trades in our model. Symmetrically, the assumption that Vayanos and Vila (2021) need to make to match the Campbell-Shiller (1991) fact in their model—that the net supply of long-term bonds is decreasing in the level of long-term yields—immediately delivers the Fama (1984) pattern for foreign exchange in our model.

Concretely, we extend the model by allowing the net supplies to depend on equilibrium prices:

$$n_t^y = s_t^y - S_y y_t, \tag{14a}$$

$$n_t^{y*} = s_t^{y*} - S_y y_t^*, \tag{14b}$$

$$n_t^q = s_t^q + S_q q_t, \tag{14c}$$

where S_q, $S_y \geq 0$. That is, we assume the net supply of each asset is increasing that asset's price. For example, the assumption that $S_q > 0$ follows Gabaix and Maggiori (2015) and is a reduced-form way of modeling balance-of-trade flows in the FX market. Specifically, assume that when foreign currency is strong, domestic exports rise and imports fall, so the domestic country runs a trade surplus of $S_q q_t$ with the foreign country: If the domestic country is running a trade surplus, domestic exporters will want to swap the foreign currency they receive from their foreign sales for domestic currency. By FX market clearing, global bond investors must take the other side of these trade-driven flows. Thus, when foreign currency is strong, the expected returns on foreign exchange must rise to induce global bond investors to increase their exposure to foreign currency, delivering the Fama (1984) result as Gabaix and Maggiori (2015) show.

Proposition A1 describes the new results.

Proposition A1: Matching Fama (1984), Campbell-Shiller (1991), and Lustig, Stathopoulos, and Verdelhan (2019). *Suppose $\rho \in [0,1)$. If (i.a) $S_q > 0$ and $S_y = 0$ or (i.b) $S_q = 0$ and $S_y > 0$ and (ii) there are no independent supply shocks ($\sigma^2_{s^y} = \sigma^2_{s^q} = 0$), then $\partial E_t \left[rx^q_{t+1}\right]/\partial i^*_t = -\partial E_t \left[rx^q_{t+1}\right]/\partial i_t > 0$. Since exchange rates are less responsive to short rates than under UIP, if one estimates the time-series regression:*

$$rx^q_{t+1} = \alpha_q + \beta_q \cdot (i^*_t - i_t) + \xi^q_{t+1}, \tag{15}$$

one obtains $\beta_q = \partial E_t[rx^q_{t+1}]/\partial i^*_t > 0$ as in Fama (1984).

Under the same conditions, we also have $\partial E_t \left[rx^y_{t+1}\right]/\partial i_t = \partial E_t \left[rx^{y*}_{t+1}\right]/\partial i^*_t < 0$. Thus, long-term yields are less responsive to movements in short rates than under the expectations hypothesis, so expected returns on long-term bonds are high when short rates are low. Furthermore, since the term spread is high when short rates are low, if one estimates the time-series regressions:

$$rx^y_{t+1} = \alpha_y + \beta_y \cdot (y_t - i_t) + \xi^y_{t+1} \quad \text{and} \quad rx^{y*}_{t+1} = \alpha_{y*} + \beta_{y*} \cdot (y^*_t - i^*_t) + \xi^{y*}_{t+1}, \tag{16}$$

one obtains $\beta_y = \beta_{y*} > 0$ as in Campbell and Shiller (1991).

Finally, if one estimates the following time-series regression:

$$rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right) = \alpha_{q,lt} + \beta_{q,lt} \cdot (i^*_t - i_t) + \xi^{q,lt}_{t+1}, \tag{17}$$

one obtains $0 < \beta_{q,lt} < \beta_q$ as in Lustig, Stathopoulos, and Verdelhan (2019). In other words, the long-term FX carry trade is less profitable than the short-term FX carry trade. Relatedly, let $rx^{q,lh}_{t\to\infty} = (1-\delta)\sum_{j=0}^{\infty} \delta^j_t \left(q_{t+j+1} - q_{t+j}\right) + (y^*_t - y_t)$ denote the return on the long-horizon or hold-to-maturity FX carry trade. Then, if one runs the following time-series regression:

$$rx^{q,lh}_{t\to\infty} = \alpha_{q,lh} + \beta_{q,lh}\left(y^*_t - y_t\right) + \xi^{q.lh}_{t+1}, \tag{18}$$

one obtains $0 < \beta_{q,lh} < \beta_q$ as in Chinn and Meredith (2004).

To see the logic, assume $\sigma^2_{s^y} = \sigma^2_{s^q} = 0$—i.e., there are no independent supply shocks, so net supplies only fluctuate because of movements in short-rates. In this case, we have

$$E_t\left[rx^q_{t+1}\right] = \tau^{-1}\left[C_{y,q} S_y \cdot (y^*_t - y_t) + V_q \cdot S_q q_t\right], \tag{19}$$

and

$$E_t\left[rx^y_{t+1} - rx^{y*}_{t+1}\right] = \tau^{-1}\left[\left(V_y - C_{y*,y}\right) S_y \cdot (y^*_t - y_t) + 2C_{y,q} S_q \cdot q_t\right]. \tag{20}$$

First, assume $S_q > 0$ and $S_y = 0$ and suppose that $i^*_t - i_t > 0$—i.e., euro short rates exceed dollar short rates. By standard UIP logic, the positive short-rate differential means the euro will be strong—i.e., q_t will be high. The assumption that $S_q > 0$ implies that global bond investors must bear greater exposure to the euro when the euro is strong, raising the expected returns on the borrow-in-dollars lend-in-euros FX trade. As a result, the expected return on the FX trade is increasing in the euro-minus-dollar short-rate differential as in Fama (1984). However, because these FX exposures mean that global bond investors will lose money if dollar short rates rise, the expected return on the dollar yield curve trade must also rise. Since standard expectations-hypothesis logic implies that the U.S. term structure will steeper when $i_t^* - i_t > 0$ due to the mean-reverting nature of short rates, the extended model will also match Campbell and Shiller's (1991) finding that a steep yield curve predicts high excess returns on long-term bonds. Finally, due to the negative relationship between the short-term interest rate and the bond term premium in each currency, the model delivers Lustig, Stathopoulos, and Verdelhan's (2019) finding that the returns on the FX carry trade are lower when borrowing long-term in currencies with low interest rates to lend long-term in currencies with high rates. And, since $E_t[rx_{t\to\infty}^{q,lh}] = (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[rx_{t+j+1}^q + rx_{t+j+1}^{y^*} - rx_{t+j+1}^y]$, we also reproduce Chinn and Meredith's finding that the long-horizon FX carry trade is less profitable than the traditional, short-run FX carry trade.[16]

Another way to simultaneously match these two facts within our model is to follow Vayanos and Vila (2021) and assume the net supply of long-term bonds is decreasing in the level of long-term yields—i.e., to assume that $S_y > 0$. This would be the case if, as in the data, firms and governments tend to borrow long-term when the level of interest rates is low, or if there are "yield-oriented investors" who tend substitute away from long-term bonds and towards equities when interest rates are low. As Vayanos and Vila (2021) show, assuming $S_y > 0$ delivers the Campbell-Shiller (1991) result for long-term bonds. Specifically, assume $S_y > 0$ and $S_q = 0$ and suppose that $i_t^* - i_t > 0$. By standard expectations hypothesis logic, euro long-term rates will be higher than dollar long-term rates, but the yield curve will be steeper in dollars since dollar short rates will be expected to rise more over time. However, since the net supply of long-term bonds is decreasing in long-term yields, the net supply of dollar long-term bonds will be higher than the supply of euro long-term bonds. This means the term premium component of long-term yields will be larger in dollars than in euros, matching Campbell-Shiller (1991). In addition, since global bond investors will have a larger exposure to dollar short-rate shocks, the expected return on the FX trade will also be positive. As a result, the expected return on the FX trade will be increasing in the difference between euro and dollar short-term rates, matching the Fama (1984) pattern.

Finally, once we link supply to prices, changes in conventional monetary policy in the eurozone (i_t^*) impact U.S. term premia ($E_t\left[rx_{t+1}^y\right]$) and vice versa, meaning the Friedman-Obstfeld-Taylor trilemma fails. In the absence of capital controls, foreign monetary policy impacts domestic financial conditions despite floating exchanges rates. The sign of this effect is ambiguous and depends on S_q, S_y, and ρ. Specifically, we have the following result:

Proposition A2: Impact of foreign short rates on domestic term premia and vice versa. *Suppose $\sigma_{sy}^2 = \sigma_{sq}^2 = 0$. (i) If $S_q > 0$, $S_y = 0$, and $\rho \in [0,1)$, $\partial E_t\left[rx_{t+1}^y\right]/\partial i_t^* = \partial E_t\left[rx_{t+1}^{y*}\right]/\partial i_t > 0$. (ii) If $S_q = 0$, $S_y > 0$, and $\rho \in (0,1]$, $\partial E_t\left[rx_{t+1}^y\right]/\partial i_t^* = \partial E_t\left[rx_{t+1}^{y*}\right]/\partial i_t < 0$.*

When $S_q > 0$, $S_y = 0$, and $\rho < 1$, raising foreign short rates raises the domestic term premium. To understand the intuition, suppose that i_t^* rises—i.e., the ECB tightens monetary policy. This results in an appreciation of the euro relative to the dollar (i.e., q_t rises) for UIP reasons. Since $S_q > 0$ and $S_y = 0$, this appreciation in turn raises global bond investors' exposure to the borrow-in-dollars lend-in-euros trade, which raises their exposure to U.S. short rate risk. Thus, the term premium on long-term U.S. bonds, $E_t\left[rx_{t+1}^y\right]$, must rise in equilibrium.

By contrast, if $S_q = 0$, $S_y > 0$, and $\rho > 0$, raising foreign short rates lowers the domestic term premium. Suppose again that short-term euro rates i_t^* rise. This raises long-term euro yields y_t^* and

[16] Indeed, $\lim_{\delta\to 1}\beta_{q,lt} = 0$ and $\lim_{\delta\to 1}\beta_{q,lh} = 0$. Specifically, as shown above, $rx_{t+1}^q + \left(rx_{t+1}^{y*} - rx_{t+1}^y\right)$ converges to zero state-by-state as the duration of long-term bonds approaches infinitity ($\delta \to 1$) and is therefore independent of the short rate (or long rate) differential.

reduces the supply of long-term euro bonds. Since excess returns on long-term U.S. bonds are positively correlated with the those on long-term euro bonds when $\rho > 0$, the term premium on long-term U.S. bonds must decline (i.e., $E_t\left[rx^y_{t+1}\right]$ must fall).

More generally, when $S_q > 0$ and $S_y > 0$, the sign of $\partial E_t\left[rx^y_{t+1}\right]/\partial i^*_t = \partial E_t\left[rx^{y*}_{t+1}\right]/\partial i_t$ is ambiguous and depends on S_q (increasing S_q raises $\partial E_t\left[rx^y_{t+1}\right]/\partial i^*_t$ when $\rho < 1$), S_y (increasing S_y lowers $\partial E_t\left[rx^y_{t+1}\right]/\partial i^*_t$ when $\rho > 0$), and ρ (raising ρ reduces $\partial E_t\left[rx^y_{t+1}\right]/\partial i^*_t$).

B.5 Relationship to consumption-based models

Discussion Our quantity-driven, segmented-markets model provides a unified way to understand term premia and exchange rates. Table A24 compares our model's implications with those of leading frictionless, consumption-based asset pricing models. The table shows that our model is able to simultaneously match many important stylized facts about long-term bonds and foreign exchange rates. By contrast, leading consumption-based models struggle to simultaneously match these empirical patterns in a unified way.

The key driver of the differences is that our assumption that the global bond and foreign exchange markets are partially segmented from financial markets more broadly. As a result, the wealth of intermediaries in these global bond markets need not be closely tied to aggregate consumption or conditions in other financial markets (e.g., equities). To be clear, we are not assuming that financial markets are highly segmented; we are simply positing that there is some segmentation at the level of broad financial asset classes.

As shown in column (1) of Table A24, the starkest implication of this assumption is that, in our model, FX rates move in response to shifts in the supply and demand for assets in different currencies—e.g., central banks' QE policies—which intermediaries must absorb. By contrast, in frictionless asset-pricing theories, a mere "reshuffling" of assets between different agents in the economy has no asset pricing implications.

A second implication of this segmentation assumption is that "bad times" for the marginal investors in global bond markets need not coincide with "bad times" for more broadly diversified investors or for the representative households in, say, the U.S. and Europe. As shown in columns (2)-(4) of Table A24, this helps us fit several features of the term structure of interest rates. Empirically, short-term real interest rates typically rise in economic expansions and fall in recessions. As a result, long-term real bonds are a macroeconomic hedge for the representative household, which leads most consumption-based models to predict negative real term premia.[17] Empirically, however, both real term and nominal term premia are positive. By contrast, in our model as in Vayanos and Vila (2021), long-term bonds are risky for specialized bond investors, who suffer capital losses when short rates rise, and real term premia are therefore positive.

Traditional complete-markets models also imply different patterns of comovement between exchange rates and real interest rates than our model, summarized in columns (5)-(7) of Table A24. In complete-markets models, foreign currency appreciates in bad times for foreign agents—i.e., $Q_{t+1}/Q_t = M^*_{t+1}/M_{t+1}$ in these models, where M^*_{t+1} and M_{t+1} are the foreign and domestic SDF, respectively. This appreciation occurs *despite* the fact that short-term foreign interest rates fall in bad foreign times (Engel [2016]) and makes domestic assets risky for foreign agents, thus rationalizing imperfect international risk sharing with complete financial markets.[18] Furthermore, since long-term bonds are hedge

[17] There are consumption-based models in which real interest rates rise in recessions, implying a positive real term premium (e.g., Wachter [2006]). Empirically, however, real interest rates tend to fall in recessions.

[18] Lustig and Verdelhan (2019) consider the implications of relaxing the "complete-spanning" assumption that $\Delta q_{t+1} = m^*_{t+1} - m_{t+1}$ and instead assume $\Delta q_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$ where η_{t+1} is wedge term that captures market incompleteness. If both domestic and foreign agents are both on their Euler equations for short-term bonds in both currencies, Lustig and Verdelhan (2019) show that this alone imposes tight restrictions on the wedge term η_{t+1}.

assets in consumption-based models, foreign long-term bond yields fall in the same bad foreign times that foreign currency appreciates. As a result, foreign currency returns are positively correlated with long-term foreign bond returns and negatively correlated with long-term domestic bond returns. Thus, in most consumption-based models, the FX risk premium is increasing in the foreign-minus-domestic term premium differential (i.e., $E_t[rx^q_{t+1}]$ is positively related to $E_t[rx^{y*}_{t+1} - rx^y_{t+1}]$). See below for additional discussion.

By contrast, in our theory and in the data, foreign currency appreciates when short-term foreign interest rates rise relative to short-term domestic interest rates (Engel [2016]). Furthermore, the realized returns on foreign currency are negatively correlated with foreign bond returns and positively correlated with domestic bond returns. This is because the realized returns on foreign exchange and long-term bonds are both driven by shocks to short-term interest rates. As a result, the expected return on foreign currency is negatively related to the foreign-minus-domestic term premium differential.

As we showed above, our model can also jointly match the Fama (1984) and Campbell-Shiller (1991) forecasting results, thereby linking expected returns to the level of short-term interest rates. While consumption-based models can match the Fama (1984) result (see, e.g., Verdelhan [2010] and Bansal and Shaliastovich [2012]), they struggle to simultaneously match the Campbell-Shiller (1991) pattern, as summarized in columns (8)-(10) of Table A24. Consider, for instance, the habit formation model of Verdelhan (2010). When domestic agents are closer to their habit level of consumption than foreign agents, domestic agents are more risk averse. Thus, the expected excess return to holding foreign currency must be positive at these times. Since the precautionary savings effect dominates the intertemporal substitution effect in Verdelhan's (2010) model, domestic short rates will be below foreign short rates at these times, thereby generating the Fama (1984) pattern. However, since interest rates decline in bad economic times in the model, long-term real bonds hedge macroeconomic risk and carry a negative term premium. Furthermore, bond risk premia are more negative when short rates are low. Thus, if the Verdelhan (2010) model is calibrated so the term structure is steeper when short rates are low, the model delivers a negative association between the term spread and bond risk premia, contrary to Campbell-Shiller (1991).[19] The same is true for Bansal and Shaliastovich (2012), a long-run risks model of foreign exchange.

While it poses a challenge for existing models, it will be possible to develop complete-markets models that, like our model, can match Lustig, Stathopoulos, and Verdelhan's (2019) finding that the FX carry trade earns lower returns when implemented with long-term bonds instead of short-term bonds. As explained in Lustig et al (2019), the resolution is to assume that the domestic and foreign SDFs share a similar permanent component but different transitory components, implying that international risk-sharing is greater in the long-run. However, to the extent that short- and long-term interest rates still fall in bad times in this next generation of consumption-based models, they will still struggle to match the correlation structure between contemporaneous returns and between different risk premia that we see in the data.

Formal details Consider a frictionless asset-pricing model featuring complete international financial markets, but imperfect risk sharing between the home and foreign countries. Since financial markets are complete, the stochastic discount factor is unique, implying:

$$M^*_{t+1} = M_{t+1} \left(Q_{t+1}/Q_t \right). \tag{21}$$

As a result, while this form of market incompleteness can help explain the volatility of exchange rates and FX risk premium, they show it cannot overturn the crucial (and arguably counterfactual) implication that foreign exchange rates appreciate in bad times for foreign agents.

[19] It is also possible to calibrate the Verdelhan (2010) model to match the Campbell-Shiller (1991) pattern, but one then needs the yield curve to be flatter (more inverted) when short rates are low, which is counterfactual.

where Q_t is the foreign exchange rate, M_{t+1} is stochastic discount factor (SDF) that price all returns in domestic currency, and M_{t+1}^* is discount factor pricing all returns in formal currency (Backus, Foresi, Telmer [2001]).

Taking logs we find:

$$q_{t+1} - q_t = m_{t+1}^* - m_{t+1}. \tag{22}$$

Thus, frictionless theories imply that foreign currency appreciates in bad times for foreign agents where m_{t+1}^* is high and depreciates in bad times for domestic agents when m_{t+1} is high. These exchange rate dynamics make domestic assets risky for foreign agents and vice versa, rationalizing imperfect international risk sharing even with complete financial markets.

As shown in Table 24, consumption-based theories typically imply that foreign interest rates decline in bad times for foreign agents, so standard uncovered-interest-rate-parity (UIP) logic pushes foreign currency toward depreciating in bad times for foreign agents. However, by construction, this UIP effect needs to more than fully offset in consumption-based models by either a temporary appreciation of foreign currency (i.e., by news that the expected returns on foreign currency will be lower going forward, perhaps, because $E_t\left[rx_{t+1}^q\right]$ is increasing in $(i_t^* - i_t)$) or by a permanent appreciation (i.e., by an innovation to a random walk component of the exchange rate).[20] Thus, many leading consumption-based models imply

$$Cov_t\left[\Delta q_{t+1}, \Delta\left(i_{t+1}^* - i_{t+1}\right)\right] = Cov_t\left[rx_{t+1}^q, i_{t+1}^* - i_{t+1}\right] < 0. \tag{23}$$

By contrast, in our theory as in the data, we have $Cov_t\left[\Delta q_{t+1}, \Delta\left(i_{t+1}^* - i_{t+1}\right)\right] > 0$.

Assuming that both the foreign and domestic SDFs are log-normally distributed, we have

$$E_t\left[rx_{t+1}^q\right] = E_t\left[q_{t+1} - q_t + (i_t^* - i_t)\right] = \frac{1}{2}\left(\sigma_t^2[m_{t+1}] - \sigma_t^2\left[m_{t+1}^*\right]\right), \tag{24}$$

which follows from the facts that $q_{t+1} - q_t = m_{t+1}^* - m_{t+1}$, $i_t = -E_t[m_{t+1}] - \sigma_t^2[m_{t+1}]/2$, and $i_t^* = -E_t[m_{t+1}^*] - \sigma_t^2[m_{t+1}^*]/2$. Thus, the expected excess return on foreign currency is one half the difference between the conditional variances of the domestic and foreign log SDFs. In other words, foreign currency risk premium will be high when domestic agents are more risk averse than foreign agents or when domestic agents are exposed to greater macroeconomic risk.

Similarly, assuming the local-currency excess returns on long-term bonds are jointly log-normal, we have:

$$E_t[rx_{t+1}^y] + \frac{1}{2}\sigma_t^2[rx_{t+1}^y] = -Corr_t[rx_{t+1}^y, m_{t+1}]\sigma_t[rx_{t+1}^y]\sigma_t[m_{t+1}], \tag{25a}$$

$$E_t[rx_{t+1}^{y^*}] + \frac{1}{2}\sigma_t^2[rx_{t+1}^{y^*}] = -Corr_t[rx_{t+1}^{y^*}, m_{t+1}^*]\sigma_t[rx_{t+1}^{y^*}]\sigma_t\left[m_{t+1}^*\right]. \tag{25b}$$

Consumption-based models almost always imply that $Corr_t[rx_{t+1}^y, m_{t+1}] > 0$ and $Corr_t[rx_{t+1}^{y^*}, m_{t+1}^*] >$

[20] We have $q_{t+1} = -\sum_{j=1}^{T}(m_{t+1+j}^* - m_{t+1+j}) + q_{t+T}$. Letting $E_{t+1}\left[q_{t+\infty}\right] \equiv \lim_{T\to\infty} E_{t+1}\left[q_{t+T}\right]$ and taking expectations and the limit as $T \to \infty$, we obtain $q_{t+1} = -\sum_{j=1}^{\infty} E_{t+1}[m_{t+1+j}^* - m_{t+1+j}] + E_{t+1}\left[q_{t+\infty}\right] = \sum_{j=0}^{\infty} E_{t+1}[i_{t+1+j}^* - i_{t+1+j} - rx_{t+2+j}^q] + E_{t+1}\left[q_{t+\infty}\right]$. Since

$$(E_{t+1} - E_t)\, q_{t+1} = \overbrace{\textstyle\sum_{j=0}^{\infty}(E_{t+1} - E_t)\left[i_{t+1+j}^* - i_{t+1+j}\right]}^{\mathcal{N}_{t+1}^{i^*-i}} - \overbrace{\textstyle\sum_{j=0}^{\infty}(E_{t+1} - E_t)\left[rx_{t+2+j}^q\right]}^{\mathcal{N}_{t+1}^{rx^q}} + \overbrace{(E_{t+1} - E_t)\left[q_{t+\infty}\right]}^{\mathcal{N}_t^{q\infty}}$$

unexpected movements in exchange rates must either reflect news about the future interest rate differentials ($\mathcal{N}_{t+1}^{i^*-i}$), news about future excess returns on foreign exchange ($\mathcal{N}_{t+1}^{rx^q}$), or permanent news about the long-run level of foreign currency ($\mathcal{N}_t^{q\infty}$).

0—i.e., long-term domestic (foreign) bonds are an attractive hedge for domestic (foreign) investors. The idea is that domestic interest rates typically decline when the domestic agents' marginal value of financial wealth is unexpectedly high (e.g., because the SDF is persistent or because the volatility of the SDF rises in bad times), leading the prices of long-term domestic bonds to rise in these states of the world.

In our model, $E_t\left[rx^q_{t+1}\right]$ is negatively related to $E_t[rx^{y^*}_{t+1} - rx^y_{t+1}]$—i.e., the expected excess returns on foreign exchange are decreasing in the foreign-minus-domestic term premium. What do leading consumption-based model imply? In modern consumption-based models, the main reason expected returns fluctuate over time is because the conditional volatilities of SDFs ($\sigma_t[m_{t+1}]$ and $\sigma_t[m^*_{t+1}]$) vary over time—e.g., due to time-varying risk aversion as in habit formation models (Campbell and Cochrane [1999]), time-varying consumption volatility as in long-run risks models (Bansal and Yaron [2004]), or a time-varying probability of a rare economic disaster (Gabaix [2012] and Wachter [2013]). Thus, since $Corr_t[rx^y_{t+1}, m_{t+1}] > 0$, an increase in $\sigma_t[m_{t+1}]$ raises $E_t[rx^q_{t+1}]$, but reduces $E_t[rx^y_{t+1}]$—i.e., $Corr\left(E_t[rx^q_{t+1}], E_t[rx^y_{t+1}]\right) < 0$. By contrast, in our model, $E_t[rx^q_{t+1}]$ tends to be high at the same time that $E_t[rx^y_{t+1}]$ is also high—i.e., $Corr\left(E_t[rx^q_{t+1}], E_t[rx^y_{t+1}]\right) > 0$. Symmetrically, since $Corr_t[rx^{y^*}_{t+1}, m^*_{t+1}] > 0$, an increase in $\sigma_t[m^*_{t+1}]$ reduces $E_t[rx^q_{t+1}]$ and also reduces $E_t[rx^{y^*}_{t+1}]$—i.e., $Corr(E_t[rx^q_{t+1}], E_t[rx^{y^*}_{t+1}]) > 0$. By contrast, in our model, we have $Corr(E_t[rx^q_{t+1}], E_t[rx^{y^*}_{t+1}]) < 0$.

This crucial difference stems from two differences between our theory and standard frictionless theories. First, we assume that the global rates market is partially segmented from the broader capital markets as well as from ultimate consumption. As a result, long-term bonds are potentially risky for the specialized bond investors who are the relevant marginal holders of long-term bonds. Second, in consumption-based models, the realized returns on foreign currency are positively correlated with those on long-term foreign bonds and negatively correlated with those on domestic bonds. By contrast, in our theory as in the data, the realized returns on foreign currency are negatively correlated with those on long-term foreign bonds and positively correlated with those on domestic bonds.

To see this juxtaposition starkly, suppose $\sigma^2_t[rx^y_{t+1}] = \sigma^2_t[rx^{y^*}_{t+1}] = \sigma^2_y$ and $Corr_t[rx^y_{t+1}, m_{t+1}] = Corr_t[rx^{y^*}_{t+1}, m^*_{t+1}] = \varrho_{y,m} > 0$ are constant over time, so

$$E_t[rx^y_{t+1}] + \frac{1}{2}\sigma^2_y = -\varrho_{y,m}\sigma_y\sigma_t[m_{t+1}], \tag{26a}$$

$$E_t[rx^{y^*}_{t+1}] + \frac{1}{2}\sigma^2_y = -\varrho_{y,m}\sigma_y\sigma_t[m^*_{t+1}]. \tag{26b}$$

Thus, all time-series variation in foreign and domestic bond risk premia is driven by time-variation in the conditional volatility of the domestic and foreign SDFs. However, this implies that

$$E_t[rx^{y*}_{t+1} - rx^y_{t+1}] = \varrho_{y,m}\sigma_y\left(\sigma_t[m_{t+1}] - \sigma_t[m^*_{t+1}]\right). \tag{27}$$

Using Eq. (24), we find that:

$$E_t\left[rx^q_{t+1}\right] = \overbrace{\left[\frac{\sigma_t[m_{t+1}] + \sigma_t[m^*_{t+1}]}{2\varrho_{y,m}\sigma_y}\right]}^{>0} \cdot E_t[rx^{y*}_{t+1} - rx^y_{t+1}]. \tag{28}$$

Thus, most consumption-based theories predict a positive relationship between FX risk premia and the difference between foreign and domestic term premia. By contrast, as emphasized in Section 3, our theory implies a negative relationship between FX risk premia and the difference between foreign and domestic bond risk premia.

Turning to the expected return to the long-term FX trade, consumption-based models in this class imply that the expected returns on the long-term carry trade are greater in magnitude than those on the short-term FX trade:

$$E_t[rx^q_{t+1} + (rx^{y^*}_{t+1} - rx^y_{t+1})] = \overbrace{\left(1 + \frac{2\varrho_{y,m}\sigma_y}{\sigma_t[m_{t+1}] + \sigma_t\left[m^*_{t+1}\right]}\right)}^{>1} \cdot E_t\left[rx^q_{t+1}\right]. \tag{29}$$

By contrast, our model is consistent with the evidence in that the return on the long-term FX trade are smaller than those on the standard, short-term FX trade.

Behavioral models Building on Frankel and Froot (1989), a set of recent papers have sought to link the predictability of exchange rate returns to expectational errors about short rates (e.g., Valente, Vasudevan, and Wu [2022], Candian and De Leo [2022], Granziera and Sihvonen [2020]). The key idea, which goes back to Gourinchas and Tornell (2004), is that investors must underreact to news about short rates.

For instance, suppose the true persistence of short-rate shocks is $\phi_i \in (0,1)$, but investors mistakenly believe the persistence is $\hat{\phi}_i \in (0,1)$. For simplicity, assume investors are risk neutral so all subjectively expected excess returns are zero Formally, letting $E^b_t[\cdot]$ denote investors' biased expectations, we have

$$\begin{aligned}
y_t &= (1-\delta)\textstyle\sum_{j=0}^{\infty}\delta^j E^b_t[i_{t+j}] = \bar{i} + \dfrac{1-\delta}{1-\delta\hat{\phi}_i}\left(i_t - \bar{i}\right) \\
y^*_t &= (1-\delta)\textstyle\sum_{j=0}^{\infty}\delta^j E^b_t[i^*_{t+j}] = \bar{i} + \dfrac{1-\delta}{1-\delta\hat{\phi}_i}\left(i^*_t - \bar{i}\right) \\
q_t &= \textstyle\sum_{j=0}^{\infty} E^b_t[i^*_{t+j} - i_{t+j}] = \dfrac{1}{1-\hat{\phi}_i}\left(i^*_t - i_t\right).
\end{aligned}$$

However, the objectively expected excess returns perceived by a rational econometrician are

$$\begin{aligned}
E_t\left[rx^y_{t+1}\right] &= -\frac{\delta}{1-\delta\hat{\phi}_i}(\phi_i - \hat{\phi}_i)\left(i_t - \bar{i}\right) \\
E_t\left[rx^{y*}_{t+1}\right] &= -\frac{\delta}{1-\delta\hat{\phi}_i}(\phi_i - \hat{\phi}_i)\left(i^*_t - \bar{i}\right) \\
E_t\left[rx^q_{t+1}\right] &= \frac{1}{1-\hat{\phi}_i}(\phi_i - \hat{\phi}_i)\left(i^*_t - i_t\right).
\end{aligned}$$

Also, note that

$$y_t - i_t = -\frac{\delta(1-\hat{\phi}_i)}{1-\delta\hat{\phi}_i}\left(i_t - \bar{i}\right).$$

It is easy to see that if $\hat{\phi}_i < \phi_i$, then (1) $E_t\left[rx^q_{t+1}\right]$ is increasing in $(i^*_t - i_t)$ as in Fama (1984) and (2) $E_t\left[rx^y_{t+1}\right]$ is increasing in $y_t - i_t$ and $E_t\left[rx^{y*}_{t+1}\right]$ is increasing in $y^*_t - i^*_t$ as in Fama and Bliss (1987) and Campbell and Shiller (1991). Intuitively, the Fama (1984) results arises because short rates mean revert more slowly than investors expect and, hence, currencies with high short rates depreciate less than investors expect. The Campbell-Shiller (1991) fact also arises because short rates mean revert more slowly than investors expect.

Furthermore, we have

$$E_t\left[rx^q_{t+1}+rx^{y*}_{t+1}-rx^y_{t+1}\right]=\overbrace{\left[\frac{1}{1-\hat{\phi}_i}-\left(\frac{\delta}{1-\delta\hat{\phi}_i}\right)\right]}^{\in\left(0,\frac{1}{1-\hat{\phi}_i}\right)}\left(\phi_i-\hat{\phi}_i\right)\left(i^*_t-i_t\right).$$

If $\hat{\phi}_i<\phi$, then (3) $E_t\left[rx^q_{t+1}+rx^{y*}_{t+1}-rx^y_{t+1}\right]$ is increasing in $(i^*_t-i_t)$, but not as strongly as $E_t\left[rx^q_{t+1}\right]$, implying that $\left|E_t\left[rx^q_{t+1}+rx^{y*}_{t+1}-rx^y_{t+1}\right]\right|\leq\left|E_t\left[rx^q_{t+1}\right]\right|$ (with strict inequality when $i^*_t\neq i_t$). In other words, this simple behavioral model can also match the evidence in Lustig, Stathopolous, and Verdelhan (2019) that the long-term FX carry trade earns lower returns that the traditional short-term FX carry trade. The three recent papers mentioned above provide evidence from surveys consistent with the idea that $\hat{\phi}_i<\phi_i$.

While these behavioral approaches match the key return predictability results that our model delivers in Section 3.4, it is unlikely that a behavioral approach would match all the facts our model delivers. In particular, an approach based purely on expectational errors would not explain our quantity-based evidence—e.g., why Quantitative Easing impacts exchange rates and term premia. In addition, our results on the relationship between CIP violations and spot exchange rates and the ways that segmentation can generate trade flows would be hard to match in an otherwise frictionless model with biased expectations.

C Model extensions

C.1 Choice of parameters for numerical illustrations

This section details the parameter choices underlying Figures 2, 3, and A1.

Each period in our numerical illustrations represents one month. For parameters that can be readily estimated, we choose parameters based on the data in our sample. The other parameters—most importantly the volatility of net supply shocks and the aggregate risk tolerance of investors (τ)—have been chosen informally to generate results seem economically plausible.

Here are the details:[21]

- Short rate parameters: σ_i, ϕ_i, and ρ.
  - The volatility of short rate shocks of $\sigma_i=0.25\%$ is chosen to roughly match the monthly volatility of changes in short-rates for the currencies in our sample.[22]
  - The short rate persistence of $\phi_i=0.985$ is chosen to match the average monthly persistence of short rates for the currencies in our sample. For reference, $\phi_i=0.985$ implies that the half-life of a shock to short rates is 46 months. Together our choices of σ_i and ϕ_i imply that the long-run volatility of short rates is $\sqrt{\sigma_i^2/\left(1-\phi_i^2\right)}=1.4\%$, matching the average short-rate volatility of the currencies in our sample.
  - We set $\delta=119/120$ which means that the long-term bonds in our illustrations have a duration of $D=1/\left(1-\delta\right)=120$ months—i.e., 10-years. Thus, our parameter choices

[21] In the prior draft, we had set $\sigma_i=0.30\%$, $\phi_i=0.98$, and $\tau=1.75$. We changed σ_i and ϕ_i slightly because we added three additional currencies to our baseline sample. This had almost no impact on our figures.

[22] Given the high monthly persistence of short rates—i.e., since $\phi_i\approx1$, we have $Var\left[i_{t+1}-i_t\right]=\left(\phi_i-1\right)^2Var\left[i_t\right]+\sigma_i^2\approx\sigma_i^2$ for monthly changes.

imply that a 1 basis point increase in the short-rate i_t should raise the yield on a coupon-bearing bond with a duration of 10-years by $(1-\delta)/(1-\delta\phi_i) \approx 0.36$ basis points through the expectations hypothesis channel. Relatedly, a 1 basis point increase in the short-rate should raise the 10-year forward rate by $\phi_i^{120} \approx 0.16$ basis points.

– We assume $\rho \equiv Corr[\varepsilon_{i_{t+1}}, \varepsilon_{i^*_{t+1}}] = 0.5$ based on the correlation between monthly changes in U.S. and foreign currency short rates in our sample.[23] Specifically, the correlations between monthly changes in 1-year USD rates and 1-year rates in the six other currencies in our sample are as follows:

Currency:	AUD	CAD	CHF	EUR	GBP	JPY	Average
$\widehat{\rho}_c$	0.40	0.73	0.49	0.52	0.58	0.29	0.50

The average across the six currencies in our sample is 0.50.

- Supply parameters: σ_{s^y}, σ_{s^q}, ϕ_{s^y}, and ϕ_{s^q}.

  – When we solve our model in the presence of supply risk, the volatility of bond supply shocks (σ_{s^y}) and FX supply shocks (σ_{s^q}) are always multiplied by the inverse of total risk tolerance—i.e., these volatilities always appear in terms like $\tau^{-1}\sigma_{s^y}$ and $\tau^{-1}\sigma_{s^q}$. Thus, investor risk tolerance (τ) and one of these supply volatilities are not separately identified based on prices alone: only their ratio is identified. Therefore, we normalize $\sigma_{s^y} = 1$. And, we also assume $\sigma_{s^q} = \sigma_{s^y}$ for the sake of simplicity—i.e., we assume that bond supply shocks and FX supply shocks are equally volatile. However, our illustrations do not change substantively if we assume a different mix of bond and FX supply shocks.

  – We assume $\phi_{s^y} = \phi_{s^q} = 0.95$, implying that the half-life of supply shocks is 14 months. This captures what we view as the empirically-relevant case where the net supply shocks that drive risk premia are less persistent than the underlying short-rate shocks.

- Other parameters: σ_{q^∞} and τ.

  – All of our illustrations assume the exchange rate contains a small random walk component whose monthly innovations have a volatility of $\sigma_{q^\infty} = 0.5\%$. Admittedly, this is chosen somewhat arbitrarily. As discussed in the paper, we add this random walk component to exchange rates so that exchange rates are not spanned by long-term bonds in the absence of supply shocks. When supply risk is small, this smooths out the illustrations in Figure 2 near the limit where bond and FX markets become perfectly segmented (i.e., near the $\mu = 1$ boundary).

  – We set $\tau = 1.80$. This has been chosen to generate results seem economically plausible.

- Parameters for individual figures:

  – In Figure 2, we assume $\pi = 1/3$, so specialists are evenly split between domestic bonds, foreign bonds, and foreign exchange. We focus on this symmetric case for the sake of

[23] Given the high persistence of short rates at monthly horizons—i.e., since $\phi_i \approx 1$, we have

$$Corr[i_{t+1} - i_t, i^*_{t+1} - i^*_t] = \frac{Cov[i_{t+1} - i_t, i^*_{t+1} - i^*_t]}{\sqrt{Var[i_{t+1} - i_t] Var\left[i^*_{t+1} - i^*_t\right]}} = \frac{(\phi_i - 1)^2 Cov\left[i_t, i^*_t\right] + \rho\sigma_i^2}{(\phi_i - 1)^2 Var\left[i_t\right] + \sigma_i^2} \approx \rho \equiv Corr[\varepsilon_{i_{t+1}}, \varepsilon_{i^*_{t+1}}]$$

for monthly changes.

simplicity: we do have any evidence suggesting that the risk bearing capacity of specialists is distributed in this symmetric fashion.

– In Figure 3, we assume bond investors are split equally between the home and foreign country. We focus on this symmetric case for simplicity and not due to any specific evidence suggesting that risk bearing capacity is symmetrically distributed.

– In Figure A1, we assume unhedged bond investors are split equally between the home and foreign country. Again, we focus on this symmetric case purely for simplicity.

C.2 Adding unhedged bond investors

A variety of frictions, including constraints on short-selling or using derivatives, may limit some investors' ability to hedge FX risk. In our third extension, we add bond investors who cannot hedge FX risk—i.e., investors who cannot separately manage the FX exposure resulting from investments they make in non-local, long-term bonds. For example, if unhedged domestic investors want to buy long-term foreign bonds to capture the foreign term premium, they must take on exposure to foreign currency. Thus, unlike global bond investors, who can separately manage their exposures to foreign currency and the foreign yield-curve trade, these unhedged domestic investors always "staple together" the returns on the FX trade and the foreign yield-curve trade. We show that adding unhedged investors is like introducing a particular form of market segmentation. Thus, adding unhedged investors amplifies the effect of supply shocks on exchange rates and leads to endogenous trading flows.

We assume there are three investor types—all with mean-variance preferences over one-period-ahead wealth and risk tolerance τ in domestic currency terms—who only differ in terms of the assets they can trade:

1. *Unhedged domestic investors* are present in mass $\eta/2$. They can trade short-term domestic bonds, long-term domestic bonds, and long-term foreign bonds, but not short-term foreign bonds. Thus, if they buy long-term foreign bonds, they must take on foreign exchange exposure, generating an excess return of $rx_{t+1}^{y^*} + rx_{t+1}^{q}$ over short-term domestic bonds.

2. *Unhedged foreign investors* are present in mass $\eta/2$ and are the mirror image of unhedged domestic investors. If they buy long-term domestic bonds, they must take on FX exposure, generating an excess return of $rx_{t+1}^{y} - rx_{t+1}^{q}$ over short-term foreign bonds.

3. *Global bond investors*, present in mass $(1-\eta)$, can hold short- and long-term bonds in both currencies and can engage in all three carry trades.

Unhedged investors will exhibit home bias in equilibrium. For instance, since an FX-unhedged position in long-term domestic bonds is always riskier than the FX-hedged position, it is particularly risky for foreign unhedged investors to invest in domestic bonds. Thus, relative to global bond investors and domestic unhedged investors, foreign unhedged investors face a comparative disadvantage in holding long-term domestic bonds.

Below, we solve for equilibrium and obtain the following results:

Proposition A3: Adding unhedged bond investors. *Suppose $\rho \in (0,1)$ and that fraction η of bond investors cannot hedge FX risk. We have the following results:*

(i.) **Price impact.** *Suppose $\sigma_{s^y}^2 = \sigma_{s^q}^2 = 0$. Increasing the fraction of unhedged investors η: (a) increases own-market price impact: $\partial^2 E_t\left[rx_{t+1}^{a}\right]/\partial s_t^a \partial \eta > 0$ for all $a \in \{y, y^*, q\}$; (b) reduces the impact of domestic bond supply shocks on long-term foreign yields and vice-versa: $\partial^2 E_t\left[rx_{t+1}^{y*}\right]/\partial s_t^y \partial \eta < 0$ and $\partial^2 E_t\left[rx_{t+1}^{y}\right]/\partial s_t^{y*} \partial \eta < 0$; (c) increases the impact of bond supply shocks on exchange* *rates:* $\partial^2 E_t\left[rx^q_{t+1}\right]/\partial s^y_t \partial\eta > 0$ *and* $\partial^2 E_t\left[rx^q_{t+1}\right]/\partial s^{y*}_t \partial\eta < 0$*; and (d) raises the expected returns on the bond market portfolio* $rx^{s_t}_{t+1} = s'_t rx_{t+1}$*:* $\partial E_t[rx^{s_t}_{t+1}]/\partial\eta > 0$ *for any* $s_t \neq 0$*.*

(ii.) **Introducing unhedged bond investors leads to endogenous trading.** *Suppose* $\sigma^2_{s^y} \geq 0$*, and* $\sigma^2_{s^q} \geq 0$*. For any* $\eta \in (0, 1]$*, a shock to the supply of any asset* $a \in \{y, y^*, q\}$ *triggers trading in all assets* $a' \neq a$*.*

Figure A1 shows how a domestic bond supply shock impacts expected returns of as a function of the fraction of unhedged investors η. In our baseline model where $\eta = 0$, an increase in domestic bond supply s^y_t raises the expected returns on all three trades. As η rises, the impact on domestic bond returns rises. Own-market price impact rises because we are replacing global bond investors with unhedged foreign investors who are at a comparative disadvantage at absorbing this domestic bond supply shock. Thus, $\partial E_t[rx^y_{t+1}]/\partial s^y_t$ must rise with η to induce unhedged domestic investors and the remaining global bond investors to pick up the slack. The same comparative advantage logic explains why the impact of a domestic supply shock on foreign bond returns declines with η: there are fewer players who are willing to elastically substitute between long-term domestic and foreign bonds. As a result, $\partial E_t[rx^{y*}_{t+1}]/\partial s^y_t$ must fall with η: otherwise unhedged foreign investors' demand for foreign bonds will exceed the (unchanged) net supply of foreign bonds. Finally, as η increases, the domestic bond supply shock has a larger impact on foreign exchange markets. To see the intuition, note that the foreign currency demands of all three investor types are increasing in $E_t[rx^q_{t+1}]$ and $E_t[rx^{y*}_{t+1}]$ and decreasing in $E_t[rx^y_{t+1}]$. Thus, with $\partial E_t[rx^y_{t+1}]/\partial s^y_t$ rising with η and $\partial E_t[rx^{y*}_{t+1}]/\partial s^y_t$ falling, $\partial E_t[rx^q_{t+1}]/\partial s^y_t$ must rise with η to keep the foreign exchange market in equilibrium.

The three plots in Panel B of Figure A1 show the trading response to a positive shock to domestic bond supply as a function of η. In keeping with their comparative advantage, unhedged domestic investors and global bond investors absorb this shock to domestic bond supply. Unhedged domestic investors buy domestic bonds and—to lower their common short-rate exposure—reduce their unhedged holdings of foreign bonds. Global bond investors buy long-term domestic bonds and hedge their increased exposure to short-term domestic rates by reducing their holdings of long-term foreign bonds and foreign exchange. Thus, both unhedged domestic investors and global bond investors sell long-term foreign bonds and foreign currency. In equilibrium, unhedged foreign investors must take the opposite side of these flows, buying both long-term foreign bonds and foreign currency. And, in order to buy foreign currency, unhedged foreign investors must reduce their holdings of long-term domestic bonds.

This extension captures one common intuition about how QE policies may impact exchange rates rates. For instance, explaining in May 2015 how he believed large-scale bond purchases by the European Central Bank had weakened the euro, President Mario Draghi commented:

> [The ECB's bond purchases] encourage investors to shift holdings into other asset classes ... and across jurisdictions, reflected in a falling of the exchange rate.

Specifically, domestic QE policies—i.e., a reduction in s^y_t—lead unhedged domestic investors to buy foreign bonds on an unhedged basis, putting additional downward pressure on domestic currency relative to our baseline model. In summary, the presence of unhedged investors gives rise to a form of segmentation in the global bond market. This segmentation implies that a reduction in domestic bond supply leads to trading flows in the FX market and a larger depreciation of domestic currency than in our baseline model.

C.3 Interest-rate insensitive assets

The key intuition in our baseline model is that foreign exchange is an "interest-rate sensitive" asset—i.e., it is highly exposed to news about future short-term interest rates. This leads shocks to the supply of other rate-sensitive assets—such as long-term domestic and foreign bonds—to impact exchange rates. However, in the absence of additional frictions, shocks to the supply of interest-rate insensitive assets—assets whose returns are not naturally exposed to short rate risk—will not impact exchange rates.

To see this idea most starkly, we can add a hypothetical set of domestic and foreign assets that have zero interest-rate exposure, which we label Z assets, to the model. We make a series of admittedly extreme assumptions to guarantee that the excess returns on domestic and foreign Z assets are naturally uncorrelated with those on long-term bonds and foreign exchange.[24] If investors in Z assets can separately manage their FX exposures and CIP holds, then shocks to the supply of Z assets will not impact equilibrium exchange rates. For instance, an increase in the supply of domestic Z assets pushes up the domestic Z asset risk premium, leaving FX premia unchanged. The shock will lead foreign investors to purchase domestic Z assets, but they will do so on a fully FX-hedged basis, leaving the FX exposure of global bond investors unchanged.

However, if there are CIP violations as in Subsection 4.2 of the paper or if some non-local investors in Z assets cannot hedge FX risk as in Subsection C.2, then shocks to Z assets will also impact spot FX rates. Under these conditions, investors in Z assets will not fully FX-hedge their non-local investments. As a result, shocks to the supply of Z assets will alter the FX exposures of non-local investors in Z assets and, thus by FX market-clearing, those of global bond investors.

In this way, our framework suggests that shocks to the supply-and-demand for even interest-rate-insensitive assets can meaningfully impact spot exchange rates when FX hedging is limited, consistent with a host of recent empirical findings (Hau and Rey [2005], Hau, Massa, and Peress [2009], Lilley, Maggiori, Neiman, and Schreger [2019], and Pandolfi and Williams [2019]). This line of reasoning suggests that the rise in bank balance sheet costs—and the corresponding CIP deviations—that have emerged since 2008 may have increased the set of capital market flows that can impact spot exchange rates. Furthermore, if bank balance sheet costs lead to CIP deviations, then the cross-border flows triggered by shocks to interest-rate-insensitive assets can lead spot exchange rates and the CIP basis to co-move positively as in Subsection 4.2.

D Solution of the baseline model

D.1 Equilibrium conjecture

There are three prices that we need to pin down in equilibrium: y_t, y_t^*, and q_t. We conjecture that prices are a linear function of a state vector $\mathbf{z}_t$:

$$\begin{aligned} y_t &= \alpha_0^y + \boldsymbol{\alpha}_1^{y\prime}\mathbf{z}_t, \\ y_t^* &= \alpha_0^{y*} + \boldsymbol{\alpha}_1^{y*\prime}\mathbf{z}_t, \\ q_t &= \alpha_0^q + \boldsymbol{\alpha}_1^{q\prime}\mathbf{z}_t. \end{aligned}$$

Given our assumptions, the 5×1 state vector $\mathbf{z}_t = \left[i_t - \bar{\imath},\ \ i_t^* - \bar{\imath},\ \ s_t^y - \bar{s}^y,\ \ s_t^{y*} - \bar{s}^y,\ \ s_t^q\right]'$ follows a VAR(1) process $\mathbf{z}_{t+1} = \boldsymbol{\Phi}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}$, with $Var_t\left[\boldsymbol{\varepsilon}_{t+1}\right] = \boldsymbol{\Sigma}$ and $\Phi = \mathbf{diag}(\phi_i, \phi_i, \phi_{s^y}, \phi_{s^y}, \phi_{s^q})$. We stack these three prices as a vector $\mathbf{y}_t = \mathbf{a} + \mathbf{A}\mathbf{z}_t$, where $\mathbf{y}_t = [y_t, y_t^*, q_t]'$, $\mathbf{a} = [\alpha_0^y, \alpha_0^{y*}, \alpha_0^q]'$, $\mathbf{A} = [\boldsymbol{\alpha}_1^y, \boldsymbol{\alpha}_1^{y*}, \boldsymbol{\alpha}_1^q]'$.

[24] Specifically, we assume news about the cashflows of Z assets perfectly offsets news about higher short-term rates. We also assume that news about future risk premia on Z assets is driven by supply-and-demand shocks that are independent of those driving bond and FX markets.

D.2 Equilibrium concept, multiplicity, and selection

A rational expectations equilibrium of our overlapping-generations model is a fixed point of a specific operator involving the "price-impact" coefficients, $\boldsymbol{\alpha} = vec(\mathbf{A})$, which show how the supplies impact bond yields and FX prices. Specifically, consider the operator $\mathbf{f}(\boldsymbol{\alpha}_0)$ which gives the price-impact coefficients that will clear the market for long-term bonds and FX when agents conjecture that $\boldsymbol{\alpha} = \boldsymbol{\alpha}_0$. Thus, a rational expectations equilibrium of our model is a fixed point $\boldsymbol{\alpha}^* = \mathbf{f}(\boldsymbol{\alpha}^*)$.

In any rational expectations equilibrium of our baseline model, bond yields always and FX prices reflect the expected path of future short rates. As a result, risk premia do not depend on short rates. This implies that an equilibrium of our baseline model is a solution to a system of 9 nonlinear equations in 9 unknowns. Specifically, we need to determine how equilibrium yields and FX prices respond to shifts in the supply of long-term bonds and the FX carry trade: this generates 9 unknowns and 9 corresponding equations.[25]

When supply is stochastic, an equilibrium solution only exists if investors are sufficiently risk tolerant (i.e., for τ sufficiently large). When an equilibrium exists, there are multiple equilibrium solutions. Equilibrium non-existence and multiplicity of this sort arise in overlapping-generations, rational-expectations models such as ours where risk-averse investors with finite investment horizons trade an infinitely-lived asset that is subject to supply shocks.[26] Different equilibria correspond to different self-fulfilling beliefs that investors can hold about the price-impact of supply shocks and, hence, the risks associated with holding long-term bonds and the FX carry trade.

The intuition for equilibrium multiplicity can be understood most clearly if short-lived investors hold a single long-lived asset. If investors are sufficiently risk tolerant there are two equilibria in this special case: a low price impact (or low return volatility) equilibrium and a high price impact (or high return volatility) equilibrium. If investors believe that supply shocks will have a large impact on prices, they will perceive the asset as being highly risky. As a result, investors will only absorb a positive supply shock if they are compensated by a large decline in prices, making the initial belief self-fulfilling. However, if investors believe that prices will be less sensitive to supply shocks, they will perceive the asset as being less risky and will absorb a supply shock even if they are only compensated by a modest decline in prices.

While our model admits multiple equilibria, we always find a unique equilibrium that is stable in the sense that equilibrium is robust to a small perturbation in investors' beliefs regarding the equilibrium that will prevail in the future. Formally, letting $\boldsymbol{\alpha}^{(1)} = \boldsymbol{\alpha}^* + \boldsymbol{\xi}$ for some small $\boldsymbol{\xi}$ and defining $\boldsymbol{\alpha}^{(n)} = \mathbf{f}(\boldsymbol{\alpha}^{(n-1)})$, an equilibrium $\boldsymbol{\alpha}^*$ is stable if $\lim_{n\to\infty} \boldsymbol{\alpha}^{(n)} = \boldsymbol{\alpha}^*$ and is unstable if $\lim_{n\to\infty} \boldsymbol{\alpha}^{(n)} \neq \boldsymbol{\alpha}^*$. Let $\{\lambda_i\}$ denote the eigenvalues of the Jacobian $\mathbf{D}_{\boldsymbol{\alpha}}\mathbf{f}(\boldsymbol{\alpha}^*)$. If $\max_i |\lambda_i| < 1$, then $\boldsymbol{\alpha}^*$ is stable; if $\max_i |\lambda_i| > 1$, then $\boldsymbol{\alpha}^*$ is unstable. We focus on this unique stable equilibrium in our numerical illustrations.

Why do we focus on the unique stable equilibrium? First, consistent with Samuelson's (1947) correspondence principle, the single stable equilibrium has local comparative statics that comport with common sense economic intuition. By contrast, the unstable equilibria feature comparative statics that conflict with standard intuition.[27] To understand the intuition for this result, consider

[25] Once we allow bond supply to depend on the level of yields and carry trade exposures to depend on the exchange rate, risk premia will depend short rates. In that case, we need to determine how equilibrium yields and FX prices respond to shifts in the supply of long-term bonds and the FX carry trade as well as short rates: this generates 15 unknowns and 15 corresponding equations.

[26] For previous treatments of these issues, see Spiegel (1998), Bacchetta and van Wincoop (2003), Watanabe (2008), Banerjee (2011), Greenwood and Vayanos (2014), and Albagli (2015).

[27] For instance, in the simple case discussed above with only a single risky asset, the low price-impact equilibrium is stable and the high price-impact equilibrium is unstable. At the stable equilibrium, an increase in the volatility of short-term rate shocks or the volatility of supply shocks is associated with an increase in the price-impact coefficient and an increase in the volatility of returns. By contrast, these comparitive statics take the opposite sign at the unstable

the impact of some parameter γ on the equilibrium. An equilibrium satisfies $\boldsymbol{\alpha}^* = \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)$. By the implicit function theorem, we have

$$\mathbf{D}_\gamma \boldsymbol{\alpha}^* = [\mathbf{I} - \mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)]^{-1} \mathbf{D}_\gamma \mathbf{f}(\boldsymbol{\alpha}^*, \gamma).$$

If an equilibrium is stable (as well as isolated and non-degenerate), then all of the eigenvalues of $\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)$ have a modulus less than 1 and we can write

$$\mathbf{D}_\gamma \boldsymbol{\alpha}^* = [\textstyle\sum_{i=0}^{\infty} (\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma))^i] \mathbf{D}_\gamma \mathbf{f}(\alpha^*, \gamma).$$

This says that comparative statics on $\boldsymbol{\alpha}^*$ have a straightforward interpretation in terms of a dynamic adjustment process. The first-round direct effect is $\mathbf{D}_\gamma \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)$. The second-round indirect effect is then $\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma) \mathbf{D}_\gamma \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)$. The third-round indirect effect is $(\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma))^2 \mathbf{D}_\gamma \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)$. The total effect is the sum across all rounds.[28] Samuelson's correspondence principle refers to this correspondence between equilibrium comparative statics and the result of this dynamic adjustment process.

When equilibrium only involves a single variable, knowledge that an equilibrium is stable (unstable) allows one to unambiguously determine the sign of equilibrium comparative statics (Samuelson (1947)).[29] Things are more complicated when an equilibrium involves multiple unknowns as it does in our general (Arrow and Hahn (1971) and Echenique (2002, 2008)). In multivariate settings, knowledge that an equilibrium is stable (or unstable) only allows one to unambiguously sign equilibrium comparative statics in very special cases. However, the fact that an equilibrium is stable, still has qualitative implications for comparative statics.

Second, the unique stable equilibrium of our general model has a well-behaved limit as investors' risk tolerance grows large ($\tau \to \infty$) in the sense that it converges to the equilibrium with risk-neutral investors. By contrast, the unstable equilibria explode in this limit with one or more price-impact coefficients going to infinity. Similarly, as the volatility of supply shocks vanishes, the stable equilibrium converges to the equilibrium with deterministic supply. Again, the unstable equilibria explode in this limit.

D.3 Equilibrium solution

To find a rational expectations equilibrium of our overlapping-generations model, we need to set up the fixed point problem discussed above. In our baseline model, one can either think of this as a fixed point problem involving $\mathbf{A}$ or $\mathbf{V}$.

To set up this problem, we begin with the market-clearing conditions $E_t[\mathbf{rx}_{t+1}] = \tau^{-1}\mathbf{V}\mathbf{s}_t$. We write the vector of excess returns as

$$\mathbf{rx}_{t+1} \equiv \begin{bmatrix} rx_{t+1}^y \\ rx_{t+1}^{y*} \\ rx_{t+1}^q \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\delta} y_t - \frac{\delta}{1-\delta} y_{t+1} - i_t \\ \frac{1}{1-\delta} y_t^* - \frac{\delta}{1-\delta} y_{t+1}^* - i_t^* \\ q_{t+1} - q_t + (i_t^* - i_t) \end{bmatrix} = \mathbf{B}_0 \mathbf{y}_t + \mathbf{B}_1 \mathbf{y}_{t+1} + \mathbf{R}_1 \mathbf{z}_t + \mathbf{r}_0,$$

equilibrium.

[28] By contrast, if an equilibrium is unstable then some of the eigenvalues of $\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)$ have a modulus greater than 1. Thus, we have $[\mathbf{I} - \mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma)]^{-1} \neq [\sum_{i=0}^{\infty} (\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma))^i]$ so $\mathbf{D}_\gamma \alpha^* \neq [\sum_{i=0}^{\infty} (\mathbf{D}_{\boldsymbol{\alpha}} \mathbf{f}(\boldsymbol{\alpha}^*, \gamma))^i] \mathbf{D}_\gamma \mathbf{f}(\alpha^*, \gamma)$ and comparative statics don't have this intuitive interptation.

[29] Consider a univariate fixed point problem of the form $\alpha^* = f(\alpha^*, \gamma)$. An equilibrium is stable if $|\partial f(\alpha^*, \gamma)/\partial \alpha| < 1$. The comparative static with respect to γ is $\partial \alpha^*/\partial \gamma = [1 - \partial f(\alpha^*, \gamma)/\partial \alpha]^{-1} [\partial f(\alpha^*, \gamma)/\partial \gamma]$. When $|\partial f(\alpha^*, \gamma)/\partial \alpha| < 1$, we have $\partial \alpha^*/\partial \gamma = [\sum_{i=0}^{\infty} (\partial f(\alpha^*, \gamma)/\partial \alpha)^i] [\partial f(\alpha^*, \gamma)/\partial \gamma] \propto \partial f(\alpha^*, \gamma)/\partial \gamma$. However, when $\partial f(\alpha^*, \gamma)/\partial \alpha > 1$, $\partial \alpha^*/\partial \gamma \propto -\partial f(\alpha^*, \gamma)/\partial \gamma$.

where

$$\mathbf{B}_0 \equiv \begin{bmatrix} \frac{1}{1-\delta} & 0 & 0 \\ 0 & \frac{1}{1-\delta} & 0 \\ 0 & 0 & -1 \end{bmatrix}, \mathbf{B}_1 \equiv \begin{bmatrix} -\frac{\delta}{1-\delta} & 0 & 0 \\ 0 & -\frac{\delta}{1-\delta} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R}_1 \equiv \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix}, \text{ and } \mathbf{r}_0 \equiv \begin{bmatrix} -\bar{\imath} \\ -\bar{\imath} \\ 0 \end{bmatrix}.$$

Substituing $\mathbf{y}_t = \mathbf{a} + \mathbf{A}\mathbf{z}_t$ and $\mathbf{y}_{t+1} = \mathbf{a} + \mathbf{A}\mathbf{z}_{t+1} = \mathbf{a} + \mathbf{A}\boldsymbol{\Phi}\mathbf{z}_t + \mathbf{A}\boldsymbol{\varepsilon}_{t+1}$, we obtain

$$\mathbf{rx}_{t+1} = [\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\boldsymbol{\Phi} + \mathbf{R}_1]\,\mathbf{z}_t + [\mathbf{B}_1\mathbf{A}]\,\boldsymbol{\varepsilon}_{t+1},$$

which implies

$$E_t\left[\mathbf{rx}_{t+1}\right] = [\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\boldsymbol{\Phi} + \mathbf{R}_1]\,\mathbf{z}_t,$$

and

$$\mathbf{V} \equiv Var_t\left[\mathbf{rx}_{t+1}\right] = \mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'.$$

Finally, the vector of net asset supplies that fixed-income investor must hold in equilibrium can be written as $\mathbf{s}_t = \mathbf{s}_0 + \mathbf{S}_1\mathbf{z}_t$, where

$$\mathbf{S}_1 \equiv \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{s}_0 \equiv \begin{bmatrix} \bar{s}^y \\ \bar{s}^y \\ 0 \end{bmatrix}.$$

Thus, the market clearing conditions $E_t\left[\mathbf{rx}_{t+1}\right] = \tau^{-1}\mathbf{V}\mathbf{s}_t$ can be written as

$$[\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\boldsymbol{\Phi} + \mathbf{R}_1]\,\mathbf{z}_t = \tau^{-1}\left(\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'\right)(\mathbf{s}_0 + \mathbf{S}_1\mathbf{z}_t)\,. \tag{30}$$

Since $\mathbf{B}_0$, $\mathbf{B}_1$, and $\boldsymbol{\Phi}$ are diagonal, it follows that $[\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\boldsymbol{\Phi}] = \mathbf{A} \circ [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}]$ where $\circ$ denotes element-wise matrix multiplication (i.e., the Hadamard product) and $\mathbf{E}$ is a 3×5 matrix of 1s. Specifically, we have

$$[\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}] = \begin{bmatrix} \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_{s^y}}{1-\delta} & \frac{1-\delta\phi_{s^y}}{1-\delta} & \frac{1-\delta\phi_{s^q}}{1-\delta} \\ \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_{s^y}}{1-\delta} & \frac{1-\delta\phi_{s^y}}{1-\delta} & \frac{1-\delta\phi_{s^q}}{1-\delta} \\ \phi_i - 1 & \phi_i - 1 & \phi_{s^y} - 1 & \phi_{s^y} - 1 & \phi_{s^q} - 1 \end{bmatrix}.$$

Thus, matching the matrices multiplying the state vector $\mathbf{z}_t$, we see that $\mathbf{A}$ must solve the following fixed point problem:

$$\mathbf{A} = \left[\tau^{-1}\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'\mathbf{S}_1 - \mathbf{R}_1\right] \oslash [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}]\,, \tag{31}$$

where $\oslash$ denotes element-wise matrix division (i.e., Hadamard division).[30] As we show below, the 6 elements for short rates are trivially pinned down, so this can be reduced to a fixed point problem involving the 9 price impact coefficients which govern how prices respond to changes in asset supply. Matching coefficients on the vector of constants, we find

$$(\mathbf{B}_0 + \mathbf{B}_1)\,\mathbf{a} = \left[\tau^{-1}\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'\mathbf{s}_0 - \mathbf{r}_0\right]. \tag{32}$$

Since the final row of $\mathbf{B}_0 + \mathbf{B}_1$ only contains zeros, the constants for the two bond yields are pinned down in equilibrium, but the constant for the exchange rate is not pinned down.

[30] The price function that will clear markets today when agents conjecture that the price function will be $\mathbf{A}$ at all future dates is $\mathbf{A} = \mathbf{B}_0^{-1}\left[\tau^{-1}\left(\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'\right)\mathbf{S}_1 - (\mathbf{B}_1\mathbf{A}\boldsymbol{\Phi} + \mathbf{R}_1)\right]$. Of course, any solution to this modified fixed point problem is a solution to the fixed point problem in equation (31) and vice versa.

Coefficients on short rates To further characterize the solution $\mathbf{A}$, we partition $\mathbf{z}_t$ as $\mathbf{z}_t = [\mathbf{z}'_{i,t}, \mathbf{z}'_{s,t}]'$ where $\mathbf{z}_{i,t} = \left[i_t - \bar{\imath}, \;\; i^*_t - \bar{\imath}\right]'$ and $\mathbf{z}_{s,t} = \left[s^y_t - \bar{s}^y, \;\; s^{y*}_t - \bar{s}^y, \;\; s^q_t\right]'$. Thus, $\mathbf{z}_{i,t}$ contains the two state variables that drive short rates and $\mathbf{z}_{s,t}$ contains the three state variables that drive asset supply. Similarly, we partition $\mathbf{A}$ as $\mathbf{A} =[\mathbf{A}_i, \, \mathbf{A}_s]$ where $\mathbf{A}_i$ is the 3×2 matrix of loadings on $\mathbf{x}_{i,t}$ and $\mathbf{A}_s$ is the 3×3 matrix of loadings on $\mathbf{x}_{s,t}$. For an arbitrary matrix $\mathbf{X}$, let $\mathbf{X}^{[n-m]}$ for $m > n$ be the submatrix consisting of columns n, $n+1$, ..., $m-1$, m of $\mathbf{X}$. Given the form of $\mathbf{R}_1$ and $\mathbf{S}_1$, we see that

$$\mathbf{A}_i = -\mathbf{R}_1^{[1-2]} \oslash [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}]^{[1-2]} = -\begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix} \oslash \begin{bmatrix} \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_i}{1-\delta} \\ \frac{1-\delta\phi_i}{1-\delta} & \frac{1-\delta\phi_i}{1-\delta} \\ \phi_i - 1 & \phi_i - 1 \end{bmatrix} = \begin{bmatrix} \frac{1-\delta}{1-\delta\phi_i} & 0 \\ 0 & \frac{1-\delta}{1-\delta\phi_i} \\ -\frac{1}{1-\phi_i} & \frac{1}{1-\phi_i} \end{bmatrix}. \tag{33}$$

Thus, the coefficients $\mathbf{A}_i$ governing how asset prices respond to interest rates are trivially pinned down in any rational expectations equilibrium.

Price impact coefficients Next, making use of the assumed orthogonality between short rate shocks and supply shocks, we partition the variance-covariance matrix of the shocks as

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_i & \mathbf{0}_{2\times 3} \\ \mathbf{0}_{3\times 2} & \mathbf{\Sigma}_s \end{bmatrix} \text{ where } \mathbf{\Sigma}_i = \begin{bmatrix} \sigma_i^2 & \rho\sigma_i^2 \\ \rho\sigma_i^2 & \sigma_i^2 \end{bmatrix} \text{ and } \mathbf{\Sigma}_s = \begin{bmatrix} \sigma^2_{s^y} & 0 & 0 \\ 0 & \sigma^2_{s^y} & 0 \\ 0 & 0 & \sigma^2_{s^q} \end{bmatrix}.$$

Thus, the variance covariance matrix of excess returns becomes

$$\mathbf{V} = (\mathbf{B}_1\mathbf{A}_i\mathbf{\Sigma}_i\mathbf{A}_i'\mathbf{B}_1') + (\mathbf{B}_1\mathbf{A}_s\mathbf{\Sigma}_s\mathbf{A}_s'\mathbf{B}_1'). \tag{34}$$

In other words, $\mathbf{V}$ is the sum of a term $(\mathbf{B}_1\mathbf{A}_i\mathbf{\Sigma}_i\mathbf{A}_i'\mathbf{B}_1')$ reflecting the fundamental risk generated by future shocks to short rates and a term $(\mathbf{B}_1\mathbf{A}_s\mathbf{\Sigma}_s\mathbf{A}_s'\mathbf{B}_1')$ reflecting the non-fundamental risk generated by future shocks to asset supply. Again, making use of the form of $\mathbf{R}_1$ and $\mathbf{S}_1$, we obtain the following fixed point problem involving $\mathbf{A}_s$ alone:

$$\mathbf{A}_s = \mathbf{F}_s(\mathbf{A}_s) \equiv \tau^{-1}\left[(\mathbf{B}_1\mathbf{A}_i\mathbf{\Sigma}_i\mathbf{A}_i'\mathbf{B}_1') + (\mathbf{B}_1\mathbf{A}_s\mathbf{\Sigma}_s\mathbf{A}_s'\mathbf{B}_1')\right] \oslash [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}]^{[3-5]}. \tag{35}$$

As discussed above, the operator $\mathbf{F}_s(\mathbf{A}_s)$ gives the price function $\mathbf{y}_t = \mathbf{g}(\mathbf{A}_s) + \mathbf{A}_i\mathbf{z}_{i,t} + \mathbf{F}_s(\mathbf{A}_s)\mathbf{z}_{s,t}$ that will clear the markets for long-term bonds and FX when agents conjecture that the risk of holding of assets is determined by the price function $\mathbf{y}_{t+1} = \mathbf{a}_0 + \mathbf{A}_i\mathbf{z}_{i,t+1} + \mathbf{A}_s\mathbf{z}_{s,t+1}$. In other words, equation (35) says that the equilibrium price impact coefficient must satisfy

$$\begin{bmatrix} \alpha^y_{s^y} & \alpha^y_{s^{y*}} & \alpha^y_{s^q} \\ \alpha^{y*}_{s^y} & \alpha^{y*}_{s^{y*}} & \alpha^{y*}_{s^q} \\ \alpha^q_{s^y} & \alpha^q_{s^{y*}} & \alpha^q_{s^q} \end{bmatrix} = \tau^{-1} \begin{bmatrix} \frac{1-\delta}{1-\delta\phi_{s^y}} V_y & \frac{1-\delta}{1-\delta\phi_{s^y}} C_{y,y*} & \frac{1-\delta}{1-\delta\phi_{s^q}} C_{y,q} \\ \frac{1-\delta}{1-\delta\phi_{s^y}} C_{y,y*} & \frac{1-\delta}{1-\delta\phi_{s^y}} V_{y*} & \frac{1-\delta}{1-\delta\phi_{s^q}} C_{y*,q} \\ -\frac{1}{1-\phi_{s^y}} C_{y,q} & -\frac{1}{1-\phi_{s^y}} C_{y*,q} & -\frac{1}{1-\phi_{s^q}} V_q \end{bmatrix}, \tag{36}$$

where $V_a \equiv Var\left[rx^a_{t+1}\right]$ and $C_{a,a'} \equiv Cov[rx^a_{t+1}, rx^{a'}_{t+1}]$ are the equilibrium return (co)variances.

The variance-covariance matrix in the absence of supply risk is $(\mathbf{B}_1\mathbf{A}_i\mathbf{\Sigma}_i\mathbf{A}_i'\mathbf{B}_1') =$

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) \end{bmatrix}. \tag{37}$$

The contribution of supply risk to the variance-covariance matrix is $(\mathbf{B}_1 \mathbf{A}_s \mathbf{\Sigma}_s \mathbf{A}_s{}' \mathbf{B}_1') =$

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta}\right)^2 \begin{pmatrix} \left(\alpha^y_{s^y}\right)^2 \sigma^2_{s^y} \\ +\left(\alpha^y_{s^{y*}}\right)^2 \sigma^2_{s^y} \\ +\left(\alpha^y_{s^q}\right)^2 \sigma^2_{s^q} \end{pmatrix} & \left(\frac{\delta}{1-\delta}\right)^2 \begin{pmatrix} \left(\alpha^y_{s^y} \alpha^{y*}_{s^y}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^{y*}} \alpha^{y*}_{s^{y*}}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^q} \alpha^{y*}_{s^q}\right) \sigma^2_{s^q} \end{pmatrix} & -\left(\frac{\delta}{1-\delta}\right) \begin{pmatrix} \left(\alpha^y_{s^y} \alpha^q_{s^y}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^{y*}} \alpha^q_{s^{y*}}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^q} \alpha^q_{s^q}\right) \sigma^2_{s^q} \end{pmatrix} \\ \left(\frac{\delta}{1-\delta}\right)^2 \begin{pmatrix} \left(\alpha^y_{s^y} \alpha^{y*}_{s^y}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^{y*}} \alpha^{y*}_{s^{y*}}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^q} \alpha^{y*}_{s^q}\right) \sigma^2_{s^q} \end{pmatrix} & \left(\frac{\delta}{1-\delta}\right)^2 \begin{pmatrix} \left(\alpha^{y*}_{s^y}\right)^2 \sigma^2_{s^y} \\ +\left(\alpha^{y*}_{s^{y*}}\right)^2 \sigma^2_{s^y} \\ +\left(\alpha^{y*}_{s^q}\right)^2 \sigma^2_{s^q} \end{pmatrix} & -\left(\frac{\delta}{1-\delta}\right) \begin{pmatrix} \left(\alpha^{y*}_{s^y} \alpha^q_{s^y}\right) \sigma^2_{s^y} \\ +\left(\alpha^{y*}_{s^{y*}} \alpha^{y*}_{s^{y*}}\right) \sigma^2_{s^y} \\ +\left(\alpha^{y*}_{s^q} \alpha^q_{s^q}\right) \sigma^2_{s^q} \end{pmatrix} \\ -\left(\frac{\delta}{1-\delta}\right) \begin{pmatrix} \left(\alpha^y_{s^y} \alpha^q_{s^y}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^{y*}} \alpha^q_{s^{y*}}\right) \sigma^2_{s^y} \\ +\left(\alpha^y_{s^q} \alpha^q_{s^q}\right) \sigma^2_{s^q} \end{pmatrix} & -\left(\frac{\delta}{1-\delta}\right) \begin{pmatrix} \left(\alpha^{y*}_{s^y} \alpha^q_{s^y}\right) \sigma^2_{s^y} \\ +\left(\alpha^{y*}_{s^{y*}} \alpha^q_{s^{y*}}\right) \sigma^2_{s^y} \\ +\left(\alpha^{y*}_{s^q} \alpha^q_{s^q}\right) \sigma^2_{s^q} \end{pmatrix} & \begin{pmatrix} \left(\alpha^q_{s^y}\right)^2 \sigma^2_{s^y} \\ +\left(\alpha^q_{s^{y*}}\right)^2 \sigma^2_{s^y} \\ +\left(\alpha^q_{s^q}\right)^2 \sigma^2_{s^q} \end{pmatrix} \end{bmatrix}. \tag{38}$$

As we will show below, any solution to this fixed point problem must satisfy $\alpha^y_{s^y} = \alpha^{y*}_{s^{y*}}$, $\alpha^y_{s^{y*}} = \alpha^{y*}_{s^y}$, $\alpha^y_{s^q} = -\alpha^{y*}_{s^q}$, and $\alpha^q_{s^y} = -\alpha^q_{s^{y*}}$.

Recasting the fixed-point problem in terms of the variance-covariance matrix We now recast this fixed point problem involving the 3×3 matrix $\mathbf{A}_s$ as a fixed point problem involving the 3×3 variance-covariance matrix of returns $\mathbf{V}$—i.e., a fixed point of the form $\mathbf{V} = \mathbf{G}(\mathbf{V})$. While $\mathbf{A}_s$ is not symmetric, $\mathbf{V}$ is symmetric, effectively reducing the fixed-point in 9 unknowns to one involving 6 unknowns. Specifically, making use of equations (34), (36), (37), and (38) and defining the constants,

$$g_y \equiv \tau^{-1} \frac{\delta}{1-\delta\phi_{s^y}} \sigma_{s^y},\; g_q \equiv \tau^{-1} \frac{\delta}{1-\delta\phi_{s^q}} \sigma_{s^q},\; h_y \equiv \tau^{-1} \frac{1}{1-\phi_{s^y}} \sigma_{s^y}, \text{ and } h_q \equiv \tau^{-1} \frac{1}{1-\phi_{s^q}} \sigma_{s^q}, \tag{39}$$

we find that $\mathbf{V}$ must satisfy the following system of 6 equations in 6 unknowns:

$$V_y = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + \left[g_y^2 (V_y)^2 + g_y^2 (C_{y,y^*})^2 + g_q^2 (C_{y,q})^2\right] \tag{40a}$$

$$V_{y^*} = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + \left[g_y^2 (V_{y^*})^2 + g_y^2 (C_{y,y^*})^2 + g_q^2 (C_{y^*,q})^2\right] \tag{40b}$$

$$V_q = \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2 (1-\rho) + \left[h_y^2 (C_{y,q})^2 + h_y^2 (C_{y^*,q})^2 + h_q^2 (V_q)^2\right] \tag{40c}$$

$$C_{y,y*} = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 + \left[g_y^2 V_y C_{y,y^*} + g_y^2 V_{y*} C_{y,y^*} + g_q^2 C_{y,q} C_{y*,q}\right] \tag{40d}$$

$$C_{y,q} = \frac{\delta}{1-\delta\phi_i} \frac{1}{1-\phi_i} \sigma_i^2 (1-\rho) + \left[g_y h_y V_y C_{y,q} + g_y h_y C_{y,y^*} C_{y*,q} + g_q h_q C_{y,q} V_q\right] \tag{40e}$$

$$C_{y*,q} = -\frac{\delta}{1-\delta\phi_i} \frac{1}{1-\phi_i} \sigma_i^2 (1-\rho) + \left[g_y h_y V_{y^*} C_{y^*,q} + g_y h_y C_{y,y^*} C_{y,q} + g_q h_q C_{y^*,q} V_q\right] \tag{40f}$$

These equations give the actual risk of holding assets when agents make specific conjectures about future asset risk and thus demand commensurate discounts to absorb supply-and-demand shocks.

It is easy to see that any solution to this fixed-point problem must satisfy $V_y = V_{y*}$ and $C_{y,q} = -C_{y*,q}$. To see this, subtract the second equation from the first to obtain $V_y - V_{y^*} = g_q^2 \cdot ((C_{y,q})^2 - (C_{y^*,q})^2)$. Next, add the fifth and sixth equations to obtain $C_{y,q} + C_{y*,q} = [g_y h_y C_{y^*,q} \cdot (V_{y^*} - V_y)] \div [1 -$

$g_y h_y(C_{y,y^*} + V_y) - g_q h_q V_q]$. Combining these two expressions, we have

$$C_{y,q} + C_{y*,q} = \overbrace{\left[\frac{g_y h_y g_q^2 C_{y^*,q}\left(C_{y,q} - C_{y*,q}\right)}{1 - g_y h_y\left(C_{y,y^*} + V_y\right) - g_q h_q V_q}\right]}^{\neq 0} \cdot \left(C_{y,q} + C_{y*,q}\right),$$

which implies that $C_{y,q} + C_{y*,q} = 0$. It then follows that $V_y = V_{y*}$.

Imposing this symmetry condition, we are left with a fixed point problem involving just four unknowns:

$$V_y = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + \left[g_y^2\left(V_y\right)^2 + g_y^2\left(C_{y,y^*}\right)^2 + g_q^2\left(C_{y,q}\right)^2\right] \tag{41a}$$

$$V_q = \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2\left(1-\rho\right) + \left[2h_y^2\left(C_{y,q}\right)^2 + h_q^2\left(V_q\right)^2\right] \tag{41b}$$

$$C_{y,y*} = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 + \left[2g_y^2 V_y C_{y,y^*} - g_q^2\left(C_{y,q}\right)^2\right] \tag{41c}$$

$$C_{y,q} = \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) + \left[h_y g_y\left(V_y - C_{y,y^*}\right) + h_q g_q V_q\right] C_{y,q} \tag{41d}$$

D.4 Characterizing the solution

We now characterize the solution to the system of equations in (41). We first discuss the solution in the limiting case where supply risk vanishes. We then discuss the solution in the general case where both $\sigma_{s_y}^2 > 0$ and $\sigma_{s_q}^2 > 0$. Finally, we discuss the solution in the special case where $\sigma_{s_y}^2 > 0$ and $\sigma_{s_q}^2 = 0$.

D.4.1 Limiting case with no supply risk

Taking the limit as supply risk grows small ($\mathbf{\Sigma}_s \to \mathbf{0}$), we have

$$\lim_{\mathbf{\Sigma}_s \to \mathbf{0}} \mathbf{V} = (\mathbf{B}_1\mathbf{A}_i\mathbf{\Sigma}_i\mathbf{A}_i'\mathbf{B}_1') = \begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2 & \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \\ \left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) & \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2\left(1-\rho\right) \end{bmatrix},$$

and $\lim_{\mathbf{\Sigma}_s \to \mathbf{0}} \mathbf{A}_s = \tau^{-1}(\mathbf{B}_1\mathbf{A}_i\mathbf{\Sigma}_i\mathbf{A}_i'\mathbf{B}_1') \oslash [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}]^{[3-5]}$

$$= \tau^{-1}\begin{bmatrix} \frac{1-\delta}{1-\delta\phi_{sy}}\left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2 & \frac{1-\delta}{1-\delta\phi_{sy}}\left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2 & \frac{1-\delta}{1-\delta\phi_{sq}}\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \\ \frac{1-\delta}{1-\delta\phi_{sy}}\left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2 & \frac{1-\delta}{1-\delta\phi_{sy}}\left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2 & -\frac{1-\delta}{1-\delta\phi_{sq}}\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \\ -\frac{1}{1-\phi_{sy}}\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) & \frac{1}{1-\phi_{sy}}\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) & -\frac{1}{1-\phi_{sq}}\left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2\left(1-\rho\right) \end{bmatrix}.$$

In the limit where supply risk grows small ($\mathbf{\Sigma}_s \to \mathbf{0}$), all of the price-impact coefficients have the expected signs. Since the model's stable equilibrium is continuous in the model's underlying parameters, this guarantees that the price-impact coefficients will always have same signs when supply risk is small.

D.4.2 General solution with both bond and FX supply shocks

When $\sigma^2_{s^y} > 0$ and $\sigma^2_{s^q} > 0$, solving the model involves reducing a system of four quadratic equations in four unknowns to an equation that behaves like a cubic in a single unknown, characterizing solutions to that equation, and then solving the rest of the system. We assume throughout that $\rho < 1$.

Step #1: Solve for $\Delta \equiv V_y - C_{y,y*}$ as a function of $C_{y,q}$. We subtract condition (41c) for $C_{y,y*}$ from condition (41a) for V_y to obtain the following quadratic in $\Delta \equiv V_y - C_{y,y*}$:

$$(V_y - C_{y,y*}) = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 (1-\rho) + g_y^2 (V_y - C_{y,y*})^2 + 2g_q^2 (C_{y,q})^2 .$$

Since the right-hand-side is always positive, we must have $\Delta = V_y - C_{y,y*} > 0$. We then solve the following quadratic equation for $\Delta \equiv (V_y - C_{y,y*})$ as a function of $C_{y,q}$:

$$0 = \overbrace{g_y^2}^{a_\Delta > 0} \Delta^2 - \Delta + \overbrace{\left[2g_q^2 (C_{y,q})^2 + \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 (1-\rho)\right]}^{c_\Delta(C_{y,q})>0}.$$

A real solution only exists if

$$\begin{aligned} 1 &> 4g_y^2 \left[2g_q^2 (C_{y,q})^2 + \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 (1-\rho)\right] \\ &= 4\left(\frac{\delta\tau^{-1}\sigma_{s^y}}{1-\delta\phi_{s^y}}\right)^2 \left[2\left(\frac{\delta\tau^{-1}\sigma_{s^q}}{1-\delta\phi_{s^q}}\right)^2 (C_{y,q})^2 + \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 (1-\rho)\right] \end{aligned}$$

or $\tau > \hat{\tau}_\Delta (C_{y,q})$—i.e., if risk tolerance is sufficiently larger—where this cutoff is an increasing function of $C_{y,q}$.

Let $\Delta (C_{y,q})$ denote the smaller solution to this quadratic:

$$\Delta (C_{y,q}) = \frac{1-\sqrt{1-4a_\Delta c_\Delta (C_{y,q})}}{2a_\Delta}, \tag{42}$$

which corresponds to the stable solution to the fixed point problem. Call the solution $\Delta (C_{y,q}) > 0$ and note that

$$\Delta' (C_{y,q}) = \frac{2g_q^2 C_{y,q}}{1-g_y^2 2\Delta (C_{y,q})} \propto C_{y,q},$$

since $1-g_y^2 2\Delta (C_{y,q}) > 0$ at the relevant solution. Thus, $\Delta' (0) = 0$, $\Delta' (C_{y,q}) > 0$ when $C_{y,q} > 0$, and $\Delta' (C_{y,q}) < 0$ when $C_{y,q} < 0$. Also note that

$$\Delta'' (C_{y,q}) = \frac{2g_q^2 \left(1-g_y^2 2\Delta (C_{y,q})\right) + 4g_y^2 g_q^2 \Delta' (C_{y,q}) C_{y,q}}{\left[1-(g_y)^2 2\Delta^* (C_{y,q})\right]^2} > 0.$$

Thus, $\Delta (C_{y,q})$ is a positive, U-shaped function of $C_{y,q}$. Also, note that we have $g_y^2 \Delta < 1$.

Step #2: Solve for V_q as a function of $C_{y,q}$. Next, we solve for V_q as a function of $C_{y,q}$. Rearranging condition (41b) for V_q, we want to solve the following quadratic for V_q as a function of

$C_{y,q}$:

$$0 = \overbrace{h_q^2}^{a_{V_q}>0} (V_q)^2 - V_q + \overbrace{\left(2h_y^2 (C_{y,q})^2 + \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2 (1-\rho)\right)}^{c_{V_q}(C_{y,q})>0}.$$

A real solution only exists if $\tau > \hat{\tau}_q (C_{y,q})$ where this cutoff is an increasing function of $C_{y,q}$. Let

$$\nu_q (C_{y,q}) = \frac{1-\sqrt{1-4a_{V_q} c_{V_q} (C_{y,q})}}{2a_{V_q}} > 0 \tag{43}$$

denote the smaller root of this quadratic which corresponds to the stable solution to the fixed point problem. Note that, at the relevant solution to this quadratic, $\nu_q' (0) = 0$, $\nu_q' (C_{y,q}) > 0$ when $C_{y,q} > 0$, and $\nu_q' (C_{y,q}) < 0$ when $C_{y,q} < 0$. Also, note that $\nu_q'' (C_{y,q}) > 0$. Thus, $\nu_q (C_{y,q})$ is also a positive, U-shaped function of $C_{y,q}$. Also, note that we have $h_q^2 v_q < 1$.

Step #3: Plug these functions back into condition (41d) for $C_{y,q}$ and solve for $C_{y,q}$. Plugging in these two functions—$\Delta (C_{y,q})$ and $\nu_q (C_{y,q})$—into condition (41d) for $C_{y,q}$, we obtain the following equation in one unknown for $C_{y,q}$:

$$C_{y,q} = F (C_{y,q}) \equiv \frac{\delta}{1-\delta\phi_i} \frac{1}{1-\phi_i} \sigma_i^2 (1-\rho) + [h_y g_y \Delta (C_{y,q}) + h_q g_q \nu_q (C_{y,q})] C_{y,q}. \tag{44}$$

We now use the properties of $F (C_{y,q})$ to characterize the solutions to $C_{y,q} = F (C_{y,q})$. Specifically, $F (C_{y,q})$ has the following properties:

- $F (0) = \frac{\delta}{1-\delta\phi_i} \frac{1}{1-\phi_i} \sigma_i^2 (1-\rho) > 0$.
- $F' (C_{y,q}) > 0$ for all $C_{y,q}$. To see this note that

  $$F' (C_{y,q}) = [h_y g_y \Delta (C_{y,q}) + h_q g_q \nu_q (C_{y,q})] + \left[h_y g_y \Delta' (C_{y,q}) + h_q g_q \nu_q' (C_{y,q})\right] C_{y,q}$$

  Since $\Delta (C_{y,q}) > 0$, $\nu_q (C_{y,q}) > 0$, and $sign (\Delta' (C_{y,q})) = sign \left(\nu_q' (C_{y,q})\right) = sign(C_{y,q})$, we have $F' (C_{y,q}) > 0$ for all $C_{y,q}$.
- $F'' (C_{y,q}) > 0$ when $C_{y,q} > 0$ and $F'' (C_{y,q}) < 0$ when $C_{y,q} < 0$. This follows from the fact that

  $$F'' (C_{y,q}) = 2 \left[h_y g_y \Delta' (C_{y,q}) + h_q g_q \nu_q' (C_{y,q})\right] + \left[h_y g_y \Delta'' (C_{y,q}) + h_q g_q \nu_q'' (C_{y,q})\right] C_{y,q}.$$

- Together, these three previous properties imply that $F (C_{y,q})$ is a "cubic-shaped" function—i.e., $F (C_{y,q})$ is shaped like $A + BX^3$ for $A, B > 0$. To be clear, $F (C_{y,q})$ is not actually a cubic function of $C_{y,q}$. It simply behaves qualitatively like a cubic function of $C_{y,q}$.

Knowing that $F (C_{y,q})$ is "cubic-shaped" allows us to characterize the solutions to $C_{y,q} = F (C_{y,q})$. We have the following results:

- **Positive solutions:** Since (i) $F (0) > 0$ and (ii) $F' (C_{y,q}) > 0$ and $F'' (C_{y,q}) > 0$ when $C_{y,q} > 0$, there are no positive solutions if $F' (0) > 1$. If $F' (0) < 1$, positive solutions may or may not exist. If there are positive solutions, there can be one or two solutions: a smaller stable solution (at which $F' (C_{y,q}) < 1$) and a larger unstable solution (at which $F' (C_{y,q}) > 1$). Since the existence of $\Delta (C_{y,q})$ and $\nu_q (C_{y,q})$ depends on $C_{y,q}$—they may exist for $(C_{y,q})^2$ small but not for $(C_{y,q})^2$ large—it is possible that no positive solutions exist. And, is possible that the smaller stable root exists, but that the larger unstable root does not exist.

- **Negative solutions:** Since (i) $F(0) > 0$ and (ii) $F'(C_{y,q}) > 0$ and $F''(C_{y,q}) < 0$ when $C_{y,q} > 0$, there is at most a single negative solution. Furthermore, if it exists, this negative solution must satisfy $F'(C_{y,q}^{*}) > 1$—i.e., it corresponds to an unstable solution to the fixed point problem. Since $F(C_{y,q})$ is cubic shaped, we have $C_{y,q} - F(C_{y,q}) < 0$ for $C_{y,q} \to -\infty$, assuming that $F(C_{y,q})$ continues to exist as $C_{y,q} \to -\infty$. However, since the existence of $\Delta(C_{y,q})$ and $\nu_q(C_{y,q})$ depends on $C_{y,q}$—they may exist for $(C_{y,q})^2$ small but not $(C_{y,q})^2$ large—it is possible that this negative unstable root does not exist.

- **Summary: Any stable solution is positive:** In summary, $C_{y,q} = F(C_{y,q})$ may have 0, 1, 2, or 3 solutions. If positive solutions exist, there is one stable positive solution (the smaller positive solution) and potentially one additional unstable positive solution (the larger positive solution). There is at most one negative solution to $C_{y,q} = F(C_{y,q})$ which is always unstable. *Thus, any stable solution is positive.* If a stable solution exists, we call this solution $\hat{C}_{y,q} > 0$.

- **Conditions to ensure existence of a positive stable solution:** A positive stable solution will never exist if $1 < F'(0)$. Therefore, a necessary—but not sufficient—condition for a positive stable solution to exist is that:

$$1 > F'(0) = g_y h_y \Delta(0,\tau) + g_q h_q \nu_q(0,\tau).$$

  Since both $\Delta(0,\tau)$ and $\nu_q(0,\tau)$ are decreasing in τ, this implies that we need τ sufficiently large. So we need $\tau > \hat{\tau}_{C_{y,q}}$ where $\hat{\tau}_{C_{y,q}}$ is implicitly defined as the solution to $1 = F'(0,\tau)$. However, knowing that $\tau > \hat{\tau}_{C_{y,q}}$ is necessary—but not sufficient—to ensure the existence of a positive stable solution. Furthermore, it is clear that only the positive stable solution exists for $\tau^{-1}\sigma_{s^y} \to 0$ and $\tau^{-1}\sigma_{s^q} \to 0$, since

$$\lim_{\tau^{-1}\sigma_{s^y}\to 0,\ \tau^{-1}\sigma_{s^q}\to 0} F(C_{y,q}) = \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) = \text{ Constant} > 0.$$

Step #4: If a positive stable solution $\hat{C}_{y,q}$ exists, use it to solve for V_y, V_q, $C_{y,y*}$. First, we solve condition (41a) for V_y. We know $\hat{C}_{y,q}$ and $\hat{\Delta} = \Delta(\hat{C}_{y,q})$. To determine V_y we use condition (41a) and solve the following quadratic equation for V_y

$$\begin{aligned} 0 &= g_y^2 (V_y)^2 + g_y^2 (V_y - \hat{\Delta})^2 - V_y + \left(\left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + g_q^2 (\hat{C}_{y,q})^2\right) \\ &= 2g_y^2 (V_y)^2 - (1 + 2g_y^2\hat{\Delta})V_y + \left(\left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + g_q^2 \left(\hat{C}_{y,q}\right)^2 + g_y^2 (\hat{\Delta})^2\right). \end{aligned}$$

A real solution only exists if $\tau > \hat{\tau}_y(\hat{C}_{y,q})$. As usual, if we want the smaller, stable solution of this quadratic. Call this solution $\hat{V}_y$.

Second, we can read off the solution to V_q: it is $\hat{V}_q = \nu_q(\hat{C}_{y,q}) > 0$.

Third, we can read off the solution to $C_{y,y*}$: it is $\hat{C}_{y,y*} = \hat{V}_y - \hat{\Delta} < \hat{V}_y$. Somewhat surprisingly, when $\sigma_{s^q}^2 > 0$, we can have $\hat{C}_{y,y*} < 0$ because foreign exchange supply shocks push domestic and long-term yields in opposite directions. For instance, if $\sigma_{s^q}^2 > 0$, $\sigma_{s^y}^2 = \sigma_{s^{y*}}^2 = 0$, and $\rho = 0$, condition (41c) becomes

$$C_{y,y*} = -g_q^2 (C_{y,q})^2 < 0,$$

so we must have $\hat{C}_{y,y*} < 0$. However, when $\rho > 0$, the two long-term yields tend to move in the same direction because domestic and foreign short rates are positively correlated. As a result, we have

$\hat{C}_{y,y*} > 0$ unless foreign exchange supply shocks are large and ρ is near zero.

Summary of solution. When $\sigma^2_{s^q} > 0$, $\sigma^2_{s^y} > 0$, and $\rho \in (0,1)$, we have

$$\hat{V}_y > \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 > 0,\ \hat{V}_q > \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) > 0, \text{ and } \hat{C}_{y,q} > \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) > 0$$

in the model's unique stable equilibrium. Naturally, once we add supply shocks, the equilibrium volatility of all three excess returns exceeds that in the absence of supply shocks. And, the equilibrium covariance between the excess returns on domestic bonds and the FX carry trade is positive—i.e., $\hat{C}_{y,q} = Cov_t\left[rx^y_{t+1}, rx^q_{t+1}\right] > 0$—and exceeds that in the absence of supply shocks. However, we can have $\hat{C}_{y,y*} < 0$ in the unique stable equilibrium. Specifically, if $\sigma^2_{s^q} > 0$ and $\sigma^2_{s^y} = \sigma^2_{s^{y*}} = 0$, then we have $\hat{C}_{y,y*} < 0$ as $\rho \to 0$.

Solution properties. Consider a stable solution of the model. We show that $\hat{V}_q - 2\hat{C}_{y,q} > 0$ and $\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y > 0$ when $\delta < 1$ and $\rho < 1$. First, using the equations in (41), note that

$$\begin{aligned}
& \hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y \\
= \ & \frac{\delta\sigma_i^2(1-\delta)(1-\rho)}{(1-\phi_i)(\delta\phi_i-1)^2} + [(h_y g_y(\hat{V}_y - \hat{C}_{y,y*}) + h_q g_q \hat{V}_q)\hat{C}_{y,q} - 2g_q^2(\hat{C}_{y,q})^2 - g_y^2(\hat{V}_y - \hat{C}_{y,y*})^2] \\
> \ & \frac{\delta\sigma_i^2(1-\delta)(1-\rho)}{(1-\phi_i)(1-\delta\phi_i)^2} + g_y^2(\hat{V}_y - \hat{C}_{y,y*})\cdot[\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y] + g_q^2\hat{C}_{y,q}\cdot[\hat{V}_q - 2\hat{C}_{y,q}],
\end{aligned}$$

where the last line follows from the facts that the hs are larger than the gs, $\hat{V}_y - \hat{C}_{y,y*} > 0$, and $\hat{C}_{y,q} > 0$. Since $g_y^2(\hat{V}_y - \hat{C}_{y,y*}) < 1$ in equilibrium (see above), we have

$$[\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y] > \overbrace{\frac{\delta\sigma_i^2(1-\delta)(1-\rho)}{(1-\phi_i)(1-\delta\phi_i)^2}\frac{1}{1-g_y^2(\hat{V}_y - \hat{C}_{y,y*})}}^{a>0} + \overbrace{\frac{g_q^2\hat{C}_{y,q}}{1-g_y^2(\hat{V}_y - \hat{C}_{y,y*})}}^{b>0}\cdot[\hat{V}_q - 2\hat{C}_{y,q}]. \quad (45)$$

Proceeding similarly, we have

$$\begin{aligned}
& [\hat{V}_q - 2\hat{C}_{y,q}] \\
= \ & \frac{2\sigma_i^2(1-\delta)(1-\rho)}{(1-\phi_i)^2(1-\delta\phi_i)} + [2h_y^2(\hat{C}_{y,q})^2 + h_q^2(\hat{V}_q)^2 - 2(h_y g_y(\hat{V}_y - \hat{C}_{y,y*}) + h_q g_q \hat{V}_q)\hat{C}_{y,q}] \\
> \ & \frac{2\sigma_i^2(1-\delta)(1-\rho)}{(1-\phi_i)^2(1-\delta\phi_i)} + 2h_y^2\hat{C}_{y,q}\cdot[\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y] + h_q^2\hat{V}_q\cdot[\hat{V}_q - 2\hat{C}_{y,q}].
\end{aligned}$$

Since $h_q^2\hat{V}_q < 1$ in equilibrium (see above), we have

$$[\hat{V}_q - 2\hat{C}_{y,q}] > \overbrace{\frac{2\sigma_i^2(1-\delta)(1-\rho)}{(1-\phi_i)^2(1-\delta\phi_i)}\frac{1}{1-h_q^2\hat{V}_q}}^{c>0} + \overbrace{\frac{2h_y^2\hat{C}_{y,q}}{1-h_q^2\hat{V}_q}}^{d>0}\cdot[\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y]. \quad (46)$$

Treating a, b, c, and d as fixed positive constants and $[\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y]$ and $[\hat{V}_q - 2\hat{C}_{y,q}]$ as unknowns to be characterized, we have shown that

$$[\hat{V}_q - 2\hat{C}_{y,q}]/d - c/d > [\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y] > a + b \cdot [\hat{V}_q - 2\hat{C}_{y,q}]. \tag{47}$$

Thus, it follows that the equilibrium solution must satisfy $[\hat{V}_q - 2\hat{C}_{y,q}] > 0$ and $[\hat{C}_{y,q} + \hat{C}_{y,y*} - \hat{V}_y] > 0$ when $\delta < 1$ and $\rho < 1$.

Additional results for Subsection 3.3.3 An analogous set of results to those presented in Section 3.3.3 also hold when $\sigma^2_{s^q} > 0$. In this case, we have

$$E_t\left[rx^q_{t+1}\right] = \overbrace{\left[\tau^{-1}C_{y,q}\right]}^{>0} \cdot (s^y_t - s^{y*}_t) + \overbrace{\left[\tau^{-1}V_q\right]}^{>0} \cdot s^q_t, \tag{48}$$

$$E_t\left[rx^y_{t+1} - rx^{y*}_{t+1}\right] = \overbrace{\left[\tau^{-1}\left(V_y - C_{y,y^*}\right)\right]}^{>0} \cdot (s^y_t - s^{y*}_t) + \overbrace{\left[2\tau^{-1}C_{y,q}\right]}^{>0} \cdot s^q_t. \tag{49}$$

Thus, both (i) the expected return on FX carry trade and (ii) difference between domestic and foreign bond risk premia are increasing in (a) the difference in domestic and foreign bond supply and (b) the supply of FX carry trade. Combining these equations, we obtain

$$E_t\left[rx^q_{t+1}\right] = \overbrace{\left[\frac{C_{y,q}}{V_y - C_{y,y^*}}\right]}^{>0} \cdot E_t\left[rx^y_{t+1} - rx^{y*}_{t+1}\right] + \overbrace{\left[\tau^{-1}\left(\frac{V_q\left(V_y - C_{y,y^*}\right) - 2\left(C_{y,q}\right)^2}{V_y - C_{y,y^*}}\right)\right]}^{>0} \cdot s^q_t. \tag{50}$$

As in Section 3.3.3, the expected return on the FX carry trade is increasing in the difference between domestic and foreign risk premia. However, there is now a second term that reflects the impact of FX supply on the FX carry trade over and above the impact that FX supply has on the difference in bond risk premia.[31]

Finally, the expected return on the long-term carry trade is

$$E_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right)\right] = \overbrace{\left[\tau^{-1}\left(C_{y,q} - \left(V_y - C_{y,y^*}\right)\right)\right]}^{\in(0,\tau^{-1}C_{q,q})} \cdot (s^y_t - s^{y*}_t) + \overbrace{\left[\tau^{-1}\left(V_q - 2C_{y,q}\right)\right]}^{\in(0,\tau^{-1}V_q)} \cdot s^q_t. \tag{51}$$

Thus, comparing equations (48) and (51), we see that the expected return on the long-term FX carry trade is less responsive to movements in $(s^y_t - s^{y*}_t)$ and s^q_t—and, hence, less variable over time—than that on the short-term FX carry trade. Specifically, since $(V_y - C_{y,y^*}) > 0$ and $C_{y,q} > 0$, we have $C_{y,q} - (V_y - C_{y,y^*}) < C_{y,q}$ and $V_q - 2C_{y,q} < V_q$. And, as shown above, both terms are still positive in the presence of supply risk.[32]

Finally, $\lim_{\delta\to1}\left[C_{y,q} + C_{y,y^*} - V_y\right] = \lim_{\rho\to1}\left[C_{y,q} + C_{y,y^*} - V_y\right] = 0$ and $\lim_{\delta\to1}\left[V_q - 2C_{y,q}\right] = \lim_{\rho\to1}\left[V_q - 2C_{y}\right.$ 0. Since $Var_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right)\right] = V_q + 2V_y - 2C_{y,y*} - 4C_{y,q}$, this implies that $\lim_{\delta\to1} Var_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} -\right.\right.$ $\lim_{\rho\to1} Var_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right)\right] = 0$. In the limiting cases where long-term bonds have infinite duration or whether the domestic and foreign short rates are perfectly correlated, the long-term FX carry trade is completely riskless and therefore earns a zero excess return.

Of course, this result assumes that there are no independent shocks to long-run FX fundamentals ($\sigma^2_{q^\infty} = 0$) as in our baseline model. When there are independent shocks to long-run FX fundamen-

[31] In the presence of supply risk, we have $\det(\mathbf{V}) = (V_y + C_{y,y^*})\left[V_q\left(V_y - C_{y,y^*}\right) - 2\left(C_{y,q}\right)^2\right] > 0$ Thus, since $(V_y + C_{y,y^*}) > 0$, we have $V_q\left(V_y - C_{y,y^*}\right) - 2\left(C_{y,q}\right)^2 > 0$ and the term multiplying s^q_t is positive.

[32] To interpret this expression, note that $\left[C_{y,q} - \left(V_y - C_{y,y^*}\right)\right] = Cov_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right), rx^y_{t+1}\right]$ and $\left[V_q - 2C_{y,q}\right] = Cov_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right), rx^q_{t+1}\right]$.

tals ($\sigma^2_{q\infty} > 0$), we still have $\lim_{\delta\to 1}\left[C_{y,q} + C_{y,y^*} - V_y\right] = \lim_{\rho\to 1}\left[C_{y,q} + C_{y,y^*} - V_y\right] = 0$. However, when $\sigma^2_{q\infty} > 0$, we now have $\lim_{\delta\to 1}\left[V_q - 2C_{y,q}\right] = \lim_{\rho\to 1}\left[V_q - 2C_{y,q}\right] > 0$ and $\lim_{\delta\to 1} Var_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right)\right] = \lim_{\rho\to 1} Var_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right)\right] > 0$.

D.4.3 Solution with only bond supply shocks

When $\sigma_{s_q} = 0$, we have $g_q = h_q = 0$ and the system of equations simplifies further. Specifically, the fixed point problem reduces to the following system of four equations in four unknowns:

$$V_y = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 + \left[g_y^2\left(V_y\right)^2 + g_y^2\left(C_{y,y^*}\right)^2\right] \tag{52a}$$

$$V_q = \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2\left(1-\rho\right) + \left[2h_y^2\left(C_{y,q}\right)^2\right] \tag{52b}$$

$$C_{y,y*} = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 + \left[2g_y^2 V_y C_{y,y^*}\right] \tag{52c}$$

$$C_{y,q} = \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) + \left[h_y g_y\left(V_y - C_{y,y^*}\right)\right] C_{y,q} \tag{52d}$$

We now assume that $\rho \in (0,1)$. This system can be solved using the following sequence of steps.

Step #1: Solve for $\Delta \equiv V_y - C_{y,y^*}$. Subtracting the condition (52c) for C_{y,y^*} the from condition (52a) for V_y, we obtain

$$V_y - C_{y,y*} = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2\left(1-\rho\right) + g_y^2\left(V_y - C_{y,y*}\right)^2 \geq 0.$$

We can solve the resulting quadratic for $\Delta \equiv V_y - C_{y,y^*} > 0$. The quadratic is

$$0 = g_y^2\Delta^2 - \Delta + \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2\left(1-\rho\right), \tag{53}$$

which has a real solution if and only if

$$\frac{\tau}{2} > \sqrt{1-\rho}\cdot\frac{\delta}{1-\delta\phi_i}\sigma_i\cdot\frac{\delta}{1-\delta\phi_{s^y}}\sigma_{s^y}. \tag{54}$$

The model's stable equilibrium corresponds to the smaller root of this quadratic and is given by

$$\hat{\Delta} = \frac{1-\sqrt{1-4g_y^2\left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2\left(1-\rho\right)}}{2g_y^2} > 0. \tag{55}$$

This stable solution $\hat{\Delta}$ converges to $\hat{\Delta} = 0$ as $\rho \to 1$ and to $\hat{\Delta} = \left(\delta\sigma_i/\left(1-\delta\phi_i\right)\right)^2\left(1-\rho\right)$ as $\tau^{-1}\sigma_{s^y} \to 0$. This stable solution also has natural comparative statics: $\partial\hat{\Delta}/\partial\tau < 0$, $\partial\hat{\Delta}/\partial\sigma_{s^y} > 0$, $\partial\hat{\Delta}/\partial\phi_{s^y} > 0$, $\partial\hat{\Delta}/\partial\sigma_i > 0$, $\partial\hat{\Delta}/\partial\phi_i > 0$, and $\partial\hat{\Delta}/\partial\rho < 0$.

Step #2: Substitute $\hat{\Delta}$ into condition (52d) for $C_{y,q}$ to obtain a solution for $C_{y,q}$. We substitute the solution for Δ into condition (52d) for $C_{y,q}$ and solve to obtain a solution for $C_{y,q}$. Doing so we obtain the following solution for $C_{y,q}$:

$$\hat{C}_{y,q} = \frac{\left(\frac{\delta\sigma_i}{1-\delta\phi_i}\right)\left(\frac{\sigma_i}{1-\phi_i}\right)(1-\rho)}{1-g_y h_y \hat{\Delta}} = \frac{\left(\frac{\delta\sigma_i}{1-\delta\phi_i}\right)\left(\frac{\sigma_i}{1-\phi_i}\right)(1-\rho)}{1-\frac{1}{2}\frac{\frac{1}{1-\phi_{sy}}}{\frac{\delta}{1-\delta\phi_{sy}}}\left(1-\sqrt{1-4\left(\frac{\delta\tau^{-1}\sigma_{sy}}{1-\delta\phi_{sy}}\right)^2\left(\frac{\delta\sigma_i}{1-\delta\phi_i}\right)^2(1-\rho)}\right)}. \tag{56}$$

We can show that we must have

$$1 > g_y h_y \hat{\Delta} = \left(\frac{\tau^{-1}\sigma_{s^y}}{1-\phi_{s^y}}\right)\left(\frac{\delta\tau^{-1}\sigma_{s^y}}{1-\delta\phi_{s^y}}\right)(\hat{V}_y - \hat{C}_{y,y^*}),$$

in any stable equilibrium. Thus, we have $\hat{C}_{y,q} \geq 0$ in any stable equilibrium and $\hat{C}_{y,q} > 0$ when $\rho \in [0,1)$. Intuitively, if this condition doesn't hold then, from condition (52d), a small perturbation to the equilibrium value of $C_{y,q}$ leads to larger and larger changes in $C_{y,q}$, indicating that the equilibrium solution is unstable. Formally, we can show that $g_y h_y \hat{\Delta} \geq 0$ is one of the eigenvalues of the Jacobian matrix of the relevant fixed point problem, so we must have $g_y h_y \hat{\Delta} < 1$ in any stable equilibrium.

Finally, it is easy to see that $\partial \hat{C}_{y,q}/\partial\rho < 0$ and that $\hat{C}_{y,q} = 0$ when $\rho = 1$.

Step #3: Substitute $\hat{\Delta}$ into condition (52c) for C_{y,y^*} to obtain a solution for C_{y,y^*}. Proceeding similarly, we substitute $V_y = \hat{\Delta} + C_{y,y^*}$ into condition (52c), we obtain

$$C_{y,y*} = \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 + 2g_y^2(\hat{\Delta} + C_{y,y^*})C_{y,y^*}.$$

Thus, we need to solve the following quadratic in C_{y,y^*}:

$$0 = 2g_y^2\left(C_{y,y^*}\right)^2 + (2g_y^2\hat{\Delta} - 1)C_{y,y^*} + \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2. \tag{57}$$

We can show that $2g_y^2\hat{\Delta} = 2g_y^2\left(V_y - C_{y,y^*}\right) < 1$ in any stable equilibrium. Specifically, we can show that $2g_y^2\Delta = 2g_y^2\left(V_y - C_{y,y^*}\right) \geq 0$ is one of the eigenvalues of the Jacobian matrix of the relevant fixed point operator, so we must have $2g_y^2\Delta = 2g_y^2\left(V_y - C_{y,y^*}\right) < 1$ in any stable equilibrium. Thus, so long as $\rho > 0$, it follows that $\hat{C}_{y,y^*} > 0$ in any stable equilibrium.

Since $1 - 2g_y^2\hat{\Delta} > 0$, a real solution exists so long as $1 - 2g_y^2\hat{\Delta} > 2\sqrt{2}g_y\left(\delta/\left(1-\delta\phi_i\right)\right)\sigma_i\sqrt{\rho}$. Using the expressions for $\hat{\Delta}$ and g_y, this is equivalent to

$$\frac{\tau}{2} > \sqrt{1+\rho}\cdot\frac{\delta}{1-\delta\phi_i}\sigma_i\cdot\frac{\delta}{1-\delta\phi_{s^y}}\sigma_{s^y}.$$

The relevant stable solution for C_{y,y^*} is

$$\hat{C}_{y,y^*} = \frac{(1-2g_y^2\hat{\Delta}) - \sqrt{(1-2g_y^2\hat{\Delta})^2 - 8g_y^2\left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2}}{4g_y^2.} > 0.$$

At this stable solution, we have $\hat{C}_{y,y^*} \to 0$ when $\rho \to 0$ and $\hat{C}_{y,y^*} \to \hat{V}_y$ when $\rho \to 1$.

Step #4: Obtain solutions for V_y and V_q. The solution for V_y is trivially given by

$$\hat{V}_y = \hat{\Delta} + \hat{C}_{y,y^*} > 0.$$

And, the solution for V_q is given by

$$\hat{V}_q = \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2 (1-\rho) + 2\left(\frac{\tau^{-1}\sigma_{s^y}}{1-\phi_{s^y}}\right)^2 (\hat{C}_{y,q})^2 > 0.$$

Solution summary. When $\sigma_{s^q}^2 = 0$, $\sigma_{s^y}^2 > 0$, and $\rho \in (0,1)$, we have $\hat{V}_y > 0$, $\hat{V}_q > 0$, $\hat{C}_{y,q} > 0$, and $\hat{C}_{y,y^*} > (\delta/(1-\delta\phi_i))^2 \rho\sigma_i^2 > 0$ in the model's unique stable equilibrium. Thus, in this case, all equilibrium variance and covariances exceed those in the absence of supply risk.

Solution properties. We are interested in the term in square brackets in

$$E_t\left[rx_{t+1}^q\right] = \left[\frac{\hat{C}_{y,q}}{\hat{V}_y - \hat{C}_{y,y^*}}\right] \cdot E_t\left[rx_{t+1}^y - rx_{t+1}^{y*}\right]. \tag{58}$$

Since $\hat{C}_{y,q} > 0$ and $\hat{V}_y - \hat{C}_{y,y^*} > 0$ in any stable equilibrium, this quantity is obviously positive. We now show that $[\hat{C}_{y,q}/(\hat{V}_y - \hat{C}_{y,y^*})] > 1$. Using equation (53), we can rewrite this term as

$$\frac{\hat{C}_{y,q}}{\hat{V}_y - \hat{C}_{y,y^*}} = \frac{1}{\hat{\Delta}} \frac{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)\left(\frac{1}{1-\phi_i}\sigma_i\right)(1-\rho)}{1 - \left(\frac{\tau^{-1}\sigma_{sy}}{1-\phi_{sy}}\right)\left(\frac{\delta\tau^{-1}\sigma_{sy}}{1-\delta\phi_{sy}}\right)\hat{\Delta}} = \frac{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)\left(\frac{1}{1-\phi_i}\sigma_i\right)}{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)^2 - \frac{1-\delta}{\delta(1-\phi_{sy})}\left(\hat{\Delta}^\rho - \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2\right)}.$$

where $\hat{\Delta}^\rho \equiv \hat{\Delta}/(1-\rho)$ is the smaller root of the following quadratic equation

$$0 = \left(\frac{\delta\tau^{-1}\sigma_{s^y}}{1-\delta\phi_{s^y}}\right)^2 (1-\rho)(\Delta^\rho)^2 - \Delta^\rho + \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2.$$

Since $\hat{\Delta}^\rho > (\delta/(1-\delta\phi_i))^2 \sigma_i^2$ when $\rho < 1$, it follows that

$$\frac{\hat{C}_{y,q}}{\hat{V}_y - \hat{C}_{y,y^*}} = \frac{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)\left(\frac{1}{1-\phi_i}\sigma_i\right)}{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)^2 - \frac{1-\delta}{\delta(1-\phi_{sy})}\left(\hat{\Delta}^\rho - \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2\right)} > \frac{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)\left(\frac{1}{1-\phi_i}\sigma_i\right)}{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)^2} = \frac{\frac{1}{1-\phi_i}}{\frac{\delta}{1-\delta\phi_i}} > 1.$$

It is also easy to see that $\partial\hat{\Delta}^\rho/\partial\rho < 0$ when $\sigma_{s^y}^2 > 0$, thus we have

$$\frac{\partial}{\partial\rho}\left[\frac{\hat{C}_{y,q}}{\hat{V}_y - \hat{C}_{y,y^*}}\right] \propto \frac{\partial\hat{\Delta}^\rho}{\partial\rho} < 0$$

when $\sigma_{s^y}^2 > 0$. Specifically, we have

$$\frac{\partial}{\partial\rho}\left[\frac{\hat{C}_{y,q}}{\hat{V}_y - \hat{C}_{y,y^*}}\right] = -\frac{\hat{C}_{y,q}}{\hat{V}_y - \hat{C}_{y,y^*}} \frac{\frac{1-\delta}{\delta(1-\phi_{sy})}}{\left(\frac{\delta}{1-\delta\phi_i}\sigma_i\right)^2 - \frac{1-\delta}{\delta(1-\phi_{sy})}\left(\hat{\Delta}^\rho - \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2\right)} \frac{\left(\frac{\delta\tau^{-1}\sigma_{sy}}{1-\delta\phi_{sy}}\right)^2(\Delta^\rho)^2}{1 - 2\left(\frac{\delta\tau^{-1}\sigma_{sy}}{1-\delta\phi_{sy}}\right)^2(1-\rho)\Delta^\rho}.$$

Finally, since $\hat{C}_{y,q} > (\hat{V}_y - \hat{C}_{y,y^*})$, it follows that

$$E_t\left[rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right)\right] = \overbrace{\left[\tau^{-1}(\hat{C}_{y,q} - (\hat{V}_y - \hat{C}_{y,y^*}))\right]}^{>0} \cdot \left(s^y_t - s^{y*}_t\right) = \overbrace{\left[1 - \frac{\hat{V}_y - \hat{C}_{y,y^*}}{\hat{C}_{y,q}}\right]}^{\in(0,1)} \cdot E_t\left[rx^q_{t+1}\right].$$

D.5 A unified approach to carry trade returns

In Section 3.4, we extend the baseline model to explain the linkages between expected carry trade returns and short-term interest rates. To do so, we assume that the total net supplies that must be absorbed by arbitrageurs are

$$\mathbf{n}_t = \mathbf{s}_t + \mathbf{S}_2\mathbf{y}_t \text{ where } \mathbf{S}_2 \equiv \begin{bmatrix} -S_y & 0 & 0 \\ 0 & -S_y & 0 \\ 0 & 0 & S_q \end{bmatrix}. \tag{59}$$

Thus, following Vayanos and Vila (2021), we assume the supplies of long-term bonds in both currencies are decreasing in the relevant long-term bond yield; and, following Gabaix and Maggiori (2015), we assume the supply of the FX carry trade is increasing in strength of the foreign currency. Since $\mathbf{s}_t = \mathbf{s}_0 + \mathbf{S}_1\mathbf{z}_t$ and $\mathbf{y}_t = \mathbf{a} + \mathbf{A}\mathbf{z}_t$, we have $\mathbf{n}_t = (\mathbf{s}_0 + \mathbf{S}_2\mathbf{a}) + (\mathbf{S_1} + \mathbf{S_2}\mathbf{A})\,\mathbf{z}_t$.

The market clearing condition for the extended model, namely $E_t\left[\mathbf{rx}_{t+1}\right] = \tau^{-1}\mathbf{V}\mathbf{n}_t$, can be written as

$$[\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0] + [\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\boldsymbol{\Phi} + \mathbf{R}_1]\,\mathbf{z}_t = \tau^{-1}\mathbf{V}\left(\mathbf{s}_0 + \mathbf{S}_2\mathbf{a}\right) + \tau^{-1}\mathbf{V}\left(\mathbf{S_1} + \mathbf{S_2}\mathbf{A}\right)\mathbf{z}_t, \tag{60}$$

where $\mathbf{V} = (\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1')$.

Matching slope coefficients in equation (60), we have

$$[\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}] \circ \mathbf{A} - \tau^{-1}\mathbf{V}\mathbf{S}_2\mathbf{A} = \tau^{-1}\mathbf{V}\mathbf{S_1} - \mathbf{R}_1.$$

To solve for $\mathbf{A}$, we vectorize this condition to obtain

$$diag\left(vec\left(\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}\right)\right) vec\left(\mathbf{A}\right) - \tau^{-1}\left(\mathbf{I}_5 \otimes \left(\mathbf{V}\mathbf{S}_2\right)\right) vec\left(\mathbf{A}\right) = vec\left(\tau^{-1}\mathbf{V}\mathbf{S_1} - \mathbf{R}_1\right)$$

where $\mathbf{I}_5$ is the 5×5 identity matrix and $\otimes$ denotes a Kronecker product. Solving this equation $vec\left(\mathbf{A}\right)$, we require

$$vec\left(\mathbf{A}\right) = \left[diag\left(vec\left(\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}\right)\right) - \tau^{-1}\left(\mathbf{I}_5 \otimes \left(\mathbf{V}\mathbf{S}_2\right)\right)\right]^{-1} vec\left(\tau^{-1}\mathbf{V}\mathbf{S_1} - \mathbf{R}_1\right), \tag{61}$$

where we note that the matrix is square brackets is block-diagonal. Finally, since $\mathbf{V} = \mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'$, we obtain the following fixed-point point problem in $\mathbf{A}$:

$$vec\left(\mathbf{A}\right) = \left[diag\left(vec\left(\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\boldsymbol{\Phi}\right)\right) - \tau^{-1}\left(\mathbf{I}_5 \otimes \left(\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'\mathbf{S}_2\right)\right)\right]^{-1} vec\left(\tau^{-1}\mathbf{B}_1\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'\mathbf{B}_1'\mathbf{S_1} - \mathbf{R}_1\right). \tag{62}$$

Natural, the fixed point problem in equation (61) reduces to that in equation (31) when $\mathbf{S}_2 = \mathbf{0}$.

Matching constant terms in equation (60), we obtain

$$\left(\mathbf{B}_0 + \mathbf{B}_1 - \tau^{-1}\mathbf{V}\mathbf{S}_2\right)\mathbf{a} = \left(\tau^{-1}\mathbf{V}\mathbf{s}_0 - \mathbf{r}_0\right).$$

This condition always allows us to pin down the steady state levels of bond yields, α^y_0 and α^{y*}_0. When

$S_q = 0$, $(\mathbf{B}_0+\mathbf{B}_1 - \tau^{-1}\mathbf{V}\mathbf{S}_2)$ is singular and the steady-state level of exchange rates α_0^q is not pinned down as in the baseline model. However, when $S_q > 0$, $(\mathbf{B}_0+\mathbf{B}_1 - \tau^{-1}\mathbf{V}\mathbf{S}_2)$ is invertible and α_0^q is pinned down. Specifically, since we have assumed the home and foreign countries are perfectly symmetric, we have $\alpha_0^q = 0$. (We would not have $\alpha_0^q = 0$ if the countries were not symmetric.) Intuitively, α_0^q is pinned down because the steady-state supply of the FX carry trade depends on the steady-state level of the exchange rate.

We are mainly interested in the loadings on i_t and i_t^*. Using equation (61), the loadings on i_t take the form

$$\begin{bmatrix} \alpha_i^y \\ \alpha_i^{y*} \\ \alpha_i^q \end{bmatrix} = \begin{bmatrix} \frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y V_y & \tau^{-1}S_y C_{y,y^*} & -\tau^{-1}S_q C_{y,q} \\ \tau^{-1}S_y C_{y,y^*} & \frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y V_y & \tau^{-1}S_q C_{y,q} \\ \tau^{-1}S_y C_{y,q} & -\tau^{-1}S_y C_{y,q} & -(1-\phi_i) - \tau^{-1}S_q V_q \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \tag{63}$$

and the loadings on i_t^* take the form

$$\begin{bmatrix} \alpha_{i^*}^y \\ \alpha_{i^*}^{y*} \\ \alpha_{i^*}^q \end{bmatrix} = \begin{bmatrix} \frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y V_y & \tau^{-1}S_y C_{y,y^*} & -\tau^{-1}S_q C_{y,q} \\ \tau^{-1}S_y C_{y,y^*} & \frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y V_y & \tau^{-1}S_q C_{y,q} \\ \tau^{-1}S_y C_{y,q} & -\tau^{-1}S_y C_{y,q} & -(1-\phi_i) - \tau^{-1}S_q V_q \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}. \tag{64}$$

Thus, it follows that

$$\alpha_{i^*}^{y*} = \alpha_i^y,\ \alpha_{i^*}^y = \alpha_i^{y*},\ \text{and } \alpha_{i^*}^q = -\alpha_i^q. \tag{65}$$

We study the extended model in two specific cases:

- **Case #1:** $S_q > 0$ and $S_y = 0$.
- **Case #2:** $S_q = 0$ and $S_y > 0$.

D.5.1 Calculations for Case #1: $S_q > 0$ and $S_y = 0$.

General analysis When $S_q > 0$ and $S_y = 0$, equation (63) implies that the loadings on i_t are

$$\begin{bmatrix} \alpha_i^y \\ \alpha_i^{y*} \\ \alpha_i^q \end{bmatrix} = \begin{bmatrix} \frac{1-\delta}{1-\delta\phi_i} \frac{1-\phi_i+\tau^{-1}S_q V_q - \tau^{-1}S_q C_{y,q}}{1-\phi_i+\tau^{-1}S_q V_q} \\ \frac{1-\delta}{1-\delta\phi_i} \frac{\tau^{-1}S_q C_{y,q}}{1-\phi_i+\tau^{-1}S_q V_q} \\ -\frac{1}{1-\phi_i+\tau^{-1}S_q V_q} \end{bmatrix} \tag{66}$$

and the loading on i_t^* again satisfy equation (65). Using equations (66) and (65), we note that

$$\alpha_i^y + \alpha_{i^*}^y = \frac{1-\delta}{1-\delta\phi_i} < 1.$$

Bounding the quantities in equation (66), we see that:

1. $\alpha_i^y \in \left(0, \frac{1-\delta}{1-\delta\phi_i}\right)$ so long as (i) $0 < C_{y,q}$ and (ii) $\tau^{-1}S_q C_{y,q} < 1 - \phi_i + \tau^{-1}S_q V_q$;
   - $\alpha_i^y < \frac{1-\delta}{1-\delta\phi_i}$ so long as (i) $0 < C_{y,q}$;
   - $\alpha_i^y > 0$ so long as (ii) $\tau^{-1}S_q C_{y,q} < 1 - \phi_i + \tau^{-1}S_q V_q$;
2. $\alpha_i^{y*} > 0$ so long as $0 < C_{y,q}$;

3. $\alpha_i^q \in \left(-\frac{1}{1-\phi_i}, 0\right)$ since $V_q > 0$.

We now explore how changes in short-term interest rates impact equilibrium expected excess returns. Since $V_q > 0$, we have

$$\gamma_{i^*}^q \equiv \partial E_t\left[rx_{t+1}^q\right]/\partial i_t^* = (\phi_i - 1)\,\alpha_{i^*}^q + 1 = \frac{\tau^{-1}S_q V_q}{1-\phi_i+\tau^{-1}S_q V_q} > 0.$$

Symmetrically, we have $\gamma_i^q \equiv \partial E_t\left[rx_{t+1}^q\right]/\partial i_t = -\gamma_{i^*}^q$. Furthermore, so long as $C_{y,q} > 0$, we have

$$\begin{aligned}
\gamma_i^y &\equiv \partial E_t\left[rx_{t+1}^y\right]/\partial i_t = \left(\frac{1-\delta\phi_i}{1-\delta}\right)\alpha_i^y - 1 = -\frac{\tau^{-1}S_q C_{y,q}}{1-\phi_i+\tau^{-1}S_q V_q} < 0, \\
\gamma_{i^*}^y &\equiv \partial E_t\left[rx_{t+1}^y\right]/\partial i_t^* = \left(\frac{1-\delta\phi_i}{1-\delta}\right)\alpha_{i^*}^y = \frac{\tau^{-1}S_q C_{y,q}}{1-\phi_i+\tau^{-1}S_q V_q} > 0.
\end{aligned}$$

Symmetrically, we have $\gamma_{i^*}^{y*} \equiv \partial E_t\left[rx_{t+1}^{y*}\right]/\partial i_t^* = \gamma_i^y$ and $\gamma_i^{y*} \equiv \partial E_t\left[rx_{t+1}^{y*}\right]/\partial i_t = \gamma_{i^*}^y$.

Closing the model in Case #1 when there are no independent supply shocks It is straightforward to analytically solve for the equilibrium variance and covariance terms in case where there are only shocks to the two short rates—i.e., when $\sigma_{s^y}^2 = \sigma_{s^q}^2 = 0$. In this case, we can show that (i) $0 < C_{y,q}$ and (ii) $\tau^{-1}S_q C_{y,q} < 1-\phi_i+\tau^{-1}S_q V_q$, confirming that the bounds discussed above indeed hold.

Assume there are only short rate shocks. Making use of the facts that $\alpha_{i^*}^{y*} = \alpha_i^y$, $\alpha_{i^*}^y = \alpha_i^{y*}$, and $\alpha_{i^*}^q = -\alpha_i^q$, we have

$$\begin{aligned}
rx_{t+1}^y - E_t\left[rx_{t+1}^y\right] &= -\frac{\delta}{1-\delta}\left(\alpha_i^y\varepsilon_{i,t+1} + \alpha_{i^*}^y\varepsilon_{i^*,t+1}\right) \\
rx_{t+1}^{y*} - E_t\left[rx_{t+1}^{y*}\right] &= -\frac{\delta}{1-\delta}\left(\alpha_{i^*}^y\varepsilon_{i,t+1} + \alpha_i^y\varepsilon_{i^*,t+1}\right) \\
rx_{t+1}^q - E_t\left[rx_{t+1}^q\right] &= \alpha_{i^*}^q\left(\varepsilon_{i^*,t+1} - \varepsilon_{i,t+1}\right).
\end{aligned}$$

It is then easy to solve for the equilibrium level of V_q. We have

$$V_q = Var\left[\alpha_{i^*}^q\left(\varepsilon_{i^*,t+1} - \varepsilon_{i,t+1}\right)\right] = \left(\alpha_{i^*}^q\right)^2 2\sigma_i^2\left(1-\rho\right).$$

Using the fact that $\alpha_{i^*}^q = 1/\left(1-\phi_i+\tau^{-1}S_q V_q\right)$, we obtain the following fixed-point condition for V_q:

$$V_q = f\left(V_q\right) = \frac{2\sigma_i^2\left(1-\rho\right)}{\left(1-\phi_i+\tau^{-1}S_q V_q\right)^2} > 0.$$

Since $f\left(V_q\right) > 0$, $f'\left(V_q\right) < 0$, and $\lim_{V_q\to\infty} f\left(V_q\right) = 0$, it follows that there is a unique solution $\hat{V}_q > 0$. It is also easy to see that $\partial\hat{V}_q/\partial S_q < 0$, so we have $\hat{V}_q < 2\sigma_i^2\left(1-\rho\right)/\left(1-\phi_i\right)^2$. However, we have $\partial(S_q\hat{V}_q)/\partial S_q > 0$.

We now solve for $C_{y,q}$. We have

$$\begin{aligned} C_{y,q} &= -\frac{\delta}{1-\delta} Cov\left[\alpha_i^y \varepsilon_{i,t+1} + \alpha_{i^*}^y \varepsilon_{i^*,t+1}, \alpha_{i^*}^q \left(\varepsilon_{i^*,t+1} - \varepsilon_{i,t+1}\right)\right] \\ &= \frac{\delta}{1-\delta} \left(1-\rho\right) \sigma_i^2 \alpha_{i^*}^q \left(\alpha_i^y - \alpha_{i^*}^y\right) \\ &= \frac{\left(1-\rho\right)\sigma_i^2}{1-\phi_i + \tau^{-1} S_q V_q} \frac{\delta}{1-\delta\phi_i} \left(\frac{1-\phi_i + \tau^{-1} S_q V_q - \tau^{-1} S_q 2 C_{y,q}}{1-\phi_i + \tau^{-1} S_q V_q}\right). \end{aligned}$$

where the third line uses the prior expressions linking $\alpha_{i^*}^q$, α_i^y, and $\alpha_{i^*}^y$ to V_q and $C_{y,q}$. Thus, the equilibrium value of $C_{y,q}$ is

$$\hat{C}_{y,q} = \frac{\frac{(1-\rho)\sigma_i^2}{1-\phi_i+\tau^{-1}S_q\hat{V}_q} \frac{\delta}{1-\delta\phi_i}}{1 + 2\tau^{-1} S_q \left(\frac{(1-\rho)\sigma_i^2}{\left(1-\phi_i+\tau^{-1}S_q\hat{V}_q\right)^2} \frac{\delta}{1-\delta\phi_i}\right)} > 0$$

where $\rho < 1$.

We now show that

$$1 - \phi_i + \tau^{-1} S_q \hat{V}_q > \tau^{-1} S_q \hat{C}_{y,q},$$

which guarantees that $\hat{\alpha}_i^y > 0$. We have

$$\begin{aligned} \hat{C}_{y,q} &= \frac{\delta}{1-\delta} \left(1-\rho\right) \sigma_i^2 \hat{\alpha}_{i^*}^q \left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right) > 0 \\ &\iff \left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right) > 0 \text{ [since } \frac{\delta}{1-\delta} \left(1-\rho\right) \sigma_i^2 \hat{\alpha}_{i^*}^q > 0] \\ &\iff \left(\frac{1-\delta}{1-\delta\phi_i} - 2\hat{\alpha}_{i^*}^y\right) > 0 \text{ [since } \hat{\alpha}_i^y = \frac{1-\delta}{1-\delta\phi_i} - \hat{\alpha}_{i^*}^y] \\ &\iff 1 > \frac{2\tau^{-1} S_q \hat{C}_{y,q}}{1-\phi_i + \tau^{-1} S_q \hat{V}_q} \text{ [since } \hat{\alpha}_{i^*}^y = \frac{1-\delta}{1-\delta\phi_i} \frac{\tau^{-1} S_q \hat{C}_{y,q}}{1-\phi_i + \tau^{-1} S_q \hat{V}_q}. \end{aligned}$$

Since $\hat{C}_{y,q} > 0$, it then follows that

$$1 - \phi_i + \tau^{-1} S_q \hat{V}_q > 2\tau^{-1} S_q \hat{C}_{y,q} > \tau^{-1} S_q \hat{C}_{y,q}.$$

Finally, we show that $\gamma_{i^*}^q + \left(\gamma_{i^*}^{y*} - \gamma_{i*}^y\right) > 0$. We have

$$\gamma_{i^*}^q + \left(\gamma_{i^*}^{y*} - \gamma_{i*}^y\right) = \frac{\tau^{-1} S_q \left(V_q - 2C_{y,q}\right)}{1 - \phi_i + \tau^{-1} S_q V_q}.$$

Thus, it suffices to show that $0 < V_q - 2C_{y,q}$. We have

$$\begin{aligned} \left[V_q - 2C_{y,q}\right] &= Cov_t\left[\left(\alpha_{i^*}^q - \frac{\delta}{1-\delta}\left(\alpha_i^y - \alpha_{i^*}^y\right)\right)\left(\varepsilon_{i^*,t+1} - \varepsilon_{i,t+1}\right), \alpha_{i^*}^q \left(\varepsilon_{i^*,t+1} - \varepsilon_{i,t+1}\right)\right] \\ &= \alpha_{i^*}^q \left(\alpha_{i^*}^q - \frac{\delta}{1-\delta}\left(\alpha_i^y - \alpha_{i^*}^y\right)\right) 2\sigma_i^2 \left(1-\rho\right) \\ &= \left(\frac{1}{1-\phi_i + \tau^{-1} S_q V_q}\right)^2 \left(1 - \delta\left(\frac{1-\phi_i + \tau^{-1} S_q \left[V_q - 2C_{y,q}\right]}{1-\delta\phi_i}\right)\right) 2\sigma_i^2 \left(1-\rho\right) \end{aligned}$$

Thus, we have

$$[V_q - 2C_{y,q}] = \frac{\left(\frac{1}{1-\phi_i+\tau^{-1}S_qV_q}\right)^2 \frac{1-\delta}{1-\delta\phi_i}2\sigma_i^2(1-\rho)}{1+\left(\frac{1}{1-\phi_i+\tau^{-1}S_q\hat{V}_q}\right)^2 \delta\left(\frac{\tau^{-1}S_q}{1-\delta\phi_i}\right)2\sigma_i^2(1-\rho)} \geq 0.$$

Furthermore, we have $[V_q - 2C_{y,q}] > 0$ when $\delta < 1$ and $\lim_{\delta\to 1}[V_q - 2C_{y,q}] = 0$.

D.5.2 Calculations for Case #2: $S_q = 0$ and $S_y > 0$.

General When $S_q = 0$ and $S_y > 0$, the loadings on i_t are

$$\begin{bmatrix} \alpha_i^y \\ \alpha_i^{y*} \\ \alpha_i^q \end{bmatrix} = \begin{bmatrix} \frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y & \tau^{-1}S_yC_{y,y^*} & 0 \\ \tau^{-1}S_yC_{y,y^*} & \frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y & 0 \\ \tau^{-1}S_yC_{y,q} & -\tau^{-1}S_yC_{y,q} & -(1-\phi_i) \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \tag{67}$$

$$= \begin{bmatrix} \left(\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y - \frac{\left(\tau^{-1}S_yC_{y,y*}\right)^2}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}\right)^{-1} \\ -\frac{\tau^{-1}S_yC_{y,y}}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}\alpha_i^y \\ -\frac{1}{1-\phi_i} + \frac{\tau^{-1}S_yC_{y,q}}{1-\phi_i}\left(\alpha_i^y - \alpha_i^{y*}\right) \end{bmatrix}$$

and the loading on i_t^* again satisfy equation (65). Since $V_y - C_{y,y*} = V_y\left(1 - Corr\left[rx_{t+1}^y, rx_{t+1}^{y*}\right]\right) > 0$, it follows that

$$\alpha_i^y - \alpha_{i*}^y = \frac{1}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y\left(V_y - C_{y,y*}\right)} > 0.$$

Bounding the quantities in equation (67), we have:

1. $\alpha_i^y = \left(\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y - \frac{\left(\tau^{-1}S_yC_{y,y*}\right)^2}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}\right)^{-1} \in \left(0, \frac{1-\delta}{1-\delta\phi_i}\right)$.

   - To show $\alpha_i^y > 0$, notice that
   $$\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y > 0$$
   and
   $$\left(\tau^{-1}S_yC_{y,y*}\right)^2 < \left(\tau^{-1}S_yV_y\right)^2 < \left(\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y\right)^2.$$
   Together these inequalities imply
   $$\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y - \frac{\left(\tau^{-1}S_yC_{y,y*}\right)^2}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y} > 0,$$
   confirming that $\alpha_i^y > 0$.
   - To show $\alpha_i^y < \frac{1-\delta}{1-\delta\phi_i}$, it suffices to show
   $$\tau^{-1}S_yV_y - \frac{\left(\tau^{-1}S_yC_{y,y*}\right)^2}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_yV_y} > 0.$$

This is equivalent to

$$\left(\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y\right)\tau^{-1}S_yV_y-\left(\tau^{-1}S_yC_{y,y*}\right)^2>0$$

which is true since $\left(\tau^{-1}S_yC_{y,y*}\right)^2<\left(\tau^{-1}S_yV_y\right)^2$ and $\frac{1-\delta\phi_i}{1-\delta}>0$.

2. $\alpha^y_{i^*}=-\frac{\tau^{-1}S_yC_{y,y^*}}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}\alpha^y_i$. Since $\alpha^y_i>0$, we have $sign\left(\alpha^y_{i^*}\right)=sign(-C_{y,y^*})$.

3. $\alpha^q_i=-\frac{1}{1-\phi_i}+\frac{\tau^{-1}S_yC_{y,q}}{1-\phi_i}(\alpha^y_i-\alpha^y_{i^*})\in\left(-\frac{1}{1-\phi_i},0\right)$ so long as (i) $C_{y,q}>0$ and (ii) $\tau^{-1}S_yC_{y,q}\left(\alpha^y_i-\alpha^{y*}_i\right)<1$.

   - Since $(\alpha^y_i-\alpha^y_{i^*})>0$, we have $\alpha^q_i>-\frac{1}{1-\phi_i}$ so long as (i) $C_{y,q}>0$.
   - And, we have $\alpha^q_i<0$ so long as (ii) $\tau^{-1}S_yC_{y,q}(\alpha^y_i-\alpha^y_{i^*})<1$.

We now explore how changes in short-term interest rates impact equilibrium expected excess returns. So long as $C_{y,q}>0$, we have

$$\gamma^q_{i^*}\equiv\partial E_t\left[rx^q_{t+1}\right]/\partial i^*_t=\left(\phi_i-1\right)\alpha^q_{i^*}+1=\tau^{-1}S_yC_{y,q}(\alpha^y_i-\alpha^y_{i^*})>0.$$

Symmetrically, we have $\gamma^q_i\equiv\partial E_t\left[rx^q_{t+1}\right]/\partial i_t=-\gamma^q_{i^*}$. Furthermore, since $\alpha^y_i<(1-\delta)/(1-\delta\phi_i)$, we have

$$\gamma^y_i\equiv\partial E_t\left[rx^y_{t+1}\right]/\partial i_t=\left(\frac{1-\delta\phi_i}{1-\delta}\right)\alpha^y_i-1<0.$$

Finally, so long as $C_{y,y^*}>0$, we have $\alpha^y_{i^*}<0$ and thus

$$\gamma^y_{i^*}\equiv\partial E_t\left[rx^y_{t+1}\right]/\partial i^*_t=\left(\frac{1-\delta\phi_i}{1-\delta}\right)\alpha^y_{i^*}<0.$$

Symmetrically, we have $\gamma^{y*}_{i*}\equiv\partial E_t\left[rx^{y*}_{t+1}\right]/\partial i^*_t=\gamma^y_i$ and $\gamma^{y*}_i\equiv\partial E_t\left[rx^{y*}_{t+1}\right]/\partial i_t=\gamma^y_{i^*}$.

Next, we show that $\gamma^y_i-\gamma^y_{i^*}<0$. To see this, note that

$$\begin{aligned}\gamma^y_i-\gamma^y_{i^*}&=\left(\frac{1-\delta\phi_i}{1-\delta}\right)(\alpha^y_i-\alpha^y_{i^*})-1\\&=\left(\frac{1-\delta\phi_i}{1-\delta}\right)\alpha^y_i\left(1+\frac{\tau^{-1}S_yC_{y,y^*}}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}\right)-1.\end{aligned}$$

Thus, we have $\gamma^y_i-\gamma^y_{i^*}<0$ so long as we have

$$\frac{1-\delta}{1-\delta\phi_i}\frac{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y+\tau^{-1}S_yC_{y,y^*}}>\alpha^y_i=\left(\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y-\frac{\left(\tau^{-1}S_yC_{y,y*}\right)^2}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_yV_y}\right)^{-1}.$$

One can show that this condition is equivalent to

$$\frac{1-\delta}{1-\delta\phi_i}\tau^{-1}S_y\left(V_y-C_{y,y^*}\right)>0,$$

which is always true since $V_y>C_{y,y^*}$.

Closing the model in Case #2 when there are no supply shocks When there are no independent supply shocks—i.e., when $\sigma_{s^y}^2 = \sigma_{s^q}^2 = 0$, it is easy to confirm that we must have $C_{y,q} > 0$ and (ii) $\tau^{-1} S_y C_{y,q}(\alpha_i^y - \alpha_{i^*}^y) < 1$. Thus, the bounds noted above must indeed hold in this case. We can also show that we must have $C_{y,y^*} > 0$ in this case.

When $\sigma_{s^y}^2 = \sigma_{s^q}^2 = 0$, we have

$$\begin{aligned} C_{y,q} &= -\frac{\delta}{1-\delta} Cov\left[\alpha_i^y \varepsilon_{i,t+1} + \alpha_{i^*}^y \varepsilon_{i^*,t+1}, \alpha_{i^*}^q \left(\varepsilon_{i^*,t+1} - \varepsilon_{i,t+1}\right)\right] \\ &= \frac{\delta}{1-\delta} \left(1-\rho\right) \sigma_i^2 \alpha_{i^*}^q \left(\alpha_i^y - \alpha_{i^*}^y\right) \\ &= \sigma_i^2 \left(1-\rho\right) \frac{\delta}{1-\delta} \frac{\left(\alpha_i^y - \alpha_{i^*}^y\right)}{1-\phi_i} \left(1 - \tau^{-1} S_y C_{y,q}(\alpha_i^y - \alpha_{i^*}^y)\right) \end{aligned}$$

where the last line follows from the fact that

$$\alpha_{i^*}^q = \frac{1}{1-\phi_i} \left(1 - \tau^{-1} S_y C_{y,q}(\alpha_i^y - \alpha_{i^*}^y)\right).$$

Thus, given an equilibrium solution for $(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y) > 0$, we have

$$\hat{C}_{y,q} = \frac{\sigma_i^2 \left(1-\rho\right) \frac{\delta}{1-\delta} \frac{\left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right)}{1-\phi_i}}{1 + \tau^{-1} S_y \sigma_i^2 \left(1-\rho\right) \frac{\delta}{1-\delta} \frac{\left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right)^2}{1-\phi_i}} > 0.$$

We also have

$$\tau^{-1} S_y \hat{C}_{y,q} \left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right) = \frac{\tau^{-1} S_y \sigma_i^2 \left(1-\rho\right) \frac{\delta}{1-\delta} \frac{\left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right)^2}{1-\phi_i}}{1 + \tau^{-1} S_y \sigma_i^2 \left(1-\rho\right) \frac{\delta}{1-\delta} \frac{\left(\hat{\alpha}_i^y - \hat{\alpha}_{i^*}^y\right)^2}{1-\phi_i}} < 1.$$

We show that we must have $C_{y,y^*} > 0$. We have

$$\begin{aligned} C_{y,y^*} &= \left(\frac{\delta}{1-\delta}\right)^2 Cov\left[\alpha_i^y \varepsilon_{i,t+1} + \alpha_{i^*}^y \varepsilon_{i^*,t+1}, \alpha_{i^*}^y \varepsilon_{i,t+1} + \alpha_i^y \varepsilon_{i^*,t+1}\right] \\ &= \left(\frac{\delta}{1-\delta}\right)^2 \left[2\alpha_i^y \alpha_{i^*}^y \sigma_i^2 + \left(\alpha_i^y\right)^2 \rho \sigma_i^2 + \left(\alpha_{i^*}^y\right)^2 \rho \sigma_i^2\right] \\ &= \left(\frac{\delta}{1-\delta}\right)^2 \left(\alpha_i^y\right)^2 \sigma_i^2 \left[\rho \left(\frac{\tau^{-1} S_y C_{y,y^*}}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1} S_y V_y}\right)^2 - 2 \frac{\tau^{-1} S_y C_{y,y^*}}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1} S_y V_y} + \rho\right]. \end{aligned}$$

Suppose that $C_{y,y^*} < 0$. If $C_{y,y^*} < 0$, then this equation implies that $C_{y,y^*} > 0$ so long as $\rho > 0$: a contradiction. Thus, when $\rho > 0$, we must have $C_{y,y^*} > 0$ in equilibrium. Of course, when $\rho = 0$, we have $C_{y,y^*} = 0$ in equilibrium.

Finally, we show that $\gamma_{i^*}^q + (\gamma_{i^*}^{y*} - \gamma_{i^*}^y) \geq 0$. We have

$$\gamma_{i^*}^q + (\gamma_{i^*}^{y*} - \gamma_{i^*}^y) = \tau^{-1} S_y C_{y,q}(\alpha_i^y - \alpha_{i^*}^y) + \left(\frac{1-\delta\phi_i}{1-\delta}\right)(\alpha_i^y - \alpha_{i^*}^y) - 1 = \frac{\tau^{-1} S_y \left(C_{y,y*} + C_{y,q} - V_y\right)}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1} S_y \left(V_y - C_{y,y*}\right)}$$

Since $V_y > C_{y,y*}$, it suffices to show that $0 < C_{y,y*} + C_{y,q} - V_y$. We have

$$\begin{aligned}
& [C_{y,y*} + C_{y,q} - V_y] \\
= \; & Cov_t\left[\left(\alpha^q_{i*} - \frac{\delta}{1-\delta}\left(\alpha^y_i - \alpha^y_{i*}\right)\right)\left(\varepsilon_{i*,t+1} - \varepsilon_{i,t+1}\right), -\frac{\delta}{1-\delta}\left(\alpha^y_i \varepsilon_{i,t+1} + \alpha^y_{i*}\varepsilon_{i*,t+1}\right)\right] \\
= \; & \left(\alpha^q_{i*} - \frac{\delta}{1-\delta}\left(\alpha^y_i - \alpha^y_{i*}\right)\right)\left(\frac{\delta}{1-\delta}\left(\alpha^y_i - \alpha^y_{i*}\right)\right)(1-\rho)\,\sigma^2_i \\
= \; & (1-\rho)\,\sigma^2_i \frac{\delta(1-\delta)}{1-\phi_i}\frac{1-\tau^{-1}S_y\left[C_{y,y*} + C_{y,q} - V_y\right]}{\left(1-\delta\phi_i + (1-\delta)\,\tau^{-1}S_y\left(V_y - C_y\right)\right)^2}.
\end{aligned}$$

Thus, we have

$$[C_{y,y*} + C_{y,q} - V_y] = \frac{(1-\rho)\,\sigma^2_i \frac{\delta(1-\delta)}{1-\phi_i}\frac{1}{(1-\delta\phi_i+(1-\delta)\tau^{-1}S_y(V_y-C_y))^2}}{1 + (1-\rho)\,\sigma^2_i \frac{\delta(1-\delta)}{1-\phi_i}\frac{\tau^{-1}S_y}{(1-\delta\phi_i+(1-\delta)\tau^{-1}S_y(V_y-C_y))^2}} \geq 0.$$

Thus, we have $[\gamma^q_{i*} + (\gamma^{y*}_{i*} - \gamma^{y^*}_i)] > 0$ when $\delta < 1$ and $\lim_{\delta\to 1}[\gamma^q_{i*} + (\gamma^{y*}_{i*} - \gamma^{y^*}_i)] = 0$. Note that this argument implies that $1 - \tau^{-1}S_y\left[C_{y,y*} + C_{y,q} - V_y\right] > 0$.

D.5.3 Regression calculations

Fama (1984) regressions Consider the excess returns on FX carry trade. Suppose we estimate the following time-series forecasting regression

$$rx^q_{t+1} = \alpha_q + \beta_q\left(i^*_t - i_t\right) + \xi^q_{t+1}.$$

We have

$$E_t\left[rx^q_{t+1}\right] = \gamma^q_{i*}\cdot\left(i^*_t - i_t\right) + \gamma^q_{s^y}\cdot\left(s^y_t - s^{y^*}_t\right) + \gamma^q_{s^q}\cdot s^q_t,$$

where $\gamma^q_f \equiv \partial E_t\left[rx^q_{t+1}\right]/\partial f_t$ for $f_t \in (i_t, i^*_t, s^y_t, s^{y*}_t, s^q_t)$ and we have made use of the fact that $\gamma^q_i = -\gamma^q_{i*}$ and $\gamma^q_{s^{y*}} = -\gamma^q_{s^y}$. Because independent movements in asset supply $(s^y_t,\, s^{y^*}_t,\, s^q_t)$ are orthogonal to the interest rate differential by assumption, it follows that

$$\beta_q = \frac{Cov\left[rx^q_{t+1}, i^*_t - i_t\right]}{Var\left[i^*_t - i_t\right]} = \frac{Cov\left[E_t\left[rx^q_{t+1}\right], i^*_t - i_t\right]}{Var\left[i^*_t - i_t\right]} = \gamma^q_{i*} = \frac{\partial E_t\left[rx^q_{t+1}\right]}{\partial i^*_t} > 0.$$

Thus, in either Case #1 or Case #2, the extended model matches Fama's (1984) finding that the expected returns on the borrow-home-lend-abroad FX carry trade are high when the foreign-minus-domestic interest rate differential is high.

Lustig, Stathopoulos, and Verdelhan (2019) regressions Consider the regression

$$rx^q_{t+1} + \left(rx^{y*}_{t+1} - rx^y_{t+1}\right) = \alpha_{q,lt} + \beta_{q,lt}\cdot\left(i^*_t - i_t\right) + \xi^{q,lt}_{t+1}. \quad (68)$$

We want to show that $0 < \beta_{q,lt} < \beta_q$ as in Lustig, Stathopoulos, and Verdelhan (2019). Since

$$\beta_{q,lt} = \beta_q + \beta_{y^*-y} = \gamma^q_{i*} + \left(\gamma^{y*}_{i*} - \gamma^{y*}_i\right),$$

it suffices to show that $-\beta_q < \beta_{y^*-y} < 0$, where β_{y^*-y} is the coefficient from the regression

$$rx_{t+1}^{y*} - rx_{t+1}^{y} = \alpha_{y^*-y} + \beta_{y^*-y} \cdot (i_t^* - i_t) + \xi_{t+1}^{y^*-y}.$$

We first consider β_{y^*-y}. We have

$$E_t \left[rx_{t+1}^{y*} - rx_{t+1}^{y}\right] = \left(\gamma_{i^*}^{y*} - \gamma_{i}^{y*}\right)(i_t^* - i_t) + \left(\gamma_{s^{y*}}^{y*} - \gamma_{s^y}^{y*}\right)(s_t^{y*} - s_t^{y}) + 2\gamma_{s^q}^{y*} s_t^q.$$

It follows that

$$\begin{aligned}\beta_{y^*-y} &= \frac{Cov\left[rx_{t+1}^{y*} - rx_{t+1}^{y}, i_t^* - i_t\right]}{Var\left[i_t^* - i_t\right]} \\ &= \frac{\partial E_t[rx_{t+1}^{y^*}]}{\partial i_t^*} - \frac{\partial E_t[rx_{t+1}^{y^*}]}{\partial i_t} = \gamma_{i^*}^{y*} - \gamma_{i}^{y*}.\end{aligned}$$

We have $\gamma_{i^*}^{y*} - \gamma_{i}^{y*} < 0$ under either Case #1 or #2. This is trivial under Case #1 since in that case $\gamma_{i^*}^{y*} < 0$ and $\gamma_{i}^{y*} > 0$. It is also negative under Case #2 since in that case we have $\gamma_{i}^{y} - \gamma_{i^*}^{y} < 0$ (even though we have $\gamma_{i^*}^{y*} < 0$ and $\gamma_{i}^{y*} < 0$). It follows that $\beta_{y^*-y} < 0$ and, therefore, that $\beta_{q,lt} = \beta_q + \beta_{y^*-y} < \beta_q$.

We now show that $\beta_{q,lt} > 0$ or, equivalently, $-\beta_q < \beta_{y^*-y}$. We have

$$-\beta_q = -\gamma_{i^*}^{q} < \gamma_{i^*}^{y*} - \gamma_{i}^{y*} = \beta_{y^*-y}.$$

It suffices to show that

$$0 < \gamma_{i^*}^{q} + \left(\gamma_{i^*}^{y*} - \gamma_{i}^{y*}\right).$$

As shown above, in both Case #1 and Case #2 we have $\gamma_{i^*}^{q} + (\gamma_{i^*}^{y*} - \gamma_{i}^{y^*}) > 0$ when $\delta < 1$ and $\lim_{\delta \to 1}[\gamma_{i^*}^{q} + (\gamma_{i^*}^{y*} - \gamma_{i}^{y^*})] = 0$.

Chinn and Meredith (2004) regressions One can also the returns on the a *long-horizon* FX carry trade that borrows long-term in domestic currency and lends long-term in foreign currency

$$rx_{t\to\infty}^{q,lh} = (1-\delta)\sum_{j=0}^{\infty} \delta_t^j \left(q_{t+j+1} - q_{t+j}\right) + \left(y_t^* - y_t\right).$$

The borrowing costs for this long-horizon or hold-to-maturity FX carry trade is known at the outset. Only the cumulative FX appreciation is unknown.

Chinn and Meredith (2004) run the following regression

$$rx_{t\to\infty}^{q,lh} = \alpha_{q,lh} + \beta_{q,lh}\left(y_t^* - y_t\right) + \xi_{t+1}^{q.lh}$$

and find that $0 < \beta_{q,lh} < \beta_q$ where

$$rx_{t+1}^{q} = \alpha_q + \beta_q\left(i_t^* - i_t\right) + \xi_{t+1}^{q}$$

is the standard Fama (1984) FX carry trade regression. This finding is intimately related to the LSV (2019) result and, naturally, our model can also replicate this fact.

The expected return on this long-horizon trade is

$$
\begin{aligned}
& (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[q_{t+j+1}-q_{t+j}]+(y_t^*-y_t) \\
= \; & (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[(i_{t+j}-i_{t+j}^*)+rx_{t+j+1}^q] \\
& +(1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[i_{t+j}^*+rx_{t+j+1}^{y^*}] \\
& -(1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[i_{t+j}+rx_{t+j+1}^{y}] \\
= \; & (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[rx_{t+j+1}^q+rx_{t+j+1}^{y^*}-rx_{t+j+1}^{y}] \\
= \; & (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[E_{t+j}[rx_{t+j+1}^q+rx_{t+j+1}^{y^*}-rx_{t+j+1}^{y}]] \\
= \; & (1-\delta)\sum_{j=0}^{\infty}\delta^j E_t[\beta_{q,lt}(i_{t+j}^*-i_{t+j})] \\
= \; & \frac{1-\delta}{1-\delta\phi_i}\beta_{q,lt}(i_t^*-i_t)
\end{aligned}
$$

where $\beta_{q,lt}$ is the coefficient from the LSV (2019) long-term FX carry trade regression:

$$rx_{t+1}^q+\left(rx_{t+1}^{y*}-rx_{t+1}^{y}\right)=\alpha_{q,lt}+\beta_{q,lt}\cdot(i_t^*-i_t)+\xi_{t+1}^{q,lt}.$$

Recall that $0<\beta_{q,lt}<\beta_q$. Noting that $y_t^*-y_t=(\alpha_{i^*}^{y^*}-\alpha_i^{y^*})(i_t^*-i_t)$, the long-horizon regression of the form $rx_{t\to\infty}^{q,lh}=\alpha_{q,lh}+\beta_{q,lh}\,(y_t^*-y_t)+\varepsilon_{t\to\infty}^q$ yields the coefficient

$$\beta_{q,lh}=\frac{\frac{1-\delta}{1-\delta\phi_i}}{\alpha_{i^*}^{y^*}-\alpha_i^{y^*}}\beta_{q,lt}.$$

We now show that $0<\beta_{q,lh}<\beta_q$.

Case #1 We have

$$\alpha_i^y-\alpha_i^{y*}=\frac{1-\delta}{1-\delta\phi_i}\frac{1-\phi_i+\tau^{-1}S_q\,(V_q-2C_{y,q})}{1-\phi_i+\tau^{-1}S_qV_q}$$

and thus

$$\beta_{q,lh}=\frac{1-\phi_i+\tau^{-1}S_qV_q}{1-\phi_i+\tau^{-1}S_q\,(V_q-2C_{y,q})}\beta_{q,lt}>\beta_{q,lt}>0.$$

We want to compare $\beta_{q,lh}$ and β_q. Since

$$\beta_{q,lt}=\frac{\tau^{-1}S_q\,(V_q-2C_{y,q})}{1-\phi_i+\tau^{-1}S_qV_q}$$

and $C_{y,q}>0$, we have

$$\beta_{q,lh}=\frac{\tau^{-1}S_q\,(V_q-2C_{y,q})}{1-\phi_i+\tau^{-1}S_q\,(V_q-2C_{y,q})}<\frac{\tau^{-1}S_qV_q}{1-\phi_i+\tau^{-1}S_qV_q}=\beta_q.$$

Case #2 We have

$$\alpha_{i^*}^{y^*}-\alpha_i^{y^*}=\frac{1}{\frac{1-\delta\phi_i}{1-\delta}+\tau^{-1}S_y\,(V_y-C_{y,y*})},$$

so

$$\beta_{q,lh} = \left(1 + \tau^{-1}\frac{1-\delta}{1-\delta\phi_i} S_y \left(V_y - C_{y,y*}\right)\right) \beta_{q,lt} > \beta_{q,lt} > 0.$$

We have

$$\begin{aligned}
\beta_{q,lt} &= \frac{\tau^{-1}S_y\left(C_{y,q} - \left(V_y - C_{y,y*}\right)\right)}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y\left(V_y - C_{y,y*}\right)} \\
\beta_q &= \frac{\tau^{-1}S_y C_{y,q}}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y\left(V_y - C_{y,y*}\right)},
\end{aligned}$$

implying that

$$\beta_{q,lh} = \frac{1-\delta}{1-\delta\phi_i}\tau^{-1}S_y\left(C_{y,q} - \left(V_y - C_{y,y*}\right)\right).$$

Thus, we have

$$\begin{aligned}
\beta_q - \beta_{q,lh} &= \frac{\tau^{-1}S_y C_{y,q}}{\frac{1-\delta\phi_i}{1-\delta} + \tau^{-1}S_y\left(V_y - C_{y,y*}\right)} - \frac{1-\delta}{1-\delta\phi_i}\tau^{-1}S_y\left(C_{y,q} - \left(V_y - C_{y,y*}\right)\right) \\
&= \tau^{-1}S_y\frac{1-\delta}{1-\delta\phi_i}\left(V_y - C_{y,y*}\right)\left(\frac{1 - \tau^{-1}\frac{1-\delta}{1-\delta\phi_i}S_y\left(C_{y,q} + C_{y,y*} - V_y\right)}{1 + \tau^{-1}\frac{1-\delta}{1-\delta\phi_i}S_y\left(V_y - C_{y,y*}\right)}\right) \\
&> \tau^{-1}S_y\frac{1-\delta}{1-\delta\phi_i}\left(V_y - C_{y,y*}\right)\left(\frac{1 - \tau^{-1}S_y\left(C_{y,q} + C_{y,y*} - V_y\right)}{1 + \tau^{-1}\frac{1-\delta}{1-\delta\phi_i}S_y\left(V_y - C_{y,y*}\right)}\right) \\
&> 0
\end{aligned}$$

since $\left(V_y - C_{y,y*}\right) > 0$, $\left(C_{y,q} + C_{y,y*} - V_y\right) > 0$, $(1-\delta)/\left(1 - \delta\phi_i\right) \in (0, 1)$, and $1 - \tau^{-1}S_y\left(C_{y,q} + C_{y,y*} - V_y\right) > 0$.

Campbell and Shiller (1991) regressions Consider the excess returns on the yield curve carry trade. Suppose we estimate the following time-series forecasting regression

$$rx_{t+1}^y = \alpha_y + \beta_y\left(y_t - i_t\right) + \xi_{t+1}^y.$$

We have

$$E_t\left[rx_{t+1}^y\right] = E\left[rx_{t+1}^y\right] + \gamma_i^y\left(i_t - \bar{\imath}\right) + \gamma_{i*}^y\left(i_t^* - \bar{\imath}\right) + \gamma_{s^y}^y\left(s_t^y - \bar{s}^y\right) + \gamma_{s^{y*}}^y\left(s_t^{y*} - \bar{s}^y\right) + \gamma_{s^q}^y s_t^q$$

where $\gamma_f^y \equiv \partial E_t\left[rx_{t+1}^y\right]/\partial f_t$ for $f_t \in \left(i_t, i_t^*, s_t^y, s_t^{y*}, s_t^q\right)$. The term spread is given by

$$\left(y_t - i_t\right) = \alpha_0^y + \left(\alpha_i^y - 1\right)\left(i_t - \bar{\imath}\right) + \alpha_{i*}^y\left(i_t^* - \bar{\imath}\right) + \alpha_{s^y}^y\left(s_t^y - \bar{s}^y\right) + \alpha_{s^{y*}}^y\left(s_t^{y*} - \bar{s}^y\right) + \alpha_{s^q}^y s_t^q.$$

We have $\beta_y = Cov\left[y_t - i_t, rx_{t+1}^y\right]/Var\left[y_t - i_t\right]$. Thus, we have

$$\begin{aligned}
\beta_y &\propto Cov\left[y_t - i_t, rx_{t+1}^y\right] \\
&= \left[\gamma_i^y\left(\alpha_i^y - 1\right) + \rho\left(\alpha_i^y - 1\right)\gamma_{i*}^y + \rho\alpha_{i*}^y\gamma_i^y + \alpha_{i*}^y\gamma_{i*}^y\right]\frac{\sigma_i^2}{1-\phi_i^2} \\
&\quad + \left[\left(\gamma_{s^y}^y\alpha_{s^y}^y + \gamma_{s^{y*}}^y\alpha_{s^{y*}}^y\right)\frac{\sigma_{s^y}^2}{1-\phi_{s_y}^2} + \left(\gamma_{s^q}^y\alpha_{s^q}^y\right)\frac{\sigma_{s^q}^2}{1-\phi_{s_q}^2}\right]
\end{aligned}$$

We wish to show that $\beta_y > 0$. For any supply factor $f_t \in (s_t^y, s_t^{y*}, s_t^q)$, we have $sign(\gamma_f^y) = sign(\alpha_f^y)$. It thus follows that $(\gamma_{s^y}^y \alpha_{s^y}^y + \gamma_{s^{y*}}^y \alpha_{s^{y*}}^y) > 0$ and $(\gamma_{s^q}^y \alpha_{s^q}^y) > 0$. Thus, the second term is square brackets is always positive. Thus, to prove that $\beta_y > 0$, it suffices to show that the first term in square brackets is positive—i.e., to show that

$$(\alpha_i^y - 1)(\gamma_i^y + \rho\gamma_{i*}^y) + \alpha_{i*}^y(\rho\gamma_i^y + \gamma_{i*}^y) > 0.$$

We prove this inequality separately in Case #1 and Case #2 below under the simplifying assumption that $\sigma_{s^y}^2 = \sigma_{s^q}^2 = 0$. Finally, letting

$$rx_{t+1}^{y*} = \alpha_{y*} + \beta_{y*}(y_t^* - i_t^*) + \xi_{t+1}^{y*},$$

we have $\beta_{y*} = \beta_y$ by symmetry.

Case #1 In Case #1 where $S_q > 0$ and $S_y = 0$, we have $\gamma_i^y = -\gamma_{i*}^y < 0$. By definition, we have

$$\gamma_i^y = \frac{1-\delta\phi_i}{1-\delta}\alpha_i^y - 1 \text{ or } \alpha_i^y = \frac{1-\delta}{1-\delta\phi_i}(1+\gamma_i^y).$$

We also have

$$\alpha_{i*}^y = \frac{1-\delta}{1-\delta\phi_i}\gamma_{i*}^y = -\frac{1-\delta}{1-\delta\phi_i}\gamma_i^y.$$

Substituting these expressions for α_i^y, α_{i*}^y, and γ_{i*}^y, we obtain

$$\begin{aligned} & (\alpha_i^y - 1)(\gamma_i^y + \rho\gamma_{i*}^y) + \alpha_{i*}^y(\rho\gamma_i^y + \gamma_{i*}^y) \\ = \; & (1-\rho)\left[\left(\frac{1-\delta}{1-\delta\phi_i} - 1\right)\gamma_i^y + 2\frac{1-\delta}{1-\delta\phi_i}(\gamma_i^y)^2\right] > 0, \end{aligned}$$

where the inequality follows because $(1-\delta)/(1-\delta\phi_i) < 1$ and $\gamma_i^y < 0$.

Case #2 In Case #2 where $S_q = 0$ and $S_y > 0$, we have $\gamma_i^y < 0$. Since we have

$$\alpha_{i*}^y = -c\alpha_i^y < 0$$

for some constant $c > 0$, we have

$$\alpha_i^y = \frac{1-\delta}{1-\delta\phi_i}(1+\gamma_i^y) \text{ and } \alpha_{i*}^y = -c\frac{1-\delta}{1-\delta\phi_i}(1+\gamma_i^y).$$

Since $\alpha_i^y > 0$ and $\gamma_i^y < 0$, we also have $0 < (1+\gamma_i^y) < 1$. Finally, we have

$$\gamma_{i*}^y = \frac{1-\delta\phi_i}{1-\delta}\alpha_{i*}^y = -c(1+\gamma_i^y) < 0.$$

Substituting these expressions for α_i^y, α_{i*}^y, and γ_{i*}^y, we obtain

$$\begin{aligned} & (\alpha_i^y - 1)(\gamma_i^y + \rho\gamma_{i*}^y) + \alpha_{i*}^y(\rho\gamma_i^y + \gamma_{i*}^y) \\ = \; & \overbrace{\left(\frac{1-\delta}{1-\delta\phi_i}(1+\gamma_i^y) - 1\right)}^{<0}\overbrace{(\gamma_i^y - \rho c(1+\gamma_i^y))}^{<0} + \overbrace{\left(-c\frac{1-\delta}{1-\delta\phi_i}(1+\gamma_i^y)\right)}^{<0}\overbrace{(\rho\gamma_i^y - c(1+\gamma_i^y))}^{<0} > 0. \end{aligned}$$

E Solutions of model extensions

E.1 Further segmenting the global bond market

In the Section 5.1, we further segment the global bond market as in Gromb and Vayanos (2002), assuming some bond investors cannot trade short- and long-term bonds in both currencies. Our extended model feature four types of bond investors. All types have mean-variance preferences over one-period-ahead wealth and a risk tolerance of τ in domestic currency terms. The types only differ in their ability to trade different assets. Specifically, the four investor types are:

1. *Domestic bond specialists*, present in mass $\mu\pi$, can only choose between short- and long-term domestic bonds—i.e., they can only engage in the domestic yield-curve carry trade. Thus, their demand for long-term domestic bonds is $b_t^y = \tau\left(Var_t\left[rx_{t+1}^y\right]\right)^{-1} E_t\left[rx_{t+1}^y\right]$.

2. *Foreign bond specialists*, also present in mass $\mu\pi$, can only choose between short- and long-term foreign bonds—i.e., they can only engage in the foreign yield-curve carry trade. Their demand for long-term foreign bonds is $b_t^{y*} = \tau\left(Var_t\left[rx_{t+1}^{y*}\right]\right)^{-1} E_t\left[rx_{t+1}^{y*}\right]$.

3. *FX specialists*, present in mass $\mu(1-2\pi)$, can only choose between short-term domestic and foreign bonds—i.e., they can only engage in the FX carry trade. Their demand for the borrow-at-home-lend-abroad FX carry trade is $b_t^q = \tau\left(Var_t\left[rx_{t+1}^q\right]\right)^{-1} E_t\left[rx_{t+1}^q\right]$.

4. *Global bond investors*, present in mass $(1-\mu)$, can hold short- and long-term bonds in both currencies and can engage in all three carry trades. Their demand for the three carry trades is $\mathbf{d}_t = \tau\left(Var_t\left[\mathbf{rx}_{t+1}\right]\right)^{-1} E_t\left[\mathbf{rx}_{t+1}\right]$.

We assume $\mu \in [0,1]$ and $\pi \in (0,1/2)$. Thus, increasing the combined mass of specialist types, μ, is equivalent to introducing greater segmentation in the global bond market. Our baseline model corresponds to the limiting case where $\mu = 0$. At the other extreme, markets are fully segmented when $\mu = 1$. And, when $\mu \in (0,1)$ markets are partially segmented.

Technical assumption: Adding FX-specific fundamental risk To solve the extended model in the absence of supply risk, we assume there is some small amount of FX-specific fundamental risk. Naturally, this implies that long-run UIP will not hold even in the $\delta \to 1$ limit. We make this assumption to study our extended model in the absence of supply risk.

Specifically, we assume $\lim_{T\to\infty} E_t\left[q_{t+T}\right] = q_t^\infty$ follows a random walk $q_{t+1}^\infty = q_t^\infty + \varepsilon_{q^\infty,t+1}$ with $Var_t\left[\varepsilon_{q^\infty,t+1}\right] = \sigma_{q^\infty}^2 > 0$, implying $q_t = q_t^\infty + \sum_{j=0}^\infty E_t[(i_{t+j}^* - i_{t+j}) - rx_{t+j+1}^q]$. Thus, in the absence of supply risk, we have:

$$\mathbf{V} = \begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \sigma_\infty^2 + \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) \end{bmatrix}. \tag{69}$$

If $\sigma_{q^\infty}^2 = 0$, then in the absence of supply risk, FX is a redundant asset: FX returns are a linear combination of those on domestic and foreign bonds. While the model can still be solved in the limit where $\sigma_{q^\infty}^2 = \sigma_{s^q}^2 = \sigma_{s^y}^2 = 0$, in this case, cross-market impact increases in μ for all $\mu \in (0,1)$ and then discontinuously vanishes at $\mu = 1$. Thus, assuming $\sigma_{q^\infty}^2 > 0$ is a technical modeling device that allows us to explore the model's qualitative behavior when $\sigma_{s^q}^2, \sigma_{s^y}^2 > 0$ under the mathematically simpler assumption that $\sigma_{s^q}^2 = \sigma_{s^y}^2 = 0$. Specifically, when $\sigma_{s^q}^2$ and $\sigma_{s^y}^2$ are small and positive the model must have the same qualitative behavior as when $\sigma_{q^\infty}^2 > 0$.

Solution under further segmentation It is straightforward to solve for equilibrium under partial segmentation. Specifically, let $\mathbf{b}_t = [b_t^y, b_t^{y*}, b_t^q]'$ denote the vector of specialist investor demands and note that

$$\mathbf{b}_t = \tau \left[\mathbf{diag}\left(\mathbf{V}\right)\right]^{-1} E_t\left[\mathbf{rx}_{t+1}\right], \tag{70}$$

where $[\mathbf{diag}(\mathbf{V})]$ is a matrix with the diagonal elements of $\mathbf{V} = Var_t[\mathbf{rx}_{t+1}]$ on its diagonal and zeros elsewhere. Also let $\mathbf{\Pi} = \mathbf{diag}(\pi, \pi, 1-2\pi)$. The market clearing condition once we further segment the global rates market is

$$\begin{aligned} \mathbf{s}_t &= \mu\mathbf{\Pi}\mathbf{b}_t + (1-\mu)\,\mathbf{d}_t \qquad (71)\\ &= \mu\mathbf{\Pi}\tau\left[\mathbf{diag}\left(\mathbf{V}\right)\right]^{-1} E_t\left[\mathbf{rx}_{t+1}\right] + (1-\mu)\,\tau\mathbf{V}^{-1}E_t\left[\mathbf{rx}_{t+1}\right]. \end{aligned}$$

As a result, equilibrium expected returns are:

$$E_t\left[\mathbf{rx}_{t+1}\right] = \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}\right)\right]^{-1} + (1-\mu)\,\mathbf{V}^{-1}\right]^{-1}\mathbf{s}_t. \tag{72}$$

Thus, adopting the notation from above, the market clearing condition under further segmentation can be expressed as

$$\begin{aligned} & \left[\mathbf{B}_0\mathbf{a} + \mathbf{B}_1\mathbf{a} + \mathbf{r}_0\right] + \left[\mathbf{B}_0\mathbf{A} + \mathbf{B}_1\mathbf{A}\mathbf{\Phi} + \mathbf{R}_1\right]\mathbf{z}_t \qquad (73)\\ = \;& \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}\right)\right]^{-1} + (1-\mu)\,\mathbf{V}^{-1}\right]^{-1}\mathbf{s}_0 + \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}\right)\right]^{-1} + (1-\mu)\,\mathbf{V}^{-1}\right]^{-1}\mathbf{S_1}\mathbf{z}_t, \end{aligned}$$

where $\mathbf{V} = (\mathbf{B}_1\mathbf{A}\mathbf{\Sigma}\mathbf{A}'\mathbf{B}_1')$.

Matching the matrices multiplying the state vector $\mathbf{x}_t$, we see that $\mathbf{A}$ must solve the following fixed point problem:

$$\mathbf{A} = \left[\tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{B}_1\mathbf{A}\mathbf{\Sigma}\mathbf{A}'\mathbf{B}_1'\right)\right]^{-1} + (1-\mu)\left(\mathbf{B}_1\mathbf{A}\mathbf{\Sigma}\mathbf{A}'\mathbf{B}_1'\right)^{-1}\right]^{-1}\mathbf{S_1} - \mathbf{R}_1\right] \oslash \left[\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}\right]. \tag{74}$$

As above, partitioning $\mathbf{A}$ as $\mathbf{A} = [\mathbf{A}_i \;\, \mathbf{A}_s]$, we find that

$$\mathbf{A}_i = -\mathbf{R}_1^{[1-2]} \oslash \left[\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}\right]^{[1-2]} = \begin{bmatrix} \frac{1-\delta}{1-\delta\phi_i} & 0 \\ 0 & \frac{1-\delta}{1-\delta\phi_i} \\ -\frac{1}{1-\phi_i} & \frac{1}{1-\phi_i} \end{bmatrix}.$$

Letting $\mathbf{V}_i = (\mathbf{B}_1\mathbf{A}_i)\,\mathbf{\Sigma}_i\,(\mathbf{B}_1\mathbf{A}_i)'$, we see that

$$\mathbf{V} = \mathbf{V}_i + (\mathbf{B}_1\mathbf{A}_s)\,\mathbf{\Sigma}_s\,(\mathbf{B}_1\mathbf{A}_s)'.$$

Thus, $\mathbf{A}_s$ must solve the following fixed point problem

$$\mathbf{A}_s = \left[\tau^{-1}\begin{bmatrix} \mu\mathbf{\Pi}\left(\mathbf{diag}\left[\mathbf{V}_i + (\mathbf{B}_1\mathbf{A}_s)\,\mathbf{\Sigma}_s\,(\mathbf{B}_1\mathbf{A}_s)'\right]\right)^{-1} \\ +(1-\mu)\left[\mathbf{V}_i + (\mathbf{B}_1\mathbf{A}_s)\,\mathbf{\Sigma}_s\,(\mathbf{B}_1\mathbf{A}_s)'\right]^{-1} \end{bmatrix}^{-1}\right] \oslash \left[\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}\right]^{[3-5]}. \tag{75}$$

We also see that $\mathbf{a}$ must satisfy

$$(\mathbf{B}_0 + \mathbf{B}_1)\,\mathbf{a} = \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{B}_1\mathbf{A}\mathbf{\Sigma}\mathbf{A}'\mathbf{B}_1'\right)\right]^{-1} + (1-\mu)\left(\mathbf{B}_1\mathbf{A}\mathbf{\Sigma}\mathbf{A}'\mathbf{B}_1'\right)^{-1}\right]^{-1}\mathbf{s}_0 - \mathbf{r}_0. \tag{76}$$

Recasting the equilibrium as a fixed point involving $\mathbf{\Omega}$ We can think of equilibrium as a fixed point problem involving the return impact matrix, $\mathbf{\Omega}$, that maps changes in asset to supply to shifts in asset returns—i.e. $E_t[\mathbf{rx}_{t+1}] = \mathbf{\Omega}\mathbf{s}_t$. This return impact matrix is given by

$$\mathbf{\Omega} = \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}\right)\right]^{-1} + (1-\mu)\mathbf{V}^{-1}\right]^{-1}.$$

Thus, $\mathbf{\Omega}$ depends on the harmonic mean of $\mathbf{\Pi}^{-1}\left[\mathbf{diag}\left(\mathbf{V}\right)\right]$ and $\mathbf{V}$. Since $\mathbf{A}_s = \mathbf{\Omega} \oslash \mathbf{Z}_s$ where $\mathbf{Z}_s = [\mathbf{B}_0\mathbf{E} + \mathbf{B}_1\mathbf{E}\mathbf{\Phi}]^{[3-5]}$, we have

$$\mathbf{V} = \mathbf{V}_i + (\mathbf{B}_1\mathbf{\Omega} \oslash \mathbf{Z}_s)\mathbf{\Sigma}_s(\mathbf{B}_1\mathbf{\Omega}(\mu) \oslash \mathbf{Z}_s)' = \mathbf{V}_i + (\mathbf{\Omega} \circ \mathbf{Z})\mathbf{\Sigma}_s(\mathbf{\Omega} \circ \mathbf{Z})'$$

where $\mathbf{Z}$ satisfies $(\mathbf{B}_1\mathbf{\Omega} \oslash \mathbf{Z}_s) = \mathbf{\Omega} \circ \mathbf{Z}$. Thus, the relevant fixed point problem is

$$\mathbf{\Omega} = \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}_i + (\mathbf{\Omega} \circ \mathbf{Z})\mathbf{\Sigma}_s(\mathbf{\Omega} \circ \mathbf{Z}')\right)\right]^{-1} + (1-\mu)\left[\mathbf{V}_i + \mathbf{Z}\circ(\mathbf{\Omega}\mathbf{\Sigma}_s\mathbf{\Omega}) \circ \mathbf{Z}'\right]^{-1}\right]^{-1}.$$

We use $\mathbf{\Omega}(\mu)$ to denote the stable solution to this fixed-point problem. We write $\mathbf{\Omega}(\mu)$ to emphasize that this solution depends on the degree of segmentation μ.

Understanding how $\mathbf{\Omega}(\mu)$ varies as a function of μ. We first want to understand how this solution $\mathbf{\Omega}(\mu)$ varies as a function of μ. Clearly, $\mathbf{\Omega}(\mu)$ is positive definite. However, we are interested in understanding when/whether $\mathbf{\Omega}'(\mu) = \partial\mathbf{\Omega}(\mu)/\partial\mu$ is itself a positive definite matrix.

Letting

$$\mathbf{\Omega}(\mu) = \tau^{-1}\left[\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}(\mu)\right)\right]^{-1} + (1-\mu)\mathbf{V}(\mu)^{-1}\right]^{-1}$$

and

$$\mathbf{V}(\mu) = \mathbf{V}_i + (\mathbf{\Omega}(\mu) \circ \mathbf{Z})\mathbf{\Sigma}_s(\mathbf{\Omega}(\mu) \circ \mathbf{Z})'$$

denote the equilibrium return-impact and variance matrices when fraction μ of investors are specialists. Using the rules of matrix differentiation, we obtain:

$$\begin{aligned}\mathbf{\Omega}'(\mu) &= \left\{\tau\mathbf{\Omega}(\mu)\left[\mathbf{V}(\mu)^{-1} - \mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}(\mu)\right)\right]^{-1}\right]\mathbf{\Omega}(\mu)\right\} \\ &\quad + \left\{\tau\mathbf{\Omega}(\mu)\left[\begin{array}{c}\mu\mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}(\mu)\right)\right]^{-1}\left[\mathbf{diag}\left(\mathbf{V}'(\mu)\right)\right]\left[\mathbf{diag}\left(\mathbf{V}(\mu)\right)\right]^{-1} \\ +(1-\mu)\mathbf{V}(\mu)^{-1}\mathbf{V}'(\mu)\mathbf{V}(\mu)^{-1}\end{array}\right]\mathbf{\Omega}(\mu)\right\}\end{aligned} \tag{77}$$

where

$$\mathbf{V}'(\mu) = (\mathbf{\Omega}'(\mu) \circ \mathbf{Z})\mathbf{\Sigma}_s(\mathbf{\Omega}(\mu) \circ \mathbf{Z})' + (\mathbf{\Omega}(\mu) \circ \mathbf{Z})\mathbf{\Sigma}_s(\mathbf{\Omega}'(\mu) \circ \mathbf{Z})'. \tag{78}$$

Below, we will show that $\mathbf{V}(\mu)^{-1} - \mathbf{\Pi}\left[\mathbf{diag}\left(\mathbf{V}(\mu)\right)\right]^{-1}$ is also positive definite, immediately implying that the first matrix in curly braces in equation (77) positive definite. This means that $\mathbf{\Omega}'(\mu)$ must always be positive definite in the absence of supply risk. Furthermore, by continuity of the stable equilibrium in the model's underlying parameters, $\mathbf{\Omega}'(\mu)$ must continue to be positive-definite when supply risk is small ($\mathbf{\Sigma}_s$ is small).

Furthermore, *if* $\mathbf{V}'(\mu)$ in equation (78) positive definite, then the second matrix in curly braces in equation (77) is also positive definite. Since the sum of positive definite matrices is also positive definite, this means that $\mathbf{\Omega}'(\mu)$ is also positive definite. (If $\mathbf{V}'(\mu)$ is positive definite, then we have $\partial Var_t[rx_{t+1}^{p_t}]/\partial\mu > 0$ for *any arbitrary* bond portfolio $\mathbf{p}_t \neq \mathbf{0}$ with returns $rx_{t+1}^{p_t} = \mathbf{p}_t'\mathbf{rx}_{t+1}$—i.e., return volatility is increasing in μ).

In our numerical solutions, we find $\partial Var_t[rx_{t+1}^{p_t}]/\partial\mu > 0$ for any portfolio $rx_{t+1}^{p_t} = \mathbf{p}_t'\mathbf{rx}_{t+1}$ so long as $\sigma_{s^y}^2$ and $\sigma_{s^q}^2$ have a similar order of magnitude. However, $\partial Var_t[rx_{t+1}^{p_t}]/\partial\mu$ can be negative when $\sigma_{s^y}^2$ and $\sigma_{s^q}^2$ have different orders of magnitude. For instance, suppose $\sigma_{s^q}^2 > 0$ and $\sigma_{s^y}^2 = 0$. Then

$Var_t[rx^y_{t+1}|\mu = 0] > Var_t[rx^y_{t+1}|\mu = 1]$ since FX supply shocks raise the volatility of bond returns in integrated markets ($\mu = 0$) but not in fully segmented markets ($\mu = 1$). As a result, there exists some $\mu^* \in [0, 1]$ such that $\partial Var_t[rx^y_{t+1}|\mu = \mu^*]/\partial\mu < 0$.

Proof that $\mathbf{\Omega}'(\mu)$ is positive definite in the absence of supply risk. We now show that $[\mathbf{V}^{-1} - \mathbf{\Pi}[\mathbf{diag}(\mathbf{V})]^{-1}]$ and therefore $\mathbf{\Omega}'(\mu) = \{\tau\mathbf{\Omega}(\mu)[\mathbf{V}^{-1} - \mathbf{\Pi}[\mathbf{diag}(\mathbf{V})]^{-1}]\mathbf{\Omega}(\mu)\}$ is positive definite in the absence of supply risk ($\mathbf{\Sigma}_s = \mathbf{0}$). This is actually true for any positive-definite covariance matrix $\mathbf{V}$ and any diagonal, positive-definite matrix $\mathbf{\Pi}$ such that $trace(\mathbf{\Pi}) = 1$.

Here will simply prove this for the special case that is relevant for us in the paper. We write $\mathbf{V} = \mathbf{S\Gamma S}$ where $\mathbf{S}$ is a diagonal matrix with the standard deviation of excess returns on its diagonals and $\mathbf{\Gamma}$ is the correlation matrix for the excess returns. Since

$$\mathbf{V}^{-1} - \mathbf{\Pi}[\mathbf{diag}(\mathbf{V})]^{-1} = \mathbf{S}^{-1}(\mathbf{\Gamma}^{-1} - \mathbf{\Pi})\mathbf{S}^{-1}$$

it suffices to show that $\mathbf{\Gamma}^{-1} - \mathbf{\Pi}$ is positive definite. We have

$$\mathbf{\Gamma}^{-1} - \mathbf{\Pi} = \left(\begin{bmatrix} 1 & \gamma_y & \gamma_q \\ \gamma_y & 1 & -\gamma_q \\ \gamma_q & -\gamma_q & 1 \end{bmatrix}^{-1} - \begin{bmatrix} \pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & 1-2\pi \end{bmatrix}\right)$$

where $\pi \in (0, 1/2)$ and $\gamma_y \in (-1, 1)$ and $\gamma_q \in (-1, 1)$.

We begin by noting that, since $\mathbf{\Gamma}$ is positive definite, the eigenvalues of $\mathbf{\Gamma}$ are positive. The eigenvalues of $\mathbf{\Gamma}$ are $1 + \gamma_y$, $1 - \frac{1}{2}\gamma_y + \frac{1}{2}\sqrt{8\gamma_q^2 + \gamma_y^2}$, $1 - \frac{1}{2}\gamma_y - \frac{1}{2}\sqrt{8\gamma_q^2 + \gamma_y^2}$. Thus, we have $2 - \gamma_y > \sqrt{8\gamma_q^2 + \gamma_y^2} > -(2 - \gamma_y)$. The fact that $1 - \frac{1}{2}\gamma_y - \frac{1}{2}\sqrt{8\gamma_q^2 + \gamma_y^2} > 0$ also implies that $1 - \gamma_y - 2\gamma_q^2 > 0$. We will use this fact repeatedly below.

Next, the three eigenvalues of $\mathbf{\Gamma}^{-1} - \mathbf{\Pi}$ are:

1. $\lambda_1 = \frac{1}{2}\frac{[1+\pi(1-\gamma_y)+(1-\pi)2\gamma_q^2]+\sqrt{[1+\pi(1-\gamma_y)+(1-\pi)2\gamma_q^2]^2-8\pi(1-\gamma_y-2\gamma_q^2)(1-\pi(1-\gamma_y)-\gamma_q^2(1-2\pi))}}{1-\gamma_y-2\gamma_q^2} > 0.$
   - As noted above, we have $1 - \gamma_y - 2\gamma_q^2 > 0$.
   - We also have $1 + \pi(1 - \gamma_y) + (1 - \pi)2\gamma_q^2 > 0$.
   - Since $\mathbf{\Gamma}^{-1} - \mathbf{\Pi}$ is symmetric, the eigenvalues of $\mathbf{\Gamma}^{-1} - \mathbf{\Pi}$ are real-valued. Thus, the term under the radical is positive.
   - Together, these three facts imply that $\lambda_1 > 0$.

2. $\lambda_2 = \frac{1}{2}\frac{[1+\pi(1-\gamma_y)+(1-\pi)2\gamma_q^2]-\sqrt{[1+\pi(1-\gamma_y)+(1-\pi)2\gamma_q^2]^2-8\pi(1-\gamma_y-2\gamma_q^2)(1-\pi(1-\gamma_y)-\gamma_q^2(1-2\pi))}}{1-\gamma_y-2\gamma_q^2} > 0.$
   - In addition to the facts listed above, this follows from the fact that

   $$8\pi(1 - \gamma_y - 2\gamma_q^2)(1 - \pi(1 - \gamma_y) - \gamma_q^2(1 - 2\pi)) > 0.$$

   Specifically, $(1 - \gamma_y - 2\gamma_q^2) > 0$ and, since $\gamma_y \in (-1, 1)$, $(1 - \pi(1 - \gamma_y) - \gamma_q^2(1 - 2\pi)) > (1 - 2\pi)(1 - \gamma_q^2) > 0$. Finally, since for $X > 0$, $Y > 0$, and $X^2 - Y > 0$ together imply $X - \sqrt{X^2 - Y} > 0$, we conclude that $\lambda_2 > 0$.

3. $\lambda_3 = \frac{1}{1+\gamma_y}(1 - \pi - \pi\gamma_y) > 0.$
   - This follows from the fact that $1 - 2\pi > 0$ and $\gamma_y \in (-1, 1)$ which together imply that $1 - \pi - \pi\gamma_y > 1 - 2\pi > 0$.

Interpreting $\mathbf{\Omega}'(\mu)$. To interpret $\mathbf{\Omega}'(\mu)$ in equation (77), we note that increasing μ—i.e., further segmenting the global rates markets—has two direct equilibrium effects. First, as we increase μ, risk sharing becomes less efficient because fewer investors can absorb a given supply shock. For instance, the fraction of investors who can absorb a shock to domestic bond supply is $\mu\pi + (1-\mu)$, which is decreasing in μ. This gives rise to the "inefficient risk-sharing" effect. Second, as we increase μ, we replace global bond investors whose demands take the correlations between the three carry trades into account with specialist investors who, taken as a group, behave as if the three carry trade returns are uncorrelated. This gives rise to the "width-of-the-pipe" effect: price impact is only transmitted across markets to the extent there are investors—"the pipe"—whose demands are impacted by shocks to other markets. Finally, there is a third indirect effect of increasing segmentation. To the extent that greater segmentation directly alters the price impact of supply shocks, greater segmentation affects equilibrium return volatility, further altering equilibrium price impact. This is an "endogenous risk effect."

Thus, we further decompose $\mathbf{\Omega}'(\mu)$ into three terms

$$\begin{aligned}\mathbf{\Omega}'(\mu) &= \overbrace{\left\{\tau\mathbf{\Omega}(\mu)\left[[\mathbf{diag}(\mathbf{V}(\mu))]^{-1} - \mathbf{\Pi}[\mathbf{diag}(\mathbf{V}(\mu))]^{-1}\right]\mathbf{\Omega}(\mu)\right\}}^{\mathbf{\Omega}'_{\text{sharing}}(\mu) = \text{Inefficient risk sharing effect}} \\ &+\overbrace{\left\{\tau\mathbf{\Omega}(\mu)\left[\mathbf{V}(\mu)^{-1} - [\mathbf{diag}(\mathbf{V}(\mu))]^{-1}\right]\mathbf{\Omega}(\mu)\right\}}^{\mathbf{\Omega}'_{\text{pipe}}(\mu) = \text{Width of pipe effect}} \\ &+\overbrace{\left\{\tau\mathbf{\Omega}(\mu)\begin{bmatrix}\mu\mathbf{\Pi}[\mathbf{diag}(\mathbf{V}(\mu))]^{-1}[\mathbf{diag}(\mathbf{V}'(\mu))][\mathbf{diag}(\mathbf{V}(\mu))]^{-1} \\ +(1-\mu)\mathbf{V}(\mu)^{-1}\mathbf{V}'(\mu)\mathbf{V}(\mu)^{-1}\end{bmatrix}\mathbf{\Omega}(\mu)\right\}}^{\mathbf{\Omega}'_{\text{risk}}(\mu) = \text{Endogenous risk effect}}.\end{aligned} \tag{79}$$

First, if the mass of investors who could buy each asset were independent of μ—i.e., if we instead had $\mathbf{\Pi} = \mathbf{I}$, we would have $\mathbf{\Omega}'_{\text{sharing}}(\mu) = \mathbf{0}$. Second, if assets returns were uncorrelated—i.e., if we instead had $\mathbf{V}(\mu) = [\mathbf{diag}(\mathbf{V}(\mu))]$, we would have $\mathbf{\Omega}'_{\text{pipe}}(\mu) = \mathbf{0}$. Finally, if there was no supply risk—i.e., if $\mathbf{V}'(\mu) = \mathbf{0}$, then would have $\mathbf{\Omega}'_{\text{risk}}(\mu) = \mathbf{0}$.

Letting $\boldsymbol{\omega}^* = vec(\mathbf{\Omega}^*)$, we can this of our equilibrium as a fixed-point problem in $\boldsymbol{\omega}$

$$\boldsymbol{\omega}^* = \mathbf{f}(\boldsymbol{\omega}^*, \mu).$$

Thus, by the Implicit Function Theorem, we have

$$\frac{\partial\boldsymbol{\omega}^*}{\partial\mu} = [\mathbf{I} - \mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)]^{-1}[\partial\mathbf{f}(\boldsymbol{\omega}^*, \mu)/\partial\mu],$$

where $\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)$ is the Jacobian matrix. Assuming we are at a stable equilibrium, we have $[\mathbf{I} - \mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)]^{-1} = \sum_{j=0}^{\infty}[\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)]^j$, so we have

$$\begin{aligned}\partial\boldsymbol{\omega}^*/\partial\mu &= \textstyle\sum_{j=0}^{\infty}[\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)]^j[\partial\mathbf{f}(\boldsymbol{\omega}^*, \mu)/\partial\mu] \\ &= [\partial\mathbf{f}(\boldsymbol{\omega}^*, \mu)/\partial\mu] + [\mathbf{I} - \mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)]^{-1}\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu)[\partial\mathbf{f}(\boldsymbol{\omega}^*, \mu)/\partial\mu]\end{aligned}$$

where

$$\partial\mathbf{f}(\boldsymbol{\omega}^*, \mu)/\partial\mu = vec\left(\tau\mathbf{\Omega}(\mu)\left[\mathbf{V}(\mu)^{-1} - \mathbf{\Pi}[\mathbf{diag}(\mathbf{V}(\mu))]^{-1}\right]\mathbf{\Omega}(\mu)\right) = vec\left(\mathbf{\Omega}'_{\text{sharing}}(\mu) + \mathbf{\Omega}'_{\text{pipe}}(\mu)\right).$$

In the absence of supply risk ($\mathbf{\Sigma}_s = \mathbf{0}$), $\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*, \mu) = \mathbf{0}$ and $\partial\boldsymbol{\omega}^*/\partial\mu = vec\left(\mathbf{\Omega}'_{\text{sharing}}(\mu) + \mathbf{\Omega}'_{\text{pipe}}(\mu)\right)$.

In the presence of supply risk, $\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*,\mu) \neq \mathbf{0}$ and

$$\partial\boldsymbol{\omega}^*/\partial\mu = \overbrace{vec[\boldsymbol{\Omega}'_{\text{sharing}}(\mu) + \boldsymbol{\Omega}'_{\text{pipe}}(\mu)]}^{\text{Direct effects}} + \overbrace{[\mathbf{I} - \mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*,\mu)]^{-1}\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}(\boldsymbol{\omega}^*,\mu)\, vec[\boldsymbol{\Omega}'_{\text{sharing}}(\mu) + \boldsymbol{\Omega}'_{\text{pipe}}(\mu)]}^{\text{Endogenous risk effect: Amplifies direct effects}}.$$

Thus, there is a clear mathematical sense in which the endogenous risk effect amplifies the two direct effects of a change in μ.

How individual elements of $\boldsymbol{\Omega}(\mu)$ behave as a function of μ.

Diagonal elements of $\boldsymbol{\Omega}(\mu)$ and $\boldsymbol{\Omega}'(\mu)$. Recall that the diagonal elements of a positive-definite matrix are positive. Then the facts that $\partial E_t[rx^a_{t+1}]/\partial s^a_t = \Omega_{aa} > 0$ and $\partial^2 E_t[rx^a_{t+1}]/\partial s^a_t \partial\mu = \partial\Omega_{aa}/\partial\mu > 0$ for any $a \in \{y, y^*, q\}$ when supply risk is small follow immediately from the facts that both $\boldsymbol{\Omega}(\mu)$ and $\boldsymbol{\Omega}'(\mu)$ are positive-definite.

Off-diagonal elements of $\boldsymbol{\Omega}(\mu)$ and $\boldsymbol{\Omega}'(\mu)$. Again, we consider the limit with zero supply risk. We show that for any $a \in \{y, y^*, q\}$ and $a' \neq a$, $|\Omega_{aa'}(\mu)| = |\partial E_t[rx^a_{t+1}]/\partial s^{a'}_t|$ is hump-shaped function of μ that satisfies $|\Omega_{aa'}(0)| > 0$ and $\Omega_{aa'}(1) = 0$.

The shape of $|\Omega_{aa'}(\mu)|$ reflects the juxtaposition of an "inefficient risk-sharing" effect which is *typically* increasing in μ and a "width-of-the-pipe" effect which is *typically* decreasing in μ. In certain special cases, we have shown that the "inefficient risk-sharing" effect is *always* increasing in μ and a "width-of-the-pipe" effect which is *always* decreasing in μ. For instance, these results hold in a model like ours if (i) there are an equal number of specialists in each asset and (ii) the returns on all assets have the same correlation with each other. However, while these results typically hold for a randomly chosen correlation matrix and a $\boldsymbol{\Pi}$ matrix, counterexamples are possible.

We assume that $\mathbf{V}$ is fixed—i.e., we ignore the endogenous risk effect—and adopt the notation that $\mathbf{V} = \mathbf{S}\boldsymbol{\Gamma}\mathbf{S}$ where $\mathbf{S}$ is a diagonal matrix of standard deviations and $\boldsymbol{\Gamma} = \mathbf{S}^{-1}\mathbf{V}\mathbf{S}^{-1}$ is the correlation matrix. Using this notation, we have

$$\boldsymbol{\Omega}(\mu) = \tau^{-1}\mathbf{S}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\mathbf{S},$$

and

$$\begin{aligned}
\boldsymbol{\Omega}'(\mu) &= \tau^{-1}\mathbf{S}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\overbrace{\left[\boldsymbol{\Gamma}^{-1} - \boldsymbol{\Pi}\right]}^{\text{Positive-definite}}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\mathbf{S} \\
\boldsymbol{\Omega}'_{\text{sharing}}(\mu) &= \tau^{-1}\mathbf{S}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\overbrace{\left[\mathbf{I} - \boldsymbol{\Pi}\right]}^{\text{Positive-definite}}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\mathbf{S} \\
\boldsymbol{\Omega}'_{\text{pipe}}(\mu) &= \tau^{-1}\mathbf{S}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\overbrace{\left[\boldsymbol{\Gamma}^{-1} - \mathbf{I}\right]}^{\text{Indefinite}}\left[\mu\boldsymbol{\Pi} + (1-\mu)\boldsymbol{\Gamma}^{-1}\right]^{-1}\mathbf{S}
\end{aligned}$$

Limit when $\mu = 0$. When $\mu = 0$, we have

$$\boldsymbol{\Omega}(0) = \tau^{-1}\mathbf{S}\boldsymbol{\Gamma}\mathbf{S} \text{ and } \boldsymbol{\Omega}'(0) = \tau^{-1}\mathbf{S}\boldsymbol{\Gamma}\left[\boldsymbol{\Gamma}^{-1} - \boldsymbol{\Pi}\right]\boldsymbol{\Gamma}\mathbf{S}.$$

Specifically, we have

$$\boldsymbol{\Omega}'(0) = \tau^{-1}\mathbf{S}\begin{bmatrix} \pi_2(1-\gamma_{12}^2) + \pi_3(1-\gamma_{13}^2) & \pi_3(\gamma_{12} - \gamma_{13}\gamma_{23}) & \pi_2(\gamma_{13} - \gamma_{12}\gamma_{23}) \\ \pi_3(\gamma_{12} - \gamma_{13}\gamma_{23}) & \pi_1(1-\gamma_{12}^2) + \pi_3(1-\gamma_{23}^2) & \pi_1(\gamma_{23} - \gamma_{12}\gamma_{13}) \\ \pi_2(\gamma_{13} - \gamma_{12}\gamma_{23}) & \pi_1(\gamma_{23} - \gamma_{12}\gamma_{13}) & \pi_1(1-\gamma_{13}^2) + \pi_2(1-\gamma_{23}^2) \end{bmatrix}\mathbf{S}.$$

Thus, the off-diagonals of $\mathbf{\Omega}(0)$ have the same signs as the corresponding univariate correlations. And, the diagonals of $\mathbf{\Omega}'(0)$ are always positive and off-diagonals have same signs as the corresponding *partial* correlations.

Limit when $\mu = 1$. We have

$$\mathbf{\Omega}(1) = \tau^{-1}\mathbf{S}\mathbf{\Pi}^{-1}\mathbf{S} \text{ and } \mathbf{\Omega}'(1) = \tau^{-1}(\mathbf{S}\mathbf{\Pi}^{-1})[\mathbf{\Gamma}^{-1} - \mathbf{\Pi}](\mathbf{\Pi}^{-1}\mathbf{S}).$$

Using the properties of the inverse correlation matrix, we can show that the diagonals of $\mathbf{\Omega}'(1)$ are always positive and the off-diagonals have opposite signs as partial correlations. Specifically, we have the following results:

- *Diagonals of* $[\mathbf{\Gamma}^{-1} - \mathbf{\Pi}]$ *are positive*: The i^{th} diagonal element of $\mathbf{\Gamma}^{-1}$ is $[\mathbf{\Gamma}^{-1}]_{ii} = (1 - R^2_{[\text{reg } i \text{ on all } i' \neq i]})^{-1} \geq 1$, where $R^2_{[\text{reg } i \text{ on all } i' \neq i]}$ is the R^2 from a regression of the ith variable on all other variables. Since $\mathbf{\Pi}$ is a diagonal matrix with diagonal elements strictly less 1 and the diagonal elements of $\mathbf{\Gamma}^{-1}$ greater than or equal to 1, the diagonal elements of $[\mathbf{\Gamma}^{-1} - \mathbf{\Pi}]$ are positive.

- *Off-diagonal elements of* $[\mathbf{\Gamma}^{-1} - \mathbf{\Pi}]$ *have signs opposite those of partial correlations*: The off-diagonals of $\mathbf{\Pi}$ are zero, so the off-diagonals of $[\mathbf{\Gamma}^{-1} - \mathbf{\Pi}]$ are just the off-diagonals of $\mathbf{\Gamma}^{-1}$. The off-diagonals are $\mathbf{\Gamma}^{-1}$ are $[\mathbf{\Gamma}^{-1}]_{ij} = -[\mathbf{\Gamma}^{-1}]_{ii} \cdot b_j^{[i]}$ where $b_j^{[i]}$ is the regression coefficient on the standardized version on variable j from a multivariate regression of standardized variable i on standardized versions all other variables. In other words, we have $(X_i - E[X_i])/\sigma[X_i] = \sum_{j \neq i} b_j^{[i]} \cdot (X_j - E[X_j])/\sigma[X_j] + \epsilon_i$.These $b_j^{[i]}$ are related to the coefficients from the more familiar multivariate regression $X_i = \sum_{j \neq i} \beta_j^{[i]} \cdot X_j + \varepsilon_i$ via $\beta_j^{[i]} = (\sigma[X_i]/\sigma[X_j]) \cdot b_j^{[i]}$. Thus, we have $[\mathbf{\Gamma}^{-1}]_{ij} = -\beta_j^{[i]}(\sigma[X_j]/\sigma[X_i])[\mathbf{\Gamma}^{-1}]_{ii}$so $[\mathbf{\Gamma}^{-1}]_{ij}$ has the opposite sign as $\beta_j^{[i]}$.

For instance, for a 3×3 correlation matrix, we have

$$\begin{bmatrix} 1 & \gamma_{12} & \gamma_{13} \\ \gamma_{12} & 1 & \gamma_{23} \\ \gamma_{13} & \gamma_{23} & 1 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 1 - \gamma_{23}^2 & -(\gamma_{12} - \gamma_{13}\gamma_{23}) & -(\gamma_{13} - \gamma_{12}\gamma_{23}) \\ -(\gamma_{12} - \gamma_{13}\gamma_{23}) & 1 - \gamma_{13}^2 & -(\gamma_{23} - \gamma_{12}\gamma_{13}) \\ -(\gamma_{13} - \gamma_{12}\gamma_{23}) & -(\gamma_{23} - \gamma_{12}\gamma_{13}) & 1 - \gamma_{12}^2 \end{bmatrix}}{1 - \gamma_{12}^2 - \gamma_{13}^2 - \gamma_{23}^2 + 2\gamma_{12}\gamma_{13}\gamma_{23}},$$

where $\det(\mathbf{\Gamma}) = 1 - \gamma_{12}^2 - \gamma_{13}^2 - \gamma_{23}^2 + 2\gamma_{12}\gamma_{13}\gamma_{23} > 0$. So we have

$$[\mathbf{\Gamma}^{-1}]_{11} = \frac{1}{1 - \frac{\gamma_{12}^2 + \gamma_{13}^2 - 2\gamma_{12}\gamma_{13}\gamma_{23}}{1 - \gamma_{23}^2}}, \; b_2^{[1]} = \frac{\gamma_{12} - \gamma_{13}\gamma_{23}}{1 - \gamma_{23}^2}, \text{ and } b_3^{[1]} = \frac{\gamma_{13} - \gamma_{12}\gamma_{23}}{1 - \gamma_{23}^2}.$$

Global behavior of off-diagonal elements on μ Using these facts, we can then characterize the global behavior of the off-diagonal elements of $\mathbf{\Omega}(\mu)$

- We always have $\Omega_{ij}(1) = 0$.
- If $sign(\gamma_{ij}) = sign(\gamma_{ij}^P)$, then $sign(\Omega_{ij}(\mu)) = sign(\gamma_{ij})$ for all $\mu \in [0, 1)$ and $\Omega_{ij}(\mu)$ is a hump-shaped function of μ.
  - If $\gamma_{ij} > 0$, then $\Omega_{ij}(\mu) > 0$ for all $\mu \in [0, 1)$, $\partial\Omega_{ij}(\mu)/\partial\mu > 0$ for μ near 0, and $\partial\Omega_{ij}(\mu)/\partial\mu < 0$ for μ near 1—i.e., $\Omega_{ij}(\mu)$ is inverse U-shaped.

– If $\gamma_{ij} < 0$, then $\Omega_{ij}(\mu) < 0$ for all $\mu \in [0,1)$, $\partial\Omega_{ij}(\mu)/\partial\mu < 0$ for μ near 0, and $\partial\Omega_{ij}(\mu)/\partial\mu > 0$ for μ near 1—i.e., a $\Omega_{ij}(\mu)$ is U-shaped.

- If $sign(\gamma_{ij}) \neq sign(\gamma_{ij}^P)$, then $sign(\Omega_{ij}(\mu)) = sign(\gamma_{ij})$ for μ near 0, $sign(\Omega_{ij}(\mu)) = sign(\gamma_{ij}^P)$ for μ near 1, and $\Omega_{ij}(\mu)$ is again a hump-shaped function of μ.

  – If $\gamma_{ij} > 0$ and $\gamma_{ij}^P < 0$, $\Omega_{ij}(\mu) > 0$ for μ near 0, $\Omega_{ij}(\mu) < 0$ for μ near 1, $\partial\Omega_{ij}(\mu)/\partial\mu < 0$ for μ near 0, and $\partial\Omega_{ij}(\mu)/\partial\mu > 0$ for μ near 1—i.e., $\Omega_{ij}(\mu)$ is U-shaped.

  – If $\gamma_{ij} < 0$ and $\gamma_{ij}^P > 0$, then $\Omega_{ij}(\mu) < 0$ for μ near 0, $\Omega_{ij}(\mu) > 0$ for μ near 1, $\partial\Omega_{ij}(\mu)/\partial\mu > 0$ for μ near 0, and $\partial\Omega_{ij}(\mu)/\partial\mu < 0$ for μ near 1—i.e., $\Omega_{ij}(\mu)$ is inverse U-shaped.

- If $\gamma_{ij}^P = 0$, then $sign(\Omega_{ij}(\mu)) = sign(\gamma_{ij})$ for all $\mu \in [0,1)$ and $|\Omega_{ij}(\mu)|$ is a monotonically decreasing function of μ.

Checking that univariate correlations equal partial correlations in our model when there is no supply risk. We now check that $sign(\gamma_{ij}) = sign(\gamma_{ij}^P)$ in our model when there is no supply risk. In the absence of supply risk, we have:

$$\mathbf{V} = \begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \sigma_{q\infty}^2 + \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) \end{bmatrix}.$$

- **Domestic bonds:** The partial correlation of domestic bonds with foreign bonds and FX are proportional to the regression coefficients:

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \sigma_{q\infty}^2 + \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) \end{bmatrix}^{-1} \begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ \delta\frac{1-\phi_i}{1-\delta\phi_i} \end{bmatrix} - \frac{(1-\phi_i)^2}{\frac{\sigma_i^2}{\sigma_{q\infty}^2}(1-\rho^2) + (1-\phi_i)^2} \begin{bmatrix} (1-\rho) \\ \delta\frac{(1-\phi_i)}{(1-\delta\phi_i)} \end{bmatrix}.$$

  These have the same signs as the univariate covariances:

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \end{bmatrix}.$$

- **Foreign bonds:** The partial correlation of foreign bonds with domestic bonds and FX are proportional to the regression coefficients:

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \sigma_{q\infty}^2 + \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) \end{bmatrix}^{-1} \begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 \\ -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -\delta\frac{1-\phi_i}{1-\delta\phi_i} \end{bmatrix} - \frac{(1-\phi_i)^2}{\frac{\sigma_i^2}{\sigma_{q\infty}^2}(1-\rho^2) + (1-\phi_i)^2} \begin{bmatrix} (1-\rho) \\ -\delta\frac{(1-\phi_i)}{(1-\delta\phi_i)} \end{bmatrix}.$$

These have the same signs as the univariate covariances:

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 \\ -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \end{bmatrix}.$$

- **Foreign exchange:** The partial correlation of foreign exchange with domestic and foreign bonds are proportional to the regression coefficients:

$$\begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 \\ \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \\ -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \end{bmatrix} = \begin{bmatrix} \frac{(1-\delta\phi_i)}{\delta(1-\phi_i)} \\ -\frac{(1-\delta\phi_i)}{\delta(1-\phi_i)} \end{bmatrix}.$$

These have the same signs as the univariate covariances:

$$\begin{bmatrix} \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \\ -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2\left(1-\rho\right) \end{bmatrix}.$$

Intuition for why cross-market price impact is hump-shaped. In fully integrated markets, price impact is determined by univariate correlations (univariate regression coefficients). In partially segmented markets, both univariate correlations and partial correlations (multivariate regression coefficients) matter. And, of course, the distinction between univariate correlations and partial correlations only arises once there are $N > 2$ assets.

When $\mu \to 1$, impact of a long-term domestic supply shock on long-term domestic returns is very large. Because of the strong hedging opportunities afforded by the opportunities to hedge in multiple asset classes, the small number of generalists trade quite aggressively, taking large short positions in foreign bonds and FX. These large positions have a large impact on prices in these markets. As markets become more integrated (μ falls), the impact on long-term domestic returns falls and generalists take smaller short positions, leading impact on foreign bonds and FX to decline.

As we increase μ, fewer investors can absorb a given supply shock. This inefficient risk sharing effect means that supply shocks have a larger impact on returns. At the same time, as we raise μ the width-of-the-pipe effect means that price impact is more localized in the specific asset class where the supply shock lands. When μ is near zero, the inefficient risk sharing effect dominates, so cross-market impact increases in μ. When μ is near 1, the width-of-the-pipe effect dominates, so cross-market impact declines with μ.

What is the intuition for the fact that $sign\left(\Omega_{ij}\left(\mu\right)\right)$ flips if $sign\left(\gamma_{ij}\right) \neq sign(\gamma_{ij}^P)$? In integrated markets ($\mu = 0$), the signs of cross-market price impact are determined by the signs of cross-market return correlations. In partially integrated markets ($\mu \in (0,1)$), partial-correlations—or multivariate regression coefficients—also play a role. For instance, suppose that the returns on asset 1 is positively correlated with those on assets 2 and 3. However, suppose that if we run a multivariate regression of 1 on 2 and 3, the coefficient on 2 is positive and that on 3 is negative.

Suppose that there are initially very few generalists—markets are close to completely segmented—and that there is a shock to the supply of asset 1. In highly segmented markets, this is going to have a large positive impact on the expected returns on 1 and smaller impacts on those of 2 and 3. Following the shock to the supply of asset 1, generalists want to go long 1 and to hedge this additional risk they want to short 2 and go *long* 3. To induce specialists to take the other side in markets 2 and 3, the return on 2 must rise and those on 3 must *fall*.

As we add more and more generalists who care about the comovement between the returns on the three assets, the shock to the supply of 1 pushes up the returns on both 2 and 3. Thus, as the number of generalists rises, the sign of the price impact on 3 flips signs. In the limit where everyone is a generalist, the rise in returns guarantees that generalists simply absorb the shock to the supply of 1 without trading 2 or 3.

E.2 Adding unhedged bond investors

E.2.1 Details and solution

Unhedged domestic investors Unhedged domestic investors are present in mass $\eta/2$. They can trade short-term domestic bonds, long-term domestic bonds, and long-term foreign bonds, but not short-term foreign bonds. Thus, if they buy long-term foreign bonds, they must take on foreign exchange exposure, generating an excess return of $rx_{t+1}^{y^*} + rx_{t+1}^{q}$ over short-term domestic bonds. Specifically, if unhedged domestic investors have a demand $b_{D,t}^{y*}$ for foreign long-term bonds, then their holdings of the foreign yield curve trade and the FX trade are $d_{D,t}^{y*} = b_{D,t}^{y*}$ and $d_{D,t}^{q} = b_{D,t}^{y*}$. So we have

$$\mathbf{d}_{D,t} = \begin{bmatrix} d_{D,t}^{y} \\ d_{D,t}^{y*} \\ d_{D,t}^{q} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_{D,t}^{y} \\ b_{D,t}^{y*} \end{bmatrix} = \mathbf{H}_D \mathbf{b}_{D,t}$$

Unhedged domestic investors choose their demands $\mathbf{b}_{D,t}$ over returns

$$\mathbf{rx}_{D,t+1} = \begin{bmatrix} rx_{t+1}^{y} \\ rx_{t+1}^{y*} + rx_{t+1}^{q} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} rx_{t+1}^{y} \\ rx_{t+1}^{y*} \\ rx_{t+1}^{q} \end{bmatrix} = \mathbf{H}_D' \mathbf{rx}_{t+1}$$

Solving

$$\max_{\mathbf{b}_{D,t}} \left\{ \mathbf{b}_{D,t}' E_t \left[\mathbf{rx}_{D,t+1} \right] - \frac{\tau}{2} \mathbf{b}_{D,t}' Var_t \left[\mathbf{rx}_{D,t+1} \right] \mathbf{b}_{D,t} \right\}.$$

Thus, we have

$$\begin{aligned} \mathbf{b}_{D,t} &= \tau \left(\mathbf{H}_D' Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_D \right)^{-1} \mathbf{H}_D' E_t \left[\mathbf{rx}_{t+1} \right] \\ &= \tau \frac{\begin{bmatrix} (V_q - 2C_q + V_y) \cdot E_t \left[rx_{t+1}^{y} \right] - (C_q + C_y) \cdot E_t \left[rx_{t+1}^{y*} + rx_{t+1}^{q} \right] \\ -(C_q + C_y) \cdot E_t \left[rx_{t+1}^{y} \right] + V_y \cdot E_t \left[rx_{t+1}^{y*} + rx_{t+1}^{q} \right] \end{bmatrix}}{V_y (V_q - 2C_q + V_y) - (C_q + C_y)^2} \end{aligned}$$

This implies that

$$\begin{aligned} \mathbf{d}_{D,t} &= \tau \mathbf{H}_D \left(\mathbf{H}_D' Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_D \right)^{-1} \mathbf{H}_D' E_t \left[\mathbf{rx}_{t+1} \right] \\ &= \tau \frac{\begin{bmatrix} (V_q - 2C_q + V_y) \cdot E_t \left[rx_{t+1}^{y} \right] - (C_q + C_y) \cdot E_t \left[rx_{t+1}^{y*} + rx_{t+1}^{q} \right] \\ -(C_q + C_y) \cdot E_t \left[rx_{t+1}^{y} \right] + V_y \cdot E_t \left[rx_{t+1}^{y*} + rx_{t+1}^{q} \right] \\ -(C_q + C_y) \cdot E_t \left[rx_{t+1}^{y} \right] + V_y \cdot E_t \left[rx_{t+1}^{y*} + rx_{t+1}^{q} \right] \end{bmatrix}}{V_y (V_q - 2C_q + V_y) - (C_q + C_y)^2} \end{aligned}$$

Unhedged foreign investors Unhedged foreign investors are present in mass $\eta/2$ and are the mirror image of unhedged domestic investors. They can buy long-term foreign bonds, long-term domestics bonds, but not short-term domestic bonds. Thus, if they buy long-term domestic bonds, they must take on FX exposure, generating an excess return of $rx_{t+1}^{y} - rx_{t+1}^{q}$ over short-term foreign bonds.

Specifically, if unhedged foreign investors have a demand $b^y_{F,t}$ for domestic long-term bonds, they we will have $d^y_{F,t} = b^y_{D,t}$ and $d^q_{F,t} = -b^y_{F,t}$. So we have

$$\mathbf{d}_{F,t} = \begin{bmatrix} d^y_{F,t} \\ d^{y*}_{F,t} \\ d^q_{F,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} b^y_{F,t} \\ b^{y*}_{F,t} \end{bmatrix} = \mathbf{H}_F \mathbf{b}_{F,t}$$

Think of unhedged foreign investors as picking demands $\mathbf{b}_{F,t}$ over returns

$$\mathbf{rx}_{F,t+1} = \begin{bmatrix} rx^y_{t+1} - rx^q_{t+1} \\ rx^{y*}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} rx^y_{t+1} \\ rx^{y*}_{t+1} \\ rx^q_{t+1} \end{bmatrix} = \mathbf{H}'_F \mathbf{rx}_{t+1}$$

Thus, unhedged foreign investors solve

$$\max_{\mathbf{b}_{F,t}} \left\{ \mathbf{b}'_{F,t} E_t \left[\mathbf{rx}_{F,t+1} \right] - \frac{\tau}{2} \mathbf{b}'_{F,t} Var_t \left[\mathbf{rx}_{F,t+1} \right] \mathbf{b}_{F,t} \right\}.$$

Thus, we have

$$\begin{aligned} \mathbf{b}_{F,t} &= \tau \left(\mathbf{H}'_F Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_F \right)^{-1} \mathbf{H}'_F E_t \left[\mathbf{rx}_{t+1} \right] \\ &= \tau \frac{\begin{bmatrix} V_y \cdot E_t \left[rx^y_{t+1} - rx^q_{t+1} \right] - \left(C_q + C_y \right) \cdot E_t \left[rx^{y*}_{t+1} \right] \\ \left(V_q - 2C_q + V_y \right) \cdot E_t \left[rx^{y*}_{t+1} \right] - \left(C_q + C_y \right) \cdot E_t \left[rx^y_{t+1} - rx^q_{t+1} \right] \end{bmatrix}}{V_y \left(V_q - 2C_q + V_y \right) - \left(C_q + C_y \right)^2} \end{aligned}$$

and

$$\begin{aligned} \mathbf{d}_{F,t} &= \tau \mathbf{H}_F \left(\mathbf{H}'_F Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_F \right)^{-1} \mathbf{H}'_F E_t \left[\mathbf{rx}_{t+1} \right] \\ &= \tau \frac{\begin{bmatrix} V_y \cdot E_t \left[rx^y_{t+1} - rx^q_{t+1} \right] - \left(C_q + C_y \right) \cdot E_t \left[rx^{y*}_{t+1} \right] \\ \left(V_q - 2C_q + V_y \right) \cdot E_t \left[rx^{y*}_{t+1} \right] - \left(C_q + C_y \right) \cdot E_t \left[rx^y_{t+1} - rx^q_{t+1} \right] \\ \left(C_q + C_y \right) \cdot E_t \left[rx^{y*}_{t+1} \right] - V_y \cdot E_t \left[rx^y_{t+1} - rx^q_{t+1} \right] \end{bmatrix}}{V_y \left(V_q - 2C_q + V_y \right) - \left(C_q + C_y \right)^2} \end{aligned}$$

Global bond investors Global bond investors, present in mass $(1 - \eta)$, can hold short- and long-term bonds in both currencies and can engage in all three carry trades. Thus, global investors have demands

$$\mathbf{d}_{G,t} = \tau \left(Var_t \left[\mathbf{rx}_{t+1} \right] \right)^{-1} E_t \left[\mathbf{rx}_{t+1} \right]$$

Market clearing and solution Letting $\mathbf{V} = Var_t \left[\mathbf{rx}_{t+1} \right]$, the market clearing condition

$$\begin{aligned} \mathbf{s}_t &= \frac{\eta}{2} \mathbf{d}_{D,t} + \frac{\eta}{2} \mathbf{d}_{F,t} + (1 - \eta) \mathbf{d}_{G,t} \\ &= \frac{\eta}{2} \tau \mathbf{H}_D \left(\mathbf{H}'_D \mathbf{V} \mathbf{H}_D \right)^{-1} \mathbf{H}'_D E_t \left[\mathbf{rx}_{t+1} \right] + \frac{\eta}{2} \tau \mathbf{H}_F \left(\mathbf{H}'_F \mathbf{V} \mathbf{H}_F \right)^{-1} \mathbf{H}'_F E_t \left[\mathbf{rx}_{t+1} \right] + (1 - \eta) \tau \mathbf{V}^{-1} E_t \left[\mathbf{rx}_{t+1} \right], \end{aligned}$$

so we have

$$E_t \left[\mathbf{rx}_{t+1} \right] = \tau^{-1} \left(\frac{\eta}{2} \mathbf{H}_D \left(\mathbf{H}'_D \mathbf{V}(\eta) \mathbf{H}_D \right)^{-1} \mathbf{H}'_D + \frac{\eta}{2} \mathbf{H}_F \left(\mathbf{H}'_F \mathbf{V}(\eta) \mathbf{H}_F \right)^{-1} \mathbf{H}'_F + (1 - \eta) \left[\mathbf{V}(\eta) \right]^{-1} \right)^{-1} \mathbf{s}_t.$$

Fixed point problem The fixed point for $\mathbf{A}$ is

$$\mathbf{A}=\left[\tau^{-1}\left(\frac{\eta}{2}\cdot\mathbf{H}_{D}\left(\mathbf{H}_{D}^{\prime}\mathbf{V}\mathbf{H}_{D}\right)^{-1}\mathbf{H}_{D}^{\prime}+\frac{\eta}{2}\cdot\mathbf{H}_{F}\left(\mathbf{H}_{F}^{\prime}\mathbf{V}\mathbf{H}_{F}\right)^{-1}\mathbf{H}_{F}^{\prime}+(1-\eta)\,\mathbf{V}^{-1}\right)^{-1}\mathbf{S}_{1}-\mathbf{R}_{1}\right]\oslash[\mathbf{B}_{0}\mathbf{E}+\mathbf{B}_{1}\mathbf{E}\mathbf{\Phi}]$$

where $\oslash$ denotes element-wise matrix division (i.e., Hadamard division) and

$$\mathbf{V}=\mathbf{B}_{1}\mathbf{A}\mathbf{\Sigma}\mathbf{A}^{\prime}\mathbf{B}_{1}^{\prime}.$$

We can think of equilibrium as a fixed point problem involving the return impact matrix, $\mathbf{\Theta}$, that maps changes in asset to supply to shifts in asset returns—i.e. $E_{t}\left[\mathbf{rx}_{t+1}\right]=\mathbf{\Theta}\mathbf{s}_{t}$. This return impact matrix is given by

$$\mathbf{\Theta}=\tau^{-1}\left(\frac{\eta}{2}\mathbf{H}_{D}\left(\mathbf{H}_{D}^{\prime}\mathbf{V}\mathbf{H}_{D}\right)^{-1}\mathbf{H}_{D}^{\prime}+\frac{\eta}{2}\mathbf{H}_{F}\left(\mathbf{H}_{F}^{\prime}\mathbf{V}\mathbf{H}_{F}\right)^{-1}\mathbf{H}_{F}^{\prime}+(1-\eta)\left[\mathbf{V}\right]^{-1}\right)^{-1}$$

Since $\mathbf{A}_{s}=\mathbf{\Theta}\oslash\mathbf{Z}_{s}$ where $\mathbf{Z}_{s}=[\mathbf{B}_{0}\mathbf{E}+\mathbf{B}_{1}\mathbf{E}\mathbf{\Phi}]^{[3-5]}$, we have

$$\mathbf{V}=\mathbf{V}_{i}+\left(\mathbf{B}_{1}\mathbf{\Theta}\oslash\mathbf{Z}_{s}\right)\mathbf{\Sigma}_{s}\left(\mathbf{B}_{1}\mathbf{\Theta}\oslash\mathbf{Z}_{s}\right)^{\prime}=\mathbf{V}_{i}+\mathbf{Z}\circ\mathbf{\Theta}\mathbf{\Sigma}_{s}\mathbf{\Theta}\circ\mathbf{Z}^{\prime},$$

where $\mathbf{Z}$ satisfies $\left(\mathbf{B}_{1}\mathbf{\Theta}\oslash\mathbf{Z}_{s}\right)=\mathbf{\Theta}\circ\mathbf{Z}$. Even though $\mathbf{\Theta}$ is symmetric, $\mathbf{Z}$ is not, so $\mathbf{\Theta}\circ\mathbf{Z}$ is not symmetric. Thus, the relevant fixed point problem in $\mathbf{\Theta}$ is

$$\mathbf{\Theta}=\tau^{-1}\left(\begin{array}{c}\frac{\eta}{2}\mathbf{H}_{D}\left(\mathbf{H}_{D}^{\prime}[\mathbf{V}_{i}+\mathbf{Z}\circ\mathbf{\Theta}\mathbf{\Sigma}_{s}\mathbf{\Theta}\circ\mathbf{Z}^{\prime}]\mathbf{H}_{D}\right)^{-1}\mathbf{H}_{D}^{\prime}\\+\frac{\eta}{2}\mathbf{H}_{F}\left(\mathbf{H}_{F}^{\prime}[\mathbf{V}_{i}+\mathbf{Z}\circ\mathbf{\Theta}\mathbf{\Sigma}_{s}\mathbf{\Theta}\circ\mathbf{Z}^{\prime}]\mathbf{H}_{F}\right)^{-1}\mathbf{H}_{F}^{\prime}\\+(1-\eta)\left[\mathbf{V}_{i}+\mathbf{Z}\circ\mathbf{\Theta}\mathbf{\Sigma}_{s}\mathbf{\Theta}\circ\mathbf{Z}^{\prime}\right]^{-1}\end{array}\right)^{-1}.$$

As always, we focus on the unique stable solution to this fixed point problem.

Trading behavior Writing out $\mathbf{\Theta}$, we have

$$\mathbf{\Theta}=\tau^{-1}\left(\frac{\eta}{2}\mathbf{H}_{D}\left(\mathbf{H}_{D}^{\prime}\mathbf{V}\mathbf{H}_{D}\right)^{-1}\mathbf{H}_{D}^{\prime}+\frac{\eta}{2}\mathbf{H}_{F}\left(\mathbf{H}_{F}^{\prime}\mathbf{V}\mathbf{H}_{F}\right)^{-1}\mathbf{H}_{F}^{\prime}+(1-\eta)\,\mathbf{V}^{-1}\right)^{-1}.$$

Thus, we have

$$\begin{aligned}\mathbf{I}&\neq\frac{\partial\mathbf{d}_{D,t}}{\partial\mathbf{s}_{t}}=\tau\mathbf{H}_{D}\left(\mathbf{H}_{D}^{\prime}\mathbf{V}(\eta\mathbf{H}_{D}\right)^{-1}\mathbf{H}_{D}^{\prime}\mathbf{\Theta},\\\mathbf{I}&\neq\frac{\partial\mathbf{d}_{F,t}}{\partial\mathbf{s}_{t}}=\tau\mathbf{H}_{F}\left(\mathbf{H}_{F}^{\prime}\mathbf{V}\mathbf{H}_{F}\right)^{-1}\mathbf{H}_{F}^{\prime}\mathbf{\Theta},\\\mathbf{I}&\neq\frac{\partial\mathbf{d}_{G,t}}{\partial\mathbf{s}_{t}}=\tau\mathbf{V}^{-1}\mathbf{\Theta},\end{aligned}$$

so the first part of the Proposition follows trivially.

Understanding how $\mathbf{\Theta}(\eta)$ varies as a function of η. We first want to understand how this solution $\mathbf{\Theta}(\eta)$ varies as a function of η. Clearly, $\mathbf{\Theta}(\eta)$ is positive definite. However, we are interested in understanding when/whether $\mathbf{\Theta}^{\prime}(\eta)=\partial\mathbf{\Theta}(\eta)/\partial\eta$ is itself a positive definite matrix.

We have

$$\mathbf{\Theta}(\eta)=\tau^{-1}\left(\frac{\eta}{2}\mathbf{H}_{D}\left(\mathbf{H}_{D}^{\prime}\mathbf{V}(\eta)\mathbf{H}_{D}\right)^{-1}\mathbf{H}_{D}^{\prime}+\frac{\eta}{2}\mathbf{H}_{F}\left(\mathbf{H}_{F}^{\prime}\mathbf{V}(\eta)\mathbf{H}_{F}\right)^{-1}\mathbf{H}_{F}^{\prime}+(1-\eta)\left[\mathbf{V}(\eta)\right]^{-1}\right)^{-1}.$$

This implies that

$$\begin{aligned}\boldsymbol{\Theta}'(\eta) &= \tau\boldsymbol{\Theta}(\eta)\left\{[\mathbf{V}(\eta)]^{-1} - \frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\mathbf{H}_D' - \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}(\eta)\mathbf{H}_F\right)^{-1}\mathbf{H}_F'\right\}\boldsymbol{\Theta}(\eta) \\ &+\tau\boldsymbol{\Theta}(\eta)\begin{bmatrix} \frac{\eta}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\left(\mathbf{H}_D'\mathbf{V}'(\eta)\mathbf{H}_D\right)\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\mathbf{H}_D' \\ +\frac{\eta}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}(\eta)\mathbf{H}_F\right)^{-1}\left(\mathbf{H}_F'\mathbf{V}'(\eta)\mathbf{H}_F\right)\left(\mathbf{H}_F'\mathbf{V}(\eta)\mathbf{H}_F\right)^{-1}\mathbf{H}_F' \\ +(1-\eta)[\mathbf{V}(\eta)]^{-1}\mathbf{V}'(\eta)[\mathbf{V}(\eta)]^{-1}\end{bmatrix}\boldsymbol{\Theta}(\eta). \end{aligned} \tag{80}$$

As above, we assume that FX rates are subject to some FX-specific fundamental risk, so $\mathbf{V}^{-1}$ exists even in the absence of supply risk. In this case, we can show that $\{\mathbf{V}^{-1} - \frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\mathbf{H}_D' - \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F'\}$ is positive semi-definite, immediately implying that the first matrix in curly braces in equation (80) positive semi-definite. This means that $\boldsymbol{\Theta}'(\eta)$ must always be positive definite in the absence of supply risk. Furthermore, by continuity of the stable equilibrium in the model's underlying parameters, $\boldsymbol{\Theta}'(\eta)$ must continue to be positive-definite when supply risk is small ($\boldsymbol{\Sigma}_s$ is small).

Proof that $\boldsymbol{\Theta}'(\eta)$ is positive semi-definite in the absence of supply risk. To prove that $\{\mathbf{V}^{-1} - \frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\mathbf{H}_D' - \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F'\}$ is positive semi-definite it suffices to prove that $[\frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\mathbf{H}_D' + \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F']^{-1} - \mathbf{V}$ is positive semi-definite.[33] We have

$$\begin{aligned}&\frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}\mathbf{H}_D\right)^{-1}\mathbf{H}_D' + \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F' \\ =& \frac{1}{V_y\left(V_y+V_q-2C_q\right)-\left(C_q+C_y\right)^2}\begin{bmatrix} \left(V_y-C_q\right)+\frac{1}{2}V_q & -\left(C_q+C_y\right) & -\frac{1}{2}\left(C_q+C_y+V_y\right) \\ -\left(C_q+C_y\right) & \left(V_y-C_q\right)+\frac{1}{2}V_q & \frac{1}{2}\left(C_q+C_y+V_y\right) \\ -\frac{1}{2}\left(C_q+C_y+V_y\right) & \frac{1}{2}\left(C_q+C_y+V_y\right) & V_y \end{bmatrix}\end{aligned}$$

Thus, we have

$$\begin{aligned}&\left[\frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}\mathbf{H}_D\right)^{-1}\mathbf{H}_D' + \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F'\right]^{-1} - \mathbf{V} \\ =& \begin{bmatrix} -\frac{C_q^2+2C_qC_y+2C_qV_y+C_y^2-V_y^2-V_qV_y}{V_q-2C_y-4C_q+2V_y} & -\frac{C_q^2-2C_qC_y-2C_qV_y-C_y^2+V_qC_y+V_y^2}{V_q-2C_y-4C_q+2V_y} & \left(C_y+V_y\right) \\ -\frac{C_q^2-2C_qC_y-2C_qV_y-C_y^2+V_qC_y+V_y^2}{V_q-2C_y-4C_q+2V_y} & -\frac{C_q^2+2C_qC_y+2C_qV_y+C_y^2-V_y^2-V_qV_y}{V_q-2C_y-4C_q+2V_y} & -\left(C_y+V_y\right) \\ \left(C_y+V_y\right) & -\left(C_y+V_y\right) & 2\left(C_y+V_y\right) \end{bmatrix}\end{aligned}$$

The eigenvalues of this later matrix are:

1. $\lambda_1 = 3\left(V_y+C_y\right) > 0$. The inequality follows from the fact that $\left(V_y+C_y\right) > 0$..
2. $\lambda_2 = [V_q\left(V_y-C_y\right) - 2C_q^2]/[2\left(V_y-C_y\right)+V_q-4C_q] > 0$.
   - Since $\mathbf{V}$ is positive definite, we have $\det(\mathbf{V}) = \left(C_y+V_y\right)\left(V_q\left(V_y-C_y\right)-2C_q^2\right) > 0$. Then, since $\left(C_y+V_y\right) > 0$, we have $V_q\left(V_y-C_y\right)-2C_q^2 > 0$.

[33] *Suppose that $\mathbf{A}$ and $\mathbf{B}$ are positive definite matrices. If $\mathbf{A} \prec \mathbf{B}$, then $\mathbf{B}^{-1} \prec \mathbf{A}^{-1}$. (We use $\mathbf{A} \prec \mathbf{B}$ to mean that $(\mathbf{B}-\mathbf{A})$ is positive-definite.) Similarly, if $\mathbf{A} \preceq \mathbf{B}$, then $\mathbf{B}^{-1} \preceq \mathbf{A}^{-1}$. (We use $\mathbf{A} \preceq \mathbf{B}$ to mean that $(\mathbf{B}-\mathbf{A})$ is positive semi-definite.)* To prove this claim, first note that $\mathbf{A} \prec \mathbf{B} \Rightarrow \mathbf{CAC}' \prec \mathbf{CBC}'$ for any conformable matrix $\mathbf{C}$. Second, note that $\mathbf{I} \prec \mathbf{B} \Rightarrow \mathbf{B}^{-1} \prec \mathbf{I}$. To see this, note that $\mathbf{B}^{-1} = \mathbf{B}^{-1/2}\mathbf{I}\mathbf{B}^{-1/2} \prec \mathbf{B}^{-1/2}\mathbf{B}\mathbf{B}^{-1/2} = \mathbf{I}$. Third, note that $\mathbf{A} \prec \mathbf{B} \Rightarrow \mathbf{0} \prec \mathbf{B}-\mathbf{A} \Rightarrow \mathbf{0} \prec \mathbf{A}^{-1/2}\left(\mathbf{B}-\mathbf{A}\right)\mathbf{A}^{-1/2} \Rightarrow \mathbf{I} \prec \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}$. Thus, we have $\mathbf{A}^{1/2}\mathbf{B}^{-1}\mathbf{A}^{1/2} = \left(\mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}\right)^{-1} \prec \mathbf{I}$. Finally, we have $\mathbf{B}^{-1} = \mathbf{A}^{-1/2}(\mathbf{A}^{1/2}\mathbf{B}^{-1}\mathbf{A}^{1/2})\mathbf{A}^{-1/2} \prec \mathbf{A}^{-1/2}(\mathbf{I})\mathbf{A}^{-1/2} = \mathbf{A}^{-1}$.

- Since $\mathbf{V}$ is positive definite, we have $2(V_y - C_y) + V_q - 4C_q = Var\left[\left(rx_{t+1}^{y*} - rx_{t+1}^{y}\right) + rx_{t+1}^{q}\right] > 0$.

3. $\lambda_3 = 0$.

Thus, we conclude that $\{\mathbf{V}^{-1} - \frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}(\eta)\mathbf{H}_D\right)^{-1}\mathbf{H}_D' - \frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F'\}$ is positive semi-definite.

How individual elements of $\boldsymbol{\Theta}(\eta)$ behave as a function of η. We consider the special case without supply risk. Here we have

$$\mathbf{V} = \begin{bmatrix} \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \rho\sigma_i^2 & \left(\frac{\delta}{1-\delta\phi_i}\right)^2 \sigma_i^2 & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) \\ \frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & -\frac{\delta}{1-\delta\phi_i}\frac{1}{1-\phi_i}\sigma_i^2(1-\rho) & \sigma_{q\infty}^2 + \left(\frac{1}{1-\phi_i}\right)^2 2\sigma_i^2(1-\rho) \end{bmatrix},$$

which is independent of η. Assuming $\sigma_{q\infty}^2 > 0$, $\mathbf{V}$ is positive definite and we have $\det(\mathbf{V}) > 0$. We have

$$\boldsymbol{\Theta}(\eta) = \tau^{-1}\left(\frac{\eta}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}\mathbf{H}_D\right)^{-1}\mathbf{H}_D' + \frac{\eta}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F' + (1-\eta)[\mathbf{V}]^{-1}\right)^{-1}.$$

$\boldsymbol{\Theta}(\eta)$ is positive definite and $\boldsymbol{\Theta}'(\eta)$ is positive semi-definite.

Diagonal elements of $\boldsymbol{\Theta}'(\eta)$ Since $\boldsymbol{\Theta}'(\eta)$ is positive semi-definite, it follows that $\boldsymbol{\Theta}'_{[1,1]}(\eta) \geq 0$, $\boldsymbol{\Theta}'_{[2,2]}(\eta) \geq 0$, $\boldsymbol{\Theta}'_{[3,3]}(\eta) \geq 0$ for all η.

Computing $\boldsymbol{\Theta}(1) - \boldsymbol{\Theta}(0) = \int_0^1 \boldsymbol{\Theta}'(\eta)\,d\eta$. We have

$$\boldsymbol{\Theta}(0) = \tau^{-1}\begin{bmatrix} V_y & C_y & C_q \\ C_y & V_y & -C_q \\ C_q & -C_q & V_q \end{bmatrix},$$

and

$$\boldsymbol{\Theta}(1) = \tau^{-1}\begin{bmatrix} \frac{3V_y^2+2V_qV_y-C_q^2-2C_qC_y-6C_qV_y-C_y^2-2C_yV_y}{2(V_y-C_y)+V_q-4C_q} & -\frac{(C_q+C_y-V_y)^2}{2(V_y-C_y)+V_q-4C_q} & C_q + (V_y + C_y) \\ -\frac{(C_q+C_y-V_y)^2}{2(V_y-C_y)+V_q-4C_q} & \frac{3V_y^2+2V_qV_y-C_q^2-2C_qC_y-6C_qV_y-C_y^2-2C_yV_y}{2(V_y-C_y)+V_q-4C_q} & -C_q - (V_y + C_y) \\ C_q + (V_y + C_y) & -C_q - (V_y + C_y) & V_q + 2(V_y + C_y) \end{bmatrix}$$

Thus, we have

$$\boldsymbol{\Theta}(1) - \boldsymbol{\Theta}(0) = \tau^{-1}\begin{bmatrix} \frac{V_y(V_y+V_q-2C_q)-(C_q+C_y)^2}{2(V_y-C_y)+V_q-4C_q} & -\frac{(C_q+C_y-V_y)^2}{2V_y+V_q-2C_y-4C_q} - C_y & C_y + V_y \\ -\frac{(C_q+C_y-V_y)^2}{2V_y+V_q-2C_y-4C_q} - C_y & \frac{V_y(V_y+V_q-2C_q)-(C_q+C_y)^2}{2(V_y-C_y)+V_q-4C_q} & -(C_y + V_y) \\ C_y + V_y & -(C_y + V_y) & 2(V_y + C_y) \end{bmatrix} \propto \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix},$$

which follows from the facts that

$$\begin{aligned}
\frac{V_y\left(V_y+V_q-2C_q\right)-\left(C_q+C_y\right)^2}{2\left(V_y-C_y\right)+V_q-4C_q} &= \frac{\sigma_{q\infty}^2\left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2+\delta^2\sigma_i^4\frac{(1-\delta)^2}{(1-\phi_i)^2}\frac{1-\rho^2}{(1-\delta\phi_i)^4}}{\sigma_{q\infty}^2+2\sigma_i^2\left(1-\rho\right)\left(\frac{\delta}{1-\delta\phi_i}-\frac{1}{1-\phi_i}\right)^2}>0,\\
-\frac{\left(C_q+C_y-V_y\right)^2}{2V_y+V_q-2C_y-4C_q}-C_y &= -\frac{\left(\frac{\sigma_i^2\delta(1-\delta)(1-\rho)}{(1-\phi_i)(1-\delta\phi_i)^2}\right)^2}{\sigma_{q\infty}^2+2\sigma_i^2\left(1-\rho\right)\left(\frac{\delta}{1-\delta\phi_i}-\frac{1}{1-\phi_i}\right)^2}-\left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2<0,\\
V_y+C_y &= \left(\frac{\delta}{1-\delta\phi_i}\right)^2\sigma_i^2\left(1+\rho\right)>0.
\end{aligned}$$

Computing $\boldsymbol{\Theta}'(0)$. Compute derivative at $\eta=0$ where $\boldsymbol{\Theta}(\eta)=\tau^{-1}\mathbf{V}$. We have

$$\begin{aligned}
\boldsymbol{\Theta}'(0) &= \tau^{-1}\mathbf{V}\left[\mathbf{V}^{-1}-\frac{1}{2}\mathbf{H}_D\left(\mathbf{H}_D'\mathbf{V}\mathbf{H}_D\right)^{-1}\mathbf{H}_D'-\frac{1}{2}\mathbf{H}_F\left(\mathbf{H}_F'\mathbf{V}\mathbf{H}_F\right)^{-1}\mathbf{H}_F'\right]\mathbf{V}\\
&= \tau^{-1}\frac{\left(C_y+V_y\right)\left(V_q\left(V_y-C_y\right)-2C_q^2\right)}{\left(C_q+C_y\right)^2+V_y\left(V_y+V_q-2C_q\right)}\begin{bmatrix}\frac{1}{2} & 0 & \frac{1}{2}\\ 0 & \frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & -\frac{1}{2} & 1\end{bmatrix}\propto\begin{bmatrix}+ & 0 & +\\ 0 & + & -\\ + & - & +\end{bmatrix},
\end{aligned}$$

which follows from that facts that $\left(C_y+V_y\right)\left(V_q\left(V_y-C_y\right)-2C_q^2\right)=\det\left(V\right)>0$ and $V_y+V_q-2C_q=Var\left[rx_{t+1}^y-rx_{t+1}^q\right]>0$.

Computing $\boldsymbol{\Theta}'(1)$. Compute the derivative at $\eta=1$. We have

$$\boldsymbol{\Theta}(1)=\tau^{-1}\begin{bmatrix}\frac{3V_y^2+2V_qV_y-C_q^2-2C_qC_y-6C_qV_y-C_y^2-2C_yV_y}{2(V_y-C_y)+V_q-4C_q} & -\frac{(C_q+C_y-V_y)^2}{2(V_y-C_y)+V_q-4C_q} & C_q+\left(V_y+C_y\right)\\ -\frac{(C_q+C_y-V_y)^2}{2(V_y-C_y)+V_q-4C_q} & \frac{3V_y^2+2V_qV_y-C_q^2-2C_qC_y-6C_qV_y-C_y^2-2C_yV_y}{2(V_y-C_y)+V_q-4C_q} & -C_q-\left(V_y+C_y\right)\\ C_q+\left(V_y+C_y\right) & -C_q-\left(V_y+C_y\right) & V_q+2\left(V_y+C_y\right)\end{bmatrix}.$$

We have

$$\begin{aligned}
\boldsymbol{\Theta}'_{[3,1]}(1) &= \tau^{-1}2\frac{\left(C_y+V_y\right)\left(V_y\left(V_y+V_q-2C_q\right)-\left(C_q+C_y\right)^2\right)}{V_q\left(V_y-C_y\right)-2C_q^2}\\
&= \tau^{-1}2\frac{\left(C_y+V_y\right)\left(Var\left[rx_{t+1}^y\right]Var\left[rx_{t+1}^{y*}+rx_{t+1}^q\right]-\left(Cov\left[rx_{t+1}^y,rx_{t+1}^{y*}+rx_{t+1}^q\right]\right)^2\right)}{V_q\left(V_y-C_y\right)-2C_q^2}>0.
\end{aligned}$$

Of course, we have $\boldsymbol{\Theta}'_{[3,1]}(1)=\boldsymbol{\Theta}'_{[1,3]}(1)=-\boldsymbol{\Theta}'_{[3,2]}(1)=-\boldsymbol{\Theta}'_{[2,3]}(1)$. We also have

$$\boldsymbol{\Theta}'_{[2,1]}(1)=-4\tau^{-1}\left(\frac{C_q^2+2C_qC_y+2C_qV_y+C_y^2-V_y^2-V_qV_y}{V_q-2C_y-4C_q+2V_y}\right)^2\frac{\left(C_q^2-2C_qC_y-2C_qV_y-C_y^2+V_qC_y+V_y^2\right)}{\left(C_y+V_y\right)\left(V_q\left(V_y-C_y\right)-2C_q^2\right)}.$$

Thus, $\boldsymbol{\Theta}'_{[2,1]}(1)$ has the opposite sign of

$$C_q^2-2C_qC_y-2C_qV_y-C_y^2+V_qC_y+V_y^2=\sigma_{q\infty}^2\left(\frac{\delta}{1-\delta\phi_i}\right)^2\rho\sigma_i^2+\delta^2\sigma_i^4\frac{\left(1-\delta\right)^2}{\left(1-\phi_i\right)^2}\frac{1-\rho^2}{\left(1-\delta\phi_i\right)^4}>0.$$

Thus, we have $\mathbf{\Theta}'_{[2,1]}(1) < 0$. In summary, we conclude that

$$\mathbf{\Theta}'(1) \propto \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}.$$

$\mathbf{\Theta}'(\eta)$ for $\eta \in [0,1]$ Although the algebra gets extremely messy, we can show that

$$\mathbf{\Theta}'(\eta) \propto \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

for $\eta \in [0,1]$. For simplicity, we show this explicitly below the case where $\delta \to 1$.

Computing $\lim_{\delta\to 1} \mathbf{\Theta}'(\eta)$ We now compute the result in the limit where $\delta \to 1$. This limit—the limit where the duration of long-term bonds becomes infinite—simplifies the algebra considerably, but does not change the underlying economics. We have $\lim_{\delta\to 1} \mathbf{\Theta}'(\eta) = \partial \left[\lim_{\delta\to 1} \mathbf{\Theta}(\eta)\right] / \partial\eta$. We now compute $\lim_{\delta\to 1} \mathbf{\Theta}(\eta)$.

$$\lim_{\delta\to 1} \mathbf{\Theta}(\eta) = \tau^{-1} \begin{bmatrix} 2\frac{\sigma_i^2}{(1-\phi_i)^2}\frac{2-\eta-\eta\rho^2}{4-4\eta+\eta^2(1-\rho^2)} & \frac{4\rho\sigma_i^2}{(1-\phi_i)^2}\frac{1-\eta}{4-4\eta+\eta^2(1-\rho^2)} & \frac{2\sigma_i^2(1-\rho)}{(1-\phi_i)^2}\frac{1}{2-\eta-\eta\rho} \\ \frac{4\rho\sigma_i^2}{(1-\phi_i)^2}\frac{1-\eta}{4-4\eta+\eta^2(1-\rho^2)} & 2\frac{\sigma_i^2}{(1-\phi_i)^2}\frac{2-\eta-\eta\rho^2}{4-4\eta+\eta^2(1-\rho^2)} & -\frac{2\sigma_i^2(1-\rho)}{(1-\phi_i)^2}\frac{1}{2-\eta-\eta\rho} \\ \frac{2\sigma_i^2(1-\rho)}{(1-\phi_i)^2}\frac{1}{2-\eta-\eta\rho} & -\frac{2\sigma_i^2(1-\rho)}{(1-\phi_i)^2}\frac{1}{2-\eta-\eta\rho} & \frac{\left(4\sigma_i^2(1-\rho)+2\sigma_{q\infty}^2(1-\phi_i)^2\right)-\eta\sigma_{q\infty}^2(1+\rho)(1-\phi_i)^2}{(1-\phi_i)^2(2-\eta(1+\rho))} \end{bmatrix}$$

Differentiating the above result, we see that

$$\lim_{\delta\to 1} \mathbf{\Theta}'(\eta) = \frac{\partial}{\partial\eta} \lim_{\delta\to 1} \mathbf{\Theta}(\eta) \propto \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}.$$

E.3 Deviations from covered-interest-rate parity

To model deviations from covered-interest rate parity (CIP), we make two assumptions.

1. We assume that the only market participants who can engage in riskless CIP arbitrage trades—i.e., borrowing at the synthetic domestic short rate to lend at the cash domestic short rate—are a set of global banks who face non-risk-based balance sheet constraints.

2. We assume that risk-averse bond investors—who are either domiciled at home or abroad—must use FX forwards if they want to make FX-hedged investments in non-local long-term bonds. This is equivalent to saying that bond investors cannot directly borrow (i.e., obtain "cash" funding) in their non-local currency.

We assume half of all global bond investors are domiciled in the home country and half are domiciled in the foreign country. Both domestic and foreign bond investors have mean-variance preferences over one-period-ahead wealth and a risk tolerance of τ in domestic currency terms.[34] Investors differ only in terms of the returns they can earn because of CIP violations.

[34] Thus, at time t, the risk tolerance of foreign bond investors is τ/Q_t in foreign currency terms, which corresponds to a risk tolerance of τ in domestic currency terms.

The vector of excess returns from t to $t+1$ to be endogenized becomes $\mathbf{rx}_{t+1} \equiv [rx^{y}_{t+1},\ rx^{y*}_{t+1},\ rx^{q}_{t+1}, x^{cip}_{t}]'$ and the vector of exogenous supplies becomes $\mathbf{s}_t \equiv [s^y_t, s^{y*}_t, s^q_t, s^{cip}_t]'$.

FX-hedged returns Consider a domestic investor taking a forward FX-hedged position at time t in a risky foreign assets with returns R^*_{t+1}. At time t, the investor converts 1 unit of domestic currency into $1/Q_t$ units of foreign currency. Suppose the investor sells forward $H_t = I^*_t$ units of foreign currency at the forward price F^q_t (this expression is valid for any H_t, but setting $H_t = I^*_t = \exp(i^*_t)$ is convenient). Then, the FX-hedged return return in domestic currency on the risky asset is

$$R^*_{H,t+1} = \frac{F^q_t I^*_t + (R^*_{t+1} - I^*_t) Q_{t+1}}{Q_t} = \frac{F^q_t}{Q_t} R^*_{t+1} + \overbrace{\frac{(Q_{t+1} - F_t)(R^*_{t+1} - I^*_t)}{Q_t}}^{\text{Basis risk}}.$$

Thus, the FX-hedged return includes a basis risk term that reflect the product of the excess return on foreign currency and the local-currency excess return on the risky asset.

We now assume that $F^q_t = (Q_t I_t)/(I^*_t X^{cip}_t)$ or $f^q_t = q_t - (i^*_t - i_t) - x^{cip}_t$ where $X^{cip}_t = \exp(x^{cip}_t)$. Thus, we have

$$R^*_{H,t+1} = \frac{F^q_t I^*_t + (R^*_{t+1} - I^*_t) Q_{t+1}}{Q_t} = \frac{I_t}{I^*_t X^{cip}_t} R^*_{t+1} + \left(\frac{Q_{t+1}}{Q_t} - \frac{I_t}{I^*_t} \frac{1}{X^{cip}_t} \right) (R^*_{t+1} - I^*_t).$$

Using a Taylor series expansion about the point where the realized basis risk term is zero, the log-hedged return is

$$r^*_{H,t+1} \approx [i_t + \left(r^*_{t+1} - i^*_t\right) - x^{cip}_t] + \left[\frac{\left(I^*_t X^{cip}_t Q_{t+1}/Q_t - I_t\right)(R^*_{t+1} - I^*_t)}{I_t R^*_{t+1}} \right].$$

The second term in square braces can be well approximated as

$$\frac{\left(I^*_t X^{cip}_t Q_{t+1}/Q_t - I_t\right)(R^*_{t+1} - I^*_t)}{I_t R^*_{t+1}} \approx (rx^q_{t+1} + x^{cip}_t) \cdot rx^*_{t+1}$$

where $rx^*_{t+1} = \ln\left(R^*_{t+1}\right) - i^*_t$. Thus, we have

$$rx^*_{H,t+1} \approx rx^*_{t+1} - x^{cip}_t + (rx^q_{t+1} + x^{cip}_t) \cdot rx^*_{t+1}.$$

We neglect this second basis risk term in our theoretical calculations in Section 4. Intuitively, this amounts to assuming that investors are regularly rebalancing their FX hedges. Formally, consider

$$E_t[(rx^q_{t+1} + x^{cip}_t) \cdot rx^*_{t+1}] = (E_t\left[rx^q_{t+1}\right] + x^{cip}_t) \cdot E_t\left[rx^*_{t+1}\right] + Cov_t\left[rx^q_{t+1}, rx^*_{t+1}\right].$$

If we let dt denote the return horizon, then $(E_t\left[rx^q_{t+1}\right] + x^{cip}_t) \cdot E_t\left[rx^*_{t+1}\right]$ will be of order $(dt)^2$ and $Cov_t\left[rx^q_{t+1}, rx^*_{t+1}\right]$ will be of order dt. Thus, in the continuous-time limit in which FX-hedges are continuously rebalanced ($dt \to 0$), the $(dt)^2$ terms vanish and we will only be left with a constant covariance term.

Domestic bond investors Domestic bond investors can obtain a riskless return of i_t from t to $t+1$ by investing in short-term domestic bonds. They can buy long-term domestic bonds, earning an excess return of rx^y_{t+1}; they can take FX-hedged positions in long-term foreign bonds, generating an excess return of $rx^{y^*}_{t+1} - x^{cip}_t$; and they can make forward investments in foreign currency, earning an excess return of $rx^q_{t+1} + x^{cip}_t$. In effect, domestic investors only have access to excess returns $\mathbf{rx}_{D,t+1} = [rx^y_{t+1}, rx^{y^*}_{t+1} - x^{cip}_t, rx^q_{t+1} + x^{cip}_t]'$.

Suppose domestic bond investors have a demand $h^y_{D,t}$ for domestic long-term bonds, $h^{y*}_{D,t}$ for FX-hedged investments in long-term foreign bonds, $h^q_{D,t}$ for forward investment in foreign currency. Thus, their positions in the four underlying long-short trades (domestic bonds, foreign bonds, cash FX investment, and CIP arbitrage) are

$$\mathbf{d}_{D,t} = \begin{bmatrix} d^y_{D,t} \\ d^{y*}_{D,t} \\ d^q_{D,t} \\ d^x_{D,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} h^y_{D,t} \\ h^{y*}_{D,t} \\ h^q_{D,t} \end{bmatrix} = \mathbf{H}_D \mathbf{h}_{D,t}.$$

Domestic bond investors choose their demands $\mathbf{h}_{D,t}$ over excess returns

$$\mathbf{rx}_{D,t+1} = \begin{bmatrix} rx^y_{t+1} \\ rx^{y^*}_{t+1} - x^{cip}_t \\ rx^q_{t+1} + x^{cip}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} rx^y_{t+1} \\ rx^{y^*}_{t+1} \\ rx^q_{t+1} \\ x^{cip}_t \end{bmatrix} = \mathbf{H}'_D \mathbf{rx}_{t+1}.$$

Thus, they solve

$$\max_{\mathbf{h}_{D,t}} \left\{ \mathbf{h}'_{D,t} E_t \left[\mathbf{rx}_{D,t+1} \right] - \frac{\tau}{2} \mathbf{h}'_{D,t} Var_t \left[\mathbf{rx}_{D,t+1} \right] \mathbf{h}_{D,t} \right\}.$$

The solution is

$$\mathbf{h}_{D,t} = \tau \left(Var_t \left[\mathbf{rx}_{D,t+1} \right] \right)^{-1} E_t \left[\mathbf{rx}_{D,t+1} \right] = \tau \left(\mathbf{H}'_D Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_D \right)^{-1} \mathbf{H}'_D E_t \left[\mathbf{rx}_{t+1} \right],$$

implying that

$$\mathbf{d}_{D,t} = \mathbf{H}_D \mathbf{h}_{D,t} = \tau \mathbf{H}_D \left(\mathbf{H}'_D Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_D \right)^{-1} \mathbf{H}'_D E_t \left[\mathbf{rx}_{t+1} \right].$$

Note that

$$\begin{aligned} \mathbf{H}'_D Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_D &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_y & C_{y,y*} & C_{y,q} & 0 \\ C_{y,y*} & V_y & -C_{y,q} & 0 \\ C_{y,q} & -C_{y,q} & V_q & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} V_y & C_{y,y*} & C_{y,q} \\ C_{y,y*} & V_y & -C_{y,q} \\ C_{y,q} & -C_{y,q} & V_q \end{bmatrix} \equiv \mathbf{V}. \end{aligned}$$

Thus, CIP basis doesn't affect the risk of their investments; just the expected returns on their investments. This implies that

$$\mathbf{d}_{D,t} = \tau \mathbf{H}_D \mathbf{V}^{-1} \mathbf{H}'_D E_t \left[\mathbf{rx}_{t+1} \right].$$

Foreign bond generalists Foreign bond investors are the mirror image of domestic investors. Foreign investors have access to excess returns $\mathbf{rx}_{F,t+1} = [rx^y_{t+1} + x^{cip}_t, rx^{y^*}_{t+1}, rx^q_{t+1} + x^{cip}_t]'$. Suppose that foreign bond investors have a demand $h^y_{F,t}$ for FX-hedged domestic long-term bonds, $h^{y*}_{F,t}$ for foreign long-term bonds, $h^q_{F,t}$ for forward-investment in FX. So their positions in the four pure long-short trades are

$$\mathbf{d}_{F,t} = \begin{bmatrix} d^{y}_{F,t} \\ d^{y*}_{F,t} \\ d^{q}_{F,t} \\ d^{x}_{F,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} h^{y}_{F,t} \\ h^{y*}_{F,t} \\ h^{q}_{F,t} \end{bmatrix} = \mathbf{H}_F \mathbf{h}_{F,t}.$$

These foreign bond investors choose their demands $\mathbf{h}_{F,t}$ over hedged returns $\mathbf{rx}_{F,t+1} = \mathbf{H}'_F \mathbf{rx}_{t+1}$. Thus, they solve

$$\max_{\mathbf{h}_{F,t}} \left\{ \mathbf{h}'_{F,t} E_t \left[\mathbf{rx}_{F,t+1} \right] - \frac{\tau}{2} \mathbf{h}'_{F,t} Var_t \left[\mathbf{rx}_{F,t+1} \right] \mathbf{h}_{F,t} \right\}.$$

As above, the solution is

$$\mathbf{h}_{F,t} = \tau \left(Var_t \left[\mathbf{rx}_{F,t+1} \right] \right)^{-1} E_t \left[\mathbf{rx}_{F,t+1} \right] = \tau \left(\mathbf{H}'_F Var_t \left[\mathbf{rx}_{t+1} \right] \mathbf{H}_F \right)^{-1} \mathbf{H}'_F E_t \left[\mathbf{rx}_{t+1} \right],$$

Following the same logic as above for domestic bond investors, we then have

$$\mathbf{d}_{F,t} = \tau \mathbf{H}_F \mathbf{V}^{-1} \mathbf{H}'_F E_t \left[\mathbf{rx}_{t+1} \right].$$

Unlike in our baseline model, we need not have $\mathbf{d}_{F,t} = \mathbf{d}_{D,t}$. This is because CIP deviations affect the hedged returns that investors earn in non-local long-term bonds.

Balance-sheet constrained banks The only players who can engage in the riskless CIP arbitrage are a set of balance-sheet constrained banks. These banks choose the value of their positions in the CIP arbitrage trade, $d^{cip}_{B,t}$, to solve

$$\max_{d^{cip}_{B,t}} \left\{ x^{cip}_t d^{cip}_{B,t} - (\kappa/2) \left(d^{cip}_{B,t} \right)^2 \right\}, \tag{81}$$

where $\kappa \geq 0$ and $(\kappa/2) \left(d^{cip}_{B,t} \right)^2$ captures non-risk-based balance sheet costs faced by banks. Thus, banks take a position in the CIP arbitrage trade equal to

$$d^{cip}_{B,t} = \kappa^{-1} x^{cip}_t,$$

or

$$\mathbf{d}_{B,t} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa^{-1} \end{bmatrix} \begin{bmatrix} E_t \left[rx^{y}_{t+1} \right] \\ E_t [rx^{y^*}_{t+1}] \\ E_t [rx^{q}_{t+1}] \\ x^{cip}_t \end{bmatrix} = \mathbf{K} E_t \left[\mathbf{rx}_{t+1} \right].$$

Market clearing In this extension, we assume that s^q_t is exogenous net supply of risky FX exposure on a *forward basis* that bond investors must hold. And we assume that s^{cip}_t is the exogenous supply of the riskless CIP arbitrage trade that banks must undertake. We parameterize the exogenous supply shocks in this way to clearly separate the supply of risky FX exposure and the supply of riskless funding that bond investors and banks must intermediate. Thus, the exogenous supply of the four underlying long-short trades are

$$\mathbf{S}^{[3-7]}_1 \mathbf{s}_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} s^{y}_t \\ s^{y*}_t \\ s^{q}_t \\ s^{cip}_t \end{bmatrix}.$$

Below, we explain how the results change if s_t^q is exogenous net supply of risky FX exposure on a cash basis.

Here we assume that $s_{t+1}^{cip} = \phi_{s^{cip}} s_t^{cip} + \varepsilon_{s_{t+1}^{cip}}$, where $Var_t[\varepsilon_{s_{t+1}^{cip}}] = \sigma_{s^{cip}}^2 \geq 0$, $\phi_{s^{cip}} \in [0, 1)$, and $\varepsilon_{s_{t+1}^{cip}}$ is orthogonal to the other shocks. The main text considers the special case where $\sigma_{s^{cip}}^2 = 0$, implying that $s_t^{cip} \equiv 0$.

The market clearing conditions are

$$\mathbf{S}_1^{[3-7]}\mathbf{s}_t = \frac{1}{2}\left(\mathbf{d}_{D,t} + \mathbf{d}_{F,t}\right) + \mathbf{d}_{B,t} = \left[\frac{1}{2}\left(\tau \mathbf{H}_D \mathbf{V}^{-1}\mathbf{H}_D' + \tau \mathbf{H}_F \mathbf{V}^{-1}\mathbf{H}_F'\right) + \mathbf{K}\right] E_t\left[\mathbf{rx}_{t+1}\right],$$

implying that

$$E_t\left[\mathbf{rx}_{t+1}\right] = \left[\frac{1}{2}\left(\tau \mathbf{H}_D \mathbf{V}^{-1}\mathbf{H}_D' + \tau \mathbf{H}_F \mathbf{V}^{-1}\mathbf{H}_F'\right) + \mathbf{K}\right]^{-1} \mathbf{S}_1^{[3-6]}\mathbf{s}_t.$$

Working through the math, we obtain

$$\begin{bmatrix} E_t\left[rx_{t+1}^y\right] \\ E_t\left[rx_{t+1}^{y*}\right] \\ E_t\left[rx_{t+1}^q\right] \\ x_t^{cip} \end{bmatrix} = \left(\tau^{-1}\begin{bmatrix} V_y & C_{y,y*} & C_{y,q} & 0 \\ C_{y,y*} & V_y & -C_{y,q} & 0 \\ C_{y,q} & -C_{y,q} & V_q & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \kappa\frac{V_y + C_{y,y*}}{2V_y + 2C_{y,y*} + \tau\kappa}\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 1 \\ 1 & -1 & 0 & -2 \\ -1 & 1 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} s_t^y \\ s_t^{y*} \\ s_t^q \\ s_t^{cip} \end{bmatrix}.$$

These are the expressions given in Proposition 5.

Given these prices, equilibrium quantities are given by

$$\begin{aligned}
\mathbf{d}_{D,t} &= \tau \mathbf{H}_D \mathbf{V}^{-1}\mathbf{H}_D' E_t\left[\mathbf{rx}_{t+1}\right] = \begin{bmatrix} \frac{4C_{y,y*}+4V_y+3\kappa\tau}{4C_{y,y*}+4V_y+2\kappa\tau} & -\frac{\kappa}{2}\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} & 0 & -\kappa\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} \\ \frac{\kappa}{2}\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} & \frac{4C_{y,y*}+4V_y+\kappa\tau}{4C_{y,y*}+4V_y+2\kappa\tau} & 0 & -\kappa\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} \\ 0 & 0 & 1 & 0 \\ -\frac{\kappa}{2}\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} & -\frac{1}{2}\frac{4C_{y,y*}+4V_y+\kappa\tau}{2C_{y,y*}+2V_y+\kappa\tau} & 1 & \kappa\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} \end{bmatrix}\begin{bmatrix} s_t^y \\ s_t^{y*} \\ s_t^q \\ s_t^{cip} \end{bmatrix} \\
\mathbf{d}_{F,t} &= \tau \mathbf{H}_F \mathbf{V}^{-1}\mathbf{H}_F' E_t\left[\mathbf{rx}_{t+1}\right] = \begin{bmatrix} \frac{4C_{y,y*}+4V_y+\kappa\tau}{4C_{y,y*}+4V_y+2\kappa\tau} & \frac{\kappa}{2}\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} & 0 & \kappa\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} \\ -\frac{\kappa}{2}\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} & \frac{4C_{y,y*}+4V_y+3\kappa\tau}{4C_{y,y*}+4V_y+2\kappa\tau} & 0 & \kappa\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} \\ 0 & 0 & 1 & 0 \\ \frac{1}{2}\frac{4C_{y,y*}+4V_y+\kappa\tau}{2C_{y,y*}+2V_y+\kappa\tau} & \frac{\kappa}{2}\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} & 1 & \kappa\frac{\tau}{2C_{y,y*}+2V_y+\kappa\tau} \end{bmatrix}\begin{bmatrix} s_t^y \\ s_t^{y*} \\ s_t^q \\ s_t^{cip} \end{bmatrix} \\
\mathbf{d}_{B,t} &= \mathbf{K} E_t\left[\mathbf{rx}_{t+1}\right] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{C_{y,y*}+V_y}{2C_{y,y*}+2V_y+\kappa\tau} & \frac{C_{y,y*}+V_y}{2C_{y,y*}+2V_y+\kappa\tau} & 0 & 2\frac{C_{y,y*}+V_y}{2C_{y,y*}+2V_y+\kappa\tau} \end{bmatrix}\begin{bmatrix} s_t^y \\ s_t^{y*} \\ s_t^q \\ s_t^{cip} \end{bmatrix}.
\end{aligned}$$

Thus, when $\kappa > 0$, we see that domestic (foreign) bond investors play an outsized role in absorbing shocks to domestic (foreign bond supply). For instance, domestic investors absorb a fraction

$$\frac{1}{2}\frac{4C_{y,y*} + 4V_y + 3\kappa\tau}{4C_{y,y*} + 4V_y + 2\kappa\tau}$$

of any domestic bond supply shock. This quantity equals 1/2 when $\kappa = 0$, is increasing in κ, and converges to 3/4 as $\kappa \to \infty$. Furthermore, since an increase in foreign bond supply (s_t^{y*}) reduces the costs that foreign investors must pay to hedge their domestic bond holding, an increase in s_t^{y*} increases the domestic bond holdings of foreign investors and reduces those of domestic investors.

As in our baseline model, the variance-covariance matrix of excess returns is an equilibrium object.

Specifically, a rational expectations equilibrium of the extended model is a fixed point of an operator involving the "price-impact" coefficients which govern how the supplies $\mathbf{s}_t = [s_t^y, s_t^{y*}, s_t^q, s_t^{cip}]'$ impact y_t, y_t^*, q_t, and x_t^{cip}. One can also recast the equilibrium as a fixed point problem involving the equilibrium variance-convariance matrix. However, as in the baseline model in Section 3, if $0 \leq \rho < 1$, $\sigma_{s^y}^2 \geq 0$, $\sigma_{s^q}^2 \geq 0$, we must have $C_{y,q} > 0$ in any stable equilibrium. Furthermore, we must have $V_y + C_{y,y*} = V_y \left(1 + Corr\left[rx_{t+1}^y, rx_{t+1}^{y*}\right]\right) > 0$ and, thus, $\left[\kappa \left(V_y + C_{y,y*}\right)\right] / \left[2V_y + 2C_{y,y*} + \tau\kappa\right] > 0$ in any equilibrium. Thus, since

$$\begin{aligned} E_t\left[rx_{t+1}^q\right] &= \tau^{-1}[\overbrace{C_{y,q}}^{>0} \cdot (s_t^y - s_t^{y*}) + \overbrace{V_q}^{>0} \cdot s_t^q] - x_t^{cip}, \\ x_t^{cip} &= \underbrace{-\kappa \frac{V_y + C_{y,y*}}{2\left(V_y + C_{y,y*}\right) + \tau\kappa}}_{<0} \cdot [(s_t^y - s_t^{y*}) - 2 \cdot s_t^{cip}], \end{aligned}$$

it follows that the three supply shocks s_t^y, s_t^{y*}, and s_t^{cip} push $E_t[rx_{t+1}^q]$ and x_t^{cip} in opposite directions. As a result, these three supply shock shocks push q_t and x_t^{cip} in the same direction.

In the special case, where there are no FX supply shocks ($s_t^q = \sigma_{s^q}^2 = 0$), we have

$$\begin{aligned} E_t\left[rx_{t+1}^{y*}\right] - E_t\left[rx_{t+1}^y\right] &= \left[\tau^{-1}\left(V_y - C_{y,y*}\right) + \kappa \frac{V_y + C_{y,y*}}{2\left(V_y + C_{y,y*}\right) + \tau\kappa}\right] \cdot \left(s_t^{y*} - s_t^y\right), \\ E_t\left[rx_{t+1}^q\right] &= -\left[\tau^{-1} C_{y,q} + \kappa \frac{V_y + C_{y,y*}}{2\left(V_y + C_{y,y*}\right) + \tau\kappa}\right] \cdot \left(s_t^{y*} - s_t^y\right), \\ x_t^{cip} &= \kappa \frac{V_y + C_{y,y*}}{2\left(V_y + C_{y,y*}\right) + \tau\kappa} \cdot \left(s_t^{y*} - s_t^y\right), \end{aligned}$$

and thus

$$\begin{aligned} E_t\left[rx_{t+1}^q\right] &= -\overbrace{\left[\frac{\tau^{-1} C_{y,q} + \kappa \frac{V_y + C_{y,y*}}{2(V_y + C_{y,y*}) + \tau\kappa}}{\tau^{-1}\left(V_y - C_{y,y*}\right) + \kappa \frac{V_y + C_{y,y*}}{2(V_y + C_{y,y*}) + \tau\kappa}}\right]}^{K_q > 0} \cdot \left(E_t\left[rx_{t+1}^{y*}\right] - E_t\left[rx_{t+1}^y\right]\right), \\ x_t^{cip} &= \underbrace{\left[\frac{\kappa \frac{V_y + C_{y,y*}}{2(V_y + C_{y,y*}) + \tau\kappa}}{\tau^{-1}\left(V_y - C_{y,y*}\right) + \kappa \frac{V_y + C_{y,y*}}{2(V_y + C_{y,y*}) + \tau\kappa}}\right]}_{K_{cip} > 0} \cdot \left(E_t\left[rx_{t+1}^{y*}\right] - E_t\left[rx_{t+1}^y\right]\right). \end{aligned}$$

Thus, we have $E_t\left[rx_{t+1}^q\right] = -K_q \cdot \left(E_t\left[rx_{t+1}^{y*}\right] - E_t\left[rx_{t+1}^y\right]\right)$ and $x_t^{cip} = K_{cip} \cdot \left(E_t\left[rx_{t+1}^{y*}\right] - E_t\left[rx_{t+1}^y\right]\right)$ where K_q,$K_{cip} > 0$.

Next, treating the variances and covariances as fixed objects—as would be appropriate in the case where supply is non-stochastic—it is easy to see that $\left[\kappa \left(V_y + C_{y,y*}\right)\right] / \left[2\left(V_y + C_{y,y*}\right) + \tau\kappa\right]$ is increasing in κ. Thus, in this case, it then immediately follows that $\partial^2 E_t[rx_{t+1}^y]/\partial s_t^y \partial\kappa = \partial^2 E_t[rx_{t+1}^{y*}]/\partial s_t^{y*} \partial\kappa > 0$, $\partial^2 E_t[rx_{t+1}^q]/\partial s_t^y \partial\kappa = -\partial^2 E_t[rx_{t+1}^q]/\partial s_t^{y*} \partial\kappa > 0$. In other words, an increase in bank balance-sheet costs (κ) raises the impact of bond supply shocks on risk premia and, hence, on market prices. Conversely, a rise in κ reduces the impact of local bond supply shocks on non-local bond risk premia:$\partial^2 E_t[rx_{t+1}^{y*}]/\partial s_t^y \partial\kappa = \partial^2 E_t[rx_{t+1}^y]/\partial s_t^{y*} \partial\kappa < 0$. Under regularity conditions, it is straightforward but tedious to show that this same argument carries through unchanged in the case where supply is stochastic.

Finally, since the model's stable equilibrium is continuous in the model's underlying parameters, it follows that if we take the limit where $\kappa \to 0$, then the extended model in Section 4 converges to the baseline model considered in Section 3. Specifically, as $\kappa \to 0$, CIP holds ($x_t^{cip} \to 0$). By contrast, in the limit where bank balance sheets costs grow without bound ($\kappa \to \infty$), one can show that prices adjust to ensure that $\frac{1}{2}(\mathbf{d}_{D,t} + \mathbf{d}_{F,t}) = \mathbf{S}_1^{[3-7]}\mathbf{s}_t$ and $\mathbf{d}_{B,t} = \mathbf{0}$. In other words, as bank balance sheet costs grow large, prices will adjust to ensure that there is zero net demand for the CIP arbitrage trade that banks must absorb in equilibrium.

Alternate assumption on FX supply If s_t^q is instead the exogenous net supply of risky FX exposure on a spot basis, then the exogenous supplies of the four underlying long-short trades are

$$\mathbf{S}_1^{[3-7]}\mathbf{s}_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_t^y \\ s_t^{y*} \\ s_t^q \\ s_t^{cip} \end{bmatrix}.$$

In this case, we still have

$$\begin{aligned} E_t\left[rx_{t+1}^y\right] &= \tau^{-1}\left[V_y \cdot s_t^y + C_{y,y*} \cdot s_t^{y*} + C_{y,q} \cdot s_t^q\right] - x_t^{cip}/2, \\ E_t\left[rx_{t+1}^{y*}\right] &= \tau^{-1}\left[C_{y,y*} \cdot s_t^y + V_y \cdot s_t^{y*} - C_{y,q} \cdot s_t^q\right] + x_t^{cip}/2, \\ E_t\left[rx_{t+1}^q\right] &= \tau^{-1}\left[C_{y,q} \cdot (s_t^y - s_t^{y*}) + V_q \cdot s_t^q\right] - x_t^{cip}, \end{aligned}$$

as above. However, we now have

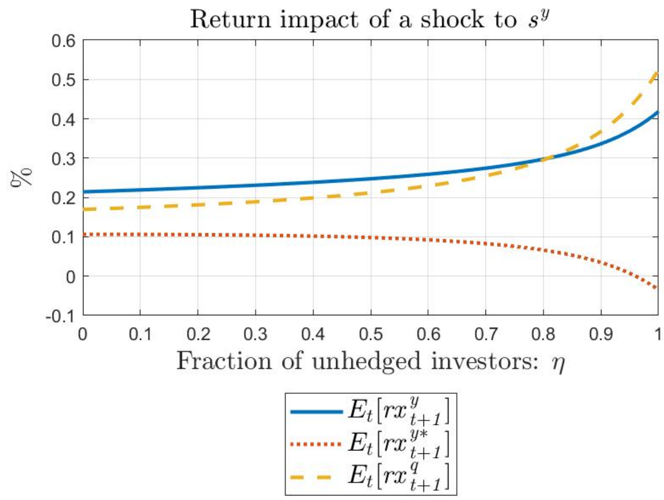
$$x_t^{cip} = \underbrace{-\kappa \frac{V_y + C_{y,y*}}{2(V_y + C_{y,y*}) + \tau\kappa}}_{<0} \cdot [(s_t^y - s_t^{y*}) - 2 \cdot (s_t^{cip} - s_t^q)].$$

Now shocks to s_t^q also push $E_t[rx_{t+1}^q]$ and x_t^{cip} in opposite directions. The intuition is simple. To clear the spot foreign exchange market at time t, investors must be willing to exchange domestic currency for foreign currency at today's spot rate with no agreement to reverse this exchange at a later date. Since $rx_{t+1}^q = (q_{t+1} - f_t^q) - x_t^{cip}$, this spot investment in foreign currency is equivalent to a (i) forward investment in foreign currency plus (ii) a reverse FX swap—i.e., an exchange of domestic for foreign currency at today's spot rate ($1/Q_t$) and simultaneous agreement to exchange foreign for domestic currency tomorrow at today's 1-period forward rate (F_t^Q). This reverse FX swap is equivalent to a reverse CIP arbitrage trade that borrows on a cash basis and lends on a synthetic basis in domestic currency.

To accommodate an inelastic demand s_t^q to swap foreign for domestic currency in the spot FX market, (i) risk-averse bond investors must make a forward investment in foreign currency in amount s_t^q and (ii) balance-sheet constrained banks must enter into a reverse CIP arbitrage trade, thereby earning $-x_t^{cip}$ in amount s_t^q. Thus, an increase in s_t^q will be associated with (i) an increase in the expected return to buying foreign currency on a forward basis, $E_t\left[q_{t+1} - f_t^q\right]$, and (ii) a decline in x_t^{cip}. The rise in $E_t\left[q_{t+1} - f_t^q\right]$ is required to induce risk-averse bond investors increase their risky forward investments in foreign currency. And, the decline in x_t^{cip} is required to induce balance-sheet constrained banks to supply reverse FX swaps. Combing these results, an increase in s_t^q must then lead $E_t\left[rx_{t+1}^q\right] = E_t\left[q_{t+1} - f_t^q\right] - x_t^{cip}$ to rise. Thus, shocks to s_t^q also push $E_t[rx_{t+1}^q]$ and x_t^{cip} in opposite directions.

Figure A1. Unhedged bond investors. This figure illustrates the model with unhedged bond investors from Subsection 4.2 in the main text and Subsection C.1 of the Online Appendix. The figure shows the impact of a shock to domestic bond supply on expected returns and investor holdings as a function of the fraction of unhedged investors, η. We chose the other parameters so each period represents one month. We assume: $\sigma_i = 0.25\%, \phi_i = 0.985,\ \rho = 0.5,\ \sigma_{s^y} = 1,\ \phi_{s^y} = 0.95,\ \sigma_{s^q} = 1,\ \phi_{s^q} = 0.95,\ \sigma_{q^\infty} = 0.5\%,\ \delta = 119/120$ (i.e., the long-term bond has a duration of 120 months or 10 years), and $\tau = 1.80$. These parameter choices are illustrative. See Section C.1 of the Online Appendix for additional details.

Panel A: Impact of a large shock (4 times σ_{s^y}) to domestic bond supply (s^y) on expected returns



Panel B: Impact of a unit shock to domestic bond supply (s^y) on investor holdings

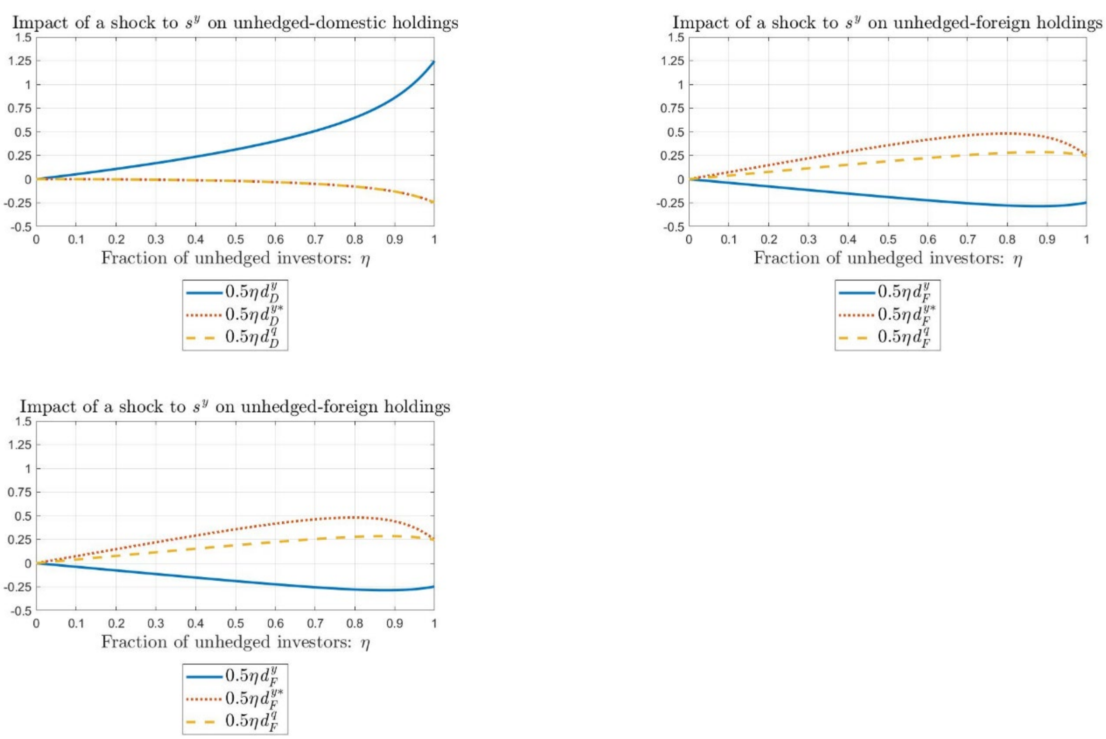


Table A1. Non-overlapping contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term interest rates. This table presents monthly panel regressions of the form:

$$\Delta_H q_{c,t} = A_c + B \times \Delta_H (i^*_{c,t} - i_t) + D \times \Delta_H (y^*_{c,t} - y_t) + \Delta_H \varepsilon_{c,t}.$$

Sampling the data every H months, we regress H-month changes in the foreign exchange rate on H-month changes in short-term interest rates and in long-term yields in both the foreign currency and in U.S. dollars—i.e., the changes are not overlapping. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. For the non-overlapping regressions, we report standard errors that cluster by time (CT). Column (2) reports the equal weighted average of the coefficient estimates and standard errors across these H non-overlapping estimates. For reference, we report the coefficient estimates and Driscoll-Kraay (1998) standard errors from the corresponding overlapping regressions in column (1). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

Panel A: H = 3-month changes (data are sampled every 3 months in non-overlapping regressions)

	Overlapping	Non-overlapping			
		EW AV	Jan, Apr, Jul, Oct	Feb, May, Aug, Nov	Mar, June, Sep, Dec
	(1)	(2)	(3)	(4)	(5)
$\Delta_3(i^*_{c,t} - i_t)$	3.22	3.15	3.75	2.87	2.84
	(1.47)**	(1.37)**	(1.41)***	(1.29)**	(1.42)**
$\Delta_3(y^*_{c,t} - y_t)$	3.13	3.26	1.33	4.27	4.18
	(1.25)**	(1.67)*	(1.73)	(1.80)**	(1.47)***
VCE	DK	CT	CT	CT	CT
N	1,512	504	504	504	504
R^2 (within)	0.14	0.15	0.12	0.17	0.15

Panel B: H = 12-month changes (data sampled every 12 months in non-overlapping regressions)

	Overlapping	Non-overlapping												
		EW AV	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$\Delta_{12}(i^*_{c,t} - i_t)$	1.26	1.16	1.07	2.33	3.29	2.15	1.49	1.84	1.74	-0.02	-0.60	-1.14	0.51	1.28
	(1.64)	(1.78)	(2.19)	(2.01)	(2.21)	(2.02)	(1.87)	(2.04)	(1.89)	(1.32)	(1.21)	(0.97)	(1.46)	(2.20)
$\Delta_{12}(y^*_{c,t} - y_t)$	5.13	5.31	7.92	5.38	1.03	2.68	3.17	2.55	1.90	6.37	6.50	11.24	8.27	6.67
	(1.87)**	(2.91)*	(2.83)**	(3.38)	(3.64)	(3.52)	(3.58)	(3.26)	(2.89)	(2.51)**	(2.22)***	(2.34)***	(1.70)***	(3.08)**
VCE	DK	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT
N	1,512	126	126	126	126	126	126	126	126	126	126	126	126	126
R^2 (within)	0.14	0.16	0.19	0.22	0.17	0.15	0.11	0.11	0.09	0.15	0.13	0.27	0.17	0.18

Table A2. Non-overlapping contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term forward rates. This table presents monthly panel regressions of the form:

$$\Delta_H q_{c,t} = A_c + B \times \Delta_H (i^*_{c,t} - i_t) + D \times \Delta_H (f^*_{c,t} - f_t) + \Delta_H \varepsilon_{c,t}.$$

Sampling the data every H months, we regress H-month changes in the foreign exchange rate on H-month changes in short-term interest rates and in distant forward rates in both the foreign currency and in U.S. dollars—i.e., the changes are not overlapping. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. For the non-overlapping regressions, we report standard errors that cluster by time (CT). Column (2) reports the equal weighted average of the coefficient estimates and standard errors across these H non-overlapping estimates. For reference, we report the coefficient estimates and Driscoll-Kraay (1998) standard errors from the corresponding overlapping regressions in column (1). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

Panel A: $H = 3$-month changes (data are sampled every 3 months in non-overlapping regressions)

	Overlapping	Non-overlapping			
		EW AV	Jan, Apr, Jul, Oct	Feb, May, Aug, Nov	Mar, June, Sep, Dec
	(1)	(2)	(3)	(4)	(5)
$\Delta_3(i^*_{c,t} - i_t)$	4.15	4.11	4.16	4.10	4.08
	(1.38)***	(1.30)***	(1.30)***	(1.34)***	(1.25)***
$\Delta_3(f^*_{c,t} - f_t)$	1.72	1.76	0.08	2.74	2.46
	(1.20)	(1.26)	(1.20)	(1.58)*	(1.00)**
VCE	DK	CT	CT	CT	CT
N	1,512	504	504	504	504
R^2 (within)	0.13	0.14	0.12	0.16	0.14

Panel B: $H = 12$-month changes (data are sampled every 12 months in non-overlapping regressions)

	Overlapping	Non-overlapping												
		EW AV	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$\Delta_{12}(i^*_{c,t} - i_t)$	2.70	2.67	3.40	3.87	3.60	2.82	2.40	2.59	2.34	1.80	1.28	2.15	2.72	3.04
	(1.42)*	(1.47)*	(1.71)*	(1.70)**	(1.65)**	(1.58)*	(1.49)	(1.58)	(1.46)	(0.98)*	(1.02)	(1.15)*	(1.41)*	(1.93)
$\Delta_{12}(f^*_{c,t} - f_t)$	2.50	2.61	4.11	3.32	0.10	1.69	0.57	0.13	-0.26	3.60	3.74	6.62	5.72	1.97
	(1.13)*	(2.22)	(1.83)**	(2.06)	(2.62)	(2.53)	(2.19)	(2.30)	(2.42)	(2.16)	(1.89)*	(1.60)***	(2.58)**	(2.47)
VCE	DK	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT
N	1,512	126	126	126	126	126	126	126	126	126	126	126	126	126
R^2 (within)	0.11	0.14	0.16	0.21	0.17	0.15	0.10	0.10	0.09	0.11	0.10	0.23	0.16	0.11

Table A3. Non-overlapping forecasts of foreign minus domestic bond excess return using short-term interest rates and long-term forward rates. This table presents monthly panel forecasting regressions of the form:

$$rx^{y*}_{c,t\to t+H} - rx^{y}_{c,t\to t+H} = A_c + B\times(i^*_{c,t} - i_t) + D\times(f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+H}.$$

Sampling the data every H months, we forecast the difference between foreign and domestic H-month bond returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollars—i.e., the returns are not overlapping. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. For the non-overlapping regressions, we report standard errors that cluster by time (CT). Column (2) reports the equal weighted average of the coefficient estimates and standard errors across these H non-overlapping estimates. For reference, we report the coefficient estimates and Driscoll-Kraay (1998) standard errors from the corresponding overlapping regressions in column (1). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

Panel A: H = 3-month returns (data are sampled every 3 months in non-overlapping regressions)

	Overlapping	Non-overlapping			
		EW AV	Jan, Apr, Jul, Oct	Feb, May, Aug, Nov	Mar, June, Sep, Dec
	(1)	(2)	(3)	(4)	(5)
$i^*_{c,t} - i_t$	-0.21	-0.21	-0.26	-0.18	-0.19
	(0.13)	(0.18)	(0.17)	(0.20)	(0.16)
$f^*_{c,t} - f_t$	1.74	1.74	1.93	1.84	1.45
	(0.22)***	(0.32)***	(0.33)***	(0.40)***	(0.24)***
VCE	DK	CT	CT	CT	CT
N	1,494	498	498	498	498
R^2 (within)	0.10	0.11	0.12	0.12	0.09

Panel B: H = 12-month returns (data are sampled every 12 months in non-overlapping regressions)

	Overlapping	Non-overlapping												
		EW AV	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$i^*_{c,t} - i_t$	-0.27	-0.28	-0.33	0.01	-0.17	-0.16	-0.21	-0.30	-0.14	-0.35	-0.45	-0.53	-0.29	-0.38
	(0.39)	(0.51)	(0.45)	(0.54)	(0.58)	(0.57)	(0.56)	(0.51)	(0.54)	(0.51)	(0.48)	(0.44)	(0.49)	(0.45)
$f^*_{c,t} - f_t$	4.31	4.35	4.16	3.62	5.02	4.53	4.57	4.89	4.27	4.59	3.75	4.46	3.63	4.66
	(0.33)***	(0.79)***	(0.77)***	(0.71)***	(0.83)***	(0.82)***	(0.88)***	(0.85)***	(0.83)***	(0.88)***	(0.62)***	(0.68)***	(0.72)***	(0.92)***
VCE	DK	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT
N	1,440	120	120	120	120	120	120	120	120	120	120	120	120	120
R^2 (within)	0.22	0.24	0.24	0.18	0.23	0.23	0.25	0.29	0.25	0.25	0.19	0.32	0.21	0.27

Table A4. Non-overlapping forecasts of foreign exchange excess return using short-term interest rates and long-term forward rates. This table presents monthly panel forecasting regressions of the form:

$$rx^{q}_{c,t\to t+H} = A_c + B\times(i^{*}_{c,t} - i_t) + D\times(f^{*}_{c,t} - f_t) + \varepsilon_{c,t\to t+H}.$$

Sampling the data every H months, we forecast H-month foreign exchange excess returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollar—i.e., the returns are not overlapping. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. For the non-overlapping regressions, we report standard errors that cluster by time (CT). Column (2) reports the equal weighted average of the coefficient estimates and standard errors across these H non-overlapping estimates. For reference, we report the coefficient estimates and Driscoll-Kraay (1998) standard errors from the corresponding overlapping regressions in column (1). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

Panel A: H = 3-month returns (data are sampled every 3 months in non-overlapping regressions)

	Overlapping	Non-overlapping			
		EW AV	Jan, Apr, Jul, Oct	Feb, May, Aug, Nov	Mar, June, Sep, Dec
	(1)	(2)	(3)	(4)	(5)
$i^{*}_{c,t} - i_t$	0.29	0.28	0.23	0.30	0.32
	(0.37)	(0.33)	(0.30)	(0.35)	(0.33)
$f^{*}_{c,t} - f_t$	-0.83	-0.82	-0.57	-1.00	-0.91
	(0.39)*	(0.47)*	(0.47)	(0.43)**	(0.49)*
VCE	DK	CT	CT	CT	CT
N	1,494	498	498	498	498
R^2 (within)	0.01	0.02	0.01	0.02	0.02

Panel B: H = 12-month returns (data are sampled every 12 months in non-overlapping regressions)

	Overlapping	Non-overlapping												
		EW AV	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$i^{*}_{c,t} - i_t$	0.96	0.96	0.99	0.37	0.16	0.81	0.80	0.62	0.75	1.42	1.57	1.54	1.34	1.15
	(1.34)	(1.37)	(1.63)	(1.92)	(1.90)	(1.52)	(1.38)	(1.28)	(1.19)	(0.95)	(0.91)	(1.01)	(1.42)	(1.28)
$f^{*}_{c,t} - f_t$	-3.59	-3.60	-5.28	-3.28	-2.07	-3.83	-2.74	-2.26	-2.77	-3.55	-3.17	-5.53	-4.31	-4.39
	(1.03)***	(1.76)**	(1.62)***	(1.92)	(2.46)	(1.90)*	(1.88)	(1.76)	(1.63)	(1.70)*	(1.42)**	(1.30)***	(1.88)**	(1.67)**
VCE	DK	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT	CT
N	1,440	120	120	120	120	120	120	120	120	120	120	120	120	120
R^2 (within)	0.04	0.07	0.08	0.04	0.03	0.06	0.05	0.04	0.06	0.10	0.11	0.15	0.08	0.08

Table A5. Country-by-country contemporaneous relationships between movements in foreign exchange, short-term interest rates, and long-term interest rates. This table presents country-level monthly time-series regressions of the form:

$$\Delta_{12} q_{c,t} = A_c + B_c \times \Delta_{12}(i^*_{c,t} - i_t) + D_c \times \Delta_{12}(y^*_{c,t} - y_t) + \Delta_{12}\varepsilon_{c,t},$$

and

$$\Delta_{12} q_{c,t} = A_c + B_{1,c} \times \Delta_{12} i^*_{c,t} + B_{2,c} \times \Delta_{12} i_t + D_{1,c} \times \Delta_{12} y^*_{c,t} + D_{2,c} \times \Delta_{12} y_t + \Delta_{12}\varepsilon_{c,t}.$$

For each for currency *c*, we separately regress 12-month changes in the foreign exchange rate on 12-month changes in short-term interest rates and in long-term yields in both the foreign currency and in U.S. dollars. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Newey-West (1987) errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-*b* asymptotic theory of Kiefer and Vogelsang (2005). For reference, we report the corresponding panel specifications in columns (1) and (2) which are a weighted average of the country-level time-series estimates.

			Country-level time-series regressions											
	Panel		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$\Delta_{12}(i^*_{c,t} - i_t)$	1.26		0.86		3.58		0.91		0.99		2.14		-0.22	
	(1.64)		(1.62)		(2.67)		(1.58)		(2.17)		(2.21)		(1.90)	
$\Delta_{12}(y^*_{c,t} - y_t)$	5.13		12.65		-3.45		3.54		3.71		4.46		5.90	
	(1.88)**		(4.27)**		(3.42)		(2.59)		(3.72)		(4.60)		(1.97)**	
$\Delta_{12} i^*_{c,t}$		3.10		5.30		5.96		-3.29		-1.00		3.96		-12.69
		(1.40)*		(1.65)***		(2.60)*		(2.71)		(2.65)		(1.47)***		(7.49)
$\Delta_{12} i_t$		-0.21		1.16		-2.73		0.19		0.85		0.55		0.58
		(1.08)		(1.08)		(2.45)		(1.25)		(1.52)		(1.03)		(1.95)
$\Delta_{12} y^*_{c,t}$		9.07		10.31		1.64		11.73		13.83		11.32		11.11
		(1.63)***		(3.86)**		(5.60)		(2.56)***		(2.65)***		(4.75)**		(4.50)**
$\Delta_{12} y_t$		-5.77		-11.18		-0.04		-5.31		-5.95		-7.61		-6.23
		(1.61)***		(3.32)***		(4.46)		(2.00)**		(2.79)*		(3.91)*		(2.04)**
NW (DK) lags	29	29	29	29	29	29	29	29	29	29	29	29	29	29
N	1,512	1,512	252	252	252	252	252	252	252	252	252	252	252	252
R^2 (within)	0.14	0.27	0.32	0.53	0.05	0.20	0.11	0.21	0.08	0.26	0.17	0.53	0.14	0.18

Table A6. Country-by-country contemporaneous relationships between movements in foreign exchange, short-term interest rates, and long-term forward rates. This table presents country-level monthly time-series regressions of the form:

$$\Delta_{12}q_{c,t} = A_c + B_c \times \Delta_{12}(i^*_{c,t} - i_t) + D_c \times \Delta_{12}(f^*_{c,t} - f_t) + \Delta_{12}\varepsilon_{c,t},$$

and

$$\Delta_{12}q_{c,t} = A_c + B_{1,c} \times \Delta_{12}i^*_{c,t} + B_{2,c} \times \Delta_{12}i_t + D_{1,c} \times \Delta_{12}f^*_{c,t} + D_{2,c} \times \Delta_{12}f_t + \Delta_{12}\varepsilon_{c,t}.$$

For each for currency *c*, we separately regress 12-month changes in the foreign exchange rate on 12-month changes in short-term interest rates and in distant forward rates in both the foreign currency and in U.S. dollars. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Newey-West (1987) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005). For reference, we report the corresponding panel specifications in columns (1) and (2) which are a weighted average of the country-level time-series estimates.

			Country-level time-series regressions											
	Panel		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$\Delta_{12}(i^*_{c,t} - i_t)$	2.70		4.37		2.44		1.99		2.07		3.37		1.19	
	(1.42)*		(1.27)***		(2.59)		(1.07)		(1.47)		(1.33)**		(1.80)	
$\Delta_{12}(f^*_{c,t} - f_t)$	2.50		6.22		-5.83		1.75		2.36		2.43		3.14	
	(1.13)*		(3.08)*		(1.64)***		(2.41)		(3.10)		(2.88)		(1.64)*	
$\Delta_{12}i^*_{c,t}$		5.54		8.24		6.03		-0.44		3.02		6.91		-6.94
		(1.12)***		(1.04)***		(2.84)*		(2.39)		(2.00)		(0.63)***		(7.98)
$\Delta_{12}i_t$		-1.72		-2.01		-2.90		-1.16		-0.54		-1.49		-0.77
		(0.87)*		(0.85)**		(2.48)		(0.88)		(0.96)		(0.45)***		(1.66)
$\Delta_{12}f^*_{c,t}$		5.38		3.81		-4.70		9.16		9.81		6.70		5.02
		(0.84)***		(2.80)		(4.13)		(2.43)***		(1.97)***		(3.78)		(4.08)
$\Delta_{12}f_t$		-2.79		-4.51		4.25		-3.48		-3.48		-4.00		-2.85
		(0.83)***		(1.63)**		(2.76)		(1.86)		(1.89)		(2.97)		(1.60)
NW (DK) lags	29	29	29	29	29	29	29	29	29	29	29	29	29	29
N	1,512	1,512	252	252	252	252	252	252	252	252	252	252	252	252
R^2 (within)	0.11	0.25	0.28	0.48	0.10	0.22	0.09	0.22	0.08	0.28	0.16	0.50	0.08	0.10

Table A7. Country-by-country results when forecasting foreign minus domestic bond excess return using short-term interest rates and long-term forward rates. This table presents country-level monthly time-series forecasting regressions of the form:

$$rx^{y*}_{c,t\to t+12} - rx^{y}_{c,t\to t+12} = A_c + B_c \times (i^*_{c,t} - i_t) + D_c \times (f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+12},$$

and

$$rx^{y*}_{c,t\to t+12} - rx^{y}_{c,t\to t+12} = A_c + B_{1,c} \times i^*_{c,t} + B_{2,c} \times i_t + D_{1,c} \times f^*_{c,t} + D_{2,c} \times f_t + \varepsilon_{c,t\to t+12}.$$

For each for currency c, we separately forecast the difference between foreign and domestic 12-month bond returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollars. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Newey-West (1987) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005). For reference, we report the corresponding panel specifications in columns (1) and (2) which are a weighted average of the country-level time-series estimates.

			Country-level time-series regressions											
	Panel		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$(i^*_{c,t} - i_t)$	-0.27		-0.62		0.65		0.84		-0.43		-0.04		-0.43	
	(0.39)		(0.57)		(0.56)		(0.59)		(0.77)		(0.53)		(0.62)	
$(f^*_{c,t} - f_t)$	4.31		5.93		3.89		8.00		2.95		5.32		3.85	
	(0.33)***		(1.28)***		(1.29)**		(0.87)***		(1.78)		(0.99)***		(0.87)***	
$i^*_{c,t}$		-0.51		0.21		1.64		-1.99		-0.71		1.11		-2.36
		(0.42)		(1.13)		(0.88)		(0.82)**		(0.95)		(0.53)*		(2.18)
i_t		-0.21		-0.45		-1.13		-0.51		-0.68		-1.29		0.42
		(0.37)		(0.82)		(0.78)		(0.52)		(0.54)		(0.62)*		(0.70)
$f^*_{c,t}$		3.94		3.84		3.33		10.79		5.46		4.29		5.40
		(0.55)***		(2.04)		(1.92)		(1.35)***		(1.46)***		(1.12)***		(1.90)**
f_t		-4.12		-5.16		-3.87		-9.13		-6.55		-5.65		-4.31
		(0.38)***		(1.38)***		(1.39)**		(0.79)***		(1.41)***		(1.16)***		(0.72)***
NW (DK) lags	28	28	28	28	28	28	28	28	28	28	28	28	28	28
N	1,440	1,440	240	240	240	240	240	240	240	240	240	240	240	240
R^2 (within)	0.22	0.28	0.26	0.36	0.17	0.20	0.36	0.47	0.08	0.45	0.40	0.51	0.27	0.28

Table A8. Country-by-country results when forecasting foreign exchange excess return using short-term interest rates and long-term forward rates. This table presents country-level monthly time-series forecasting regressions of the form:

$$rx^{q}_{c,t\to t+12} = A_c + B_c \times (i^{*}_{c,t} - i_t) + D_c \times (f^{*}_{c,t} - f_t) + \varepsilon_{c,t\to t+12},$$

and

$$rx^{q}_{c,t\to t+12} = A_c + B_{1,c} \times i^{*}_{c,t} + B_{2,c} \times i_t + D_{1,c} \times f^{*}_{c,t} + D_{2,c} \times f_t + \varepsilon_{c,t\to t+12}.$$

For each for currency c, we separately forecast 12-month foreign exchange excess returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollar. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Newey-West (1987) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005). For reference, we report the corresponding panel specifications in columns (1) and (2) which are a weighted average of the country-level time-series estimates.

			Country-level time-series regressions											
	Panel		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$(i^{*}_{c,t} - i_t)$	0.96		1.59		-0.83		1.02		0.52		0.26		0.62	
	(1.34)		(1.49)		(2.62)		(1.07)		(2.15)		(1.80)		(1.18)	
$(f^{*}_{c,t} - f_t)$	-3.59		-11.06		6.75		-5.50		2.26		-5.60		-4.56	
	$(1.03)^{***}$		$(4.06)^{**}$		$(3.16)^{*}$		$(2.84)^{*}$		(2.66)		(3.04)		$(1.59)^{**}$	
$i^{*}_{c,t}$		0.11		-3.20		-5.63		2.54		1.75		-1.18		14.95
		(1.76)		(3.41)		(3.34)		(1.71)		(2.83)		(2.53)		$(5.15)^{**}$
i_t		-0.02		0.46		3.74		-0.75		0.25		0.14		-0.93
		(1.22)		(2.23)		(2.59)		(0.97)		(2.01)		(2.66)		(1.19)
$f^{*}_{c,t}$		-1.86		-3.40		10.82		-5.05		-0.14		-6.20		-8.80
		$(0.84)^{*}$		(5.33)		$(3.19)^{***}$		(3.15)		(3.08)		$(3.02)^{*}$		$(2.69)^{***}$
f_t		3.51		9.74		-7.21		5.26		0.16		7.88		5.49
		$(1.09)^{***}$		$(3.61)^{***}$		$(3.08)^{**}$		(3.00)		(3.75)		$(3.62)^{*}$		$(1.65)^{***}$
NW (DK) lags	28	28	28	28	28	28	28	28	28	28	28	28	28	28
N	1,440	1,440	240	240	240	240	240	240	240	240	240	240	240	240
R^2 (within)	0.04	0.09	0.14	0.24	0.06	0.25	0.07	0.15	0.03	0.11	0.10	0.13	0.15	0.23

Table A9. Contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term interest rates for the EUR, GBP, and JPY. This table presents monthly panel regressions of the form:

$$\Delta_H q_{c,t} = A_c + B \times \Delta_H (i^*_{c,t} - i_t) + D \times \Delta_H (y^*_{c,t} - y_t) + \Delta_H \varepsilon_{c,t},$$

and

$$\Delta_H q_{c,t} = A_c + B_1 \times \Delta_H i^*_{c,t} + B_2 \times \Delta_H i_t + D_1 \times \Delta_H y^*_{c,t} + D_2 \times \Delta_H y_t + \Delta_H \varepsilon_{c,t}.$$

We regress H-month changes in the foreign exchange rate on H-month changes in short-term interest rates and in long-term yields in both the foreign currency and in U.S. dollars. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes three currency pairs: EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	$H=$ 3-month changes				$H=$ 12-month changes			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta_H(i^*_{c,t} - i_t)$	4.00	2.90			2.14	0.69		
	(1.63)**	(1.91)			(1.44)	(1.74)		
$\Delta_H(y^*_{c,t} - y_t)$		3.39				5.30		
		(1.36)**				(2.10)**		
$\Delta_H i^*_{c,t}$			6.64	5.33			5.45	2.18
			(1.48)***	(1.69)***			(1.38)***	(1.94)
$\Delta_H i_t$			-3.26	-1.86			-1.70	0.18
			(1.15)**	(1.32)			(1.02)	(1.29)
$\Delta_H y^*_{c,t}$				4.82				11.00
				(1.41)***				(2.27)***
$\Delta_H y_t$				-4.09				-6.06
				(1.06)***				(1.95)***
DK lags	18	18	18	18	29	29	29	29
N	756	756	756	756	756	756	756	756
R^2 (within)	0.11	0.14	0.16	0.21	0.06	0.12	0.15	0.27

Table A10. Contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term forward rates for the EUR, GBP, and JPY. This table presents monthly panel regressions of the form:

$$\Delta_H q_{c,t} = A_c + B \times \Delta_H (i^*_{c,t} - i_t) + D \times \Delta_H (f^*_{c,t} - f_t) + \Delta_H \varepsilon_{c,t},$$

and

$$\Delta_H q_{c,t} = A_c + B_1 \times \Delta_H i^*_{c,t} + B_2 \times \Delta_H i_t + D_1 \times \Delta_H f^*_{c,t} + D_2 \times \Delta_H f_t + \Delta_H \varepsilon_{c,t}.$$

We regress H-month changes in the foreign exchange rate on H-month changes in short-term interest rates and in distant forward rates in both the foreign currency and in U.S. dollars. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes three currency pairs: EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	H = 3-month changes				H = 12-month changes			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta_H(i^*_{c,t} - i_t)$	4.00	3.87			2.14	2.11		
	(1.63)**	(1.68)**			(1.44)	(1.44)		
$\Delta_H(f^*_{c,t} - f_t)$		2.15				2.98		
		(0.91)**				(1.21)**		
$\Delta_H i^*_{c,t}$			6.64	6.49			5.45	5.13
			(1.48)***	(1.51)***			(1.38)***	(1.31)***
$\Delta_H i_t$			-3.26	-3.07			-1.70	-1.22
			(1.15)**	(1.21)**			(1.02)	(0.95)
$\Delta_H f^*_{c,t}$				3.09				7.17
				(1.06)***				(1.26)***
$\Delta_H f_t$				-2.36				-3.29
				(0.78)***				(1.01)***
DK lags	18	18	18	18	29	29	29	29
N	756	756	756	756	756	756	756	756
R^2 (within)	0.11	0.13	0.16	0.19	0.06	0.09	0.15	0.24

Table A11. Forecasting foreign minus domestic bond excess return using short-term interest rates and long-term forward rates for the EUR, GBP, and JPY. This table presents monthly panel forecasting regressions of the form:

$$rx^{y*}_{c,t\to t+H} - rx^{y}_{c,t\to t+H} = A_c + B\times(i^*_{c,t} - i_t) + D\times(f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+H},$$

and

$$rx^{y*}_{c,t\to t+H} - rx^{y}_{c,t\to t+H} = A_c + B_1\times i^*_{c,t} + B_2\times i_t + D_1\times f^*_{c,t} + D_2\times f_t + \varepsilon_{c,t\to t+H}.$$

We forecast the difference between foreign and domestic H-month bond returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollars. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes three currency pairs: EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	$H=$ 3-month excess returns				$H=$ 12-month excess returns			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$i^*_{c,t} - i_t$	-0.08	-0.28			0.05	-0.44		
	(0.14)	(0.13)*			(0.51)	(0.42)		
$f^*_{c,t} - f_t$		1.42				3.89		
		(0.20)***				(0.36)***		
$i^*_{c,t}$			-0.22	-0.28			-0.48	-0.64
			(0.16)	(0.14)*			(0.55)	(0.43)
i_t			-0.08	0.12			-0.67	-0.15
			(0.18)	(0.14)			(0.61)	(0.41)
$f^*_{c,t}$				1.28				3.42
				(0.19)***				(0.46)***
f_t				-1.41				-3.69
				(0.26)***				(0.40)***
DK lags	17	17	17	17	28	28	28	28
N	747	747	747	747	720	720	720	720
R^2 (within)	0.00	0.10	0.02	0.11	0.00	0.23	0.10	0.30

Table A12. Forecasting foreign exchange excess return using short-term interest rates and long-term forward rates for the EUR, GBP, and JPY. This table presents monthly panel forecasting regressions of the form:

$$rx^{q}_{c,t\to t+H} = A_c + B\times(i^*_{c,t} - i_t) + D\times(f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+H},$$

and

$$rx^{q}_{c,t\to t+H} = A_c + B_1\times i^*_{c,t} + B_2\times i_t + D_1\times f^*_{c,t} + D_2\times f_t + \varepsilon_{c,t\to t+H}.$$

In words, we forecast H-month foreign exchange excess returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollar. The sample runs from 2001m1 to 2021m12 and includes three currency pairs: EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	$H = 3$-month excess returns				$H = 12$-month excess returns			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$i^*_{c,t} - i_t$	0.15	0.26			0.56	0.97		
	(0.35)	(0.36)			(1.30)	(1.33)		
$f^*_{c,t} - f_t$		-0.83				-3.20		
		(0.33)**				(0.97)***		
$i^*_{c,t}$			0.26	0.01			1.12	0.77
			(0.41)	(0.54)			(1.39)	(1.62)
i_t			-0.02	-0.07			0.08	-0.21
			(0.32)	(0.35)			(1.11)	(1.20)
$f^*_{c,t}$				-0.47				-2.27
				(0.32)				(0.92)**
f_t				0.94				3.14
				(0.37)**				(1.04)**
DK lags	17	17	17	17	28	28	28	28
N	747	747	747	747	720	720	720	720
R^2 (within)	0.00	0.01	0.01	0.03	0.01	0.05	0.04	0.08

Table A13. Varying the base currency when examining the contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term interest rates. This table presents monthly panel regressions of the form:

$$\Delta_{12}q_{c,t} = A_c + B\times\Delta_{12}(i^*_{c,t} - i_t) + D\times\Delta_{12}(y^*_{c,t} - y_t) + \Delta_{12}\varepsilon_{c,t},$$

and

$$\Delta_{12}q_{c,t} = A_c + B_1\times\Delta_{12}i^*_{c,t} + B_2\times\Delta_{12}i_t + D_1\times\Delta_{12}y^*_{c,t} + D_2\times\Delta_{12}y_t + \Delta_{12}\varepsilon_{c,t}.$$

We regress 12-month changes in the foreign exchange rate on 12-month changes in short-term interest rates and in long-term yields in both the foreign currency and in the base (domestic) currency. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and the currencies in our sample are the USD, AUD, CAD, CHF, EUR, GBP, JPY. Moving across the columns shows the effect of varying the base (domestic) currency—a.k.a., the numeraire—in our regressions. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

Base currency:	USD		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$\Delta_{12}(i^*_{c,t} - i_t)$	1.26		3.98		2.48		1.44		2.03		4.22		2.99	
	(1.64)		(1.32)**		(1.84)		(1.83)		(2.02)		(1.94)*		(2.40)	
$\Delta_{12}(y^*_{c,t} - y_t)$	5.13		4.65		2.98		3.97		4.98		5.44		5.88	
	(1.88)**		(2.10)*		(2.37)		(1.39)**		(2.19)*		(2.20)**		(1.14)***	
$\Delta_{12}i^*_{c,t}$		3.10		1.71		1.40		1.88		2.31		2.42		1.85
		(1.40)*		(1.30)		(1.72)		(1.24)		(1.93)		(1.54)		(2.45)
$\Delta_{12}i_t$		-0.21		-5.33		-2.99		2.43		-0.69		-5.11		11.22
		(1.08)		(0.91)***		(2.05)		(1.80)		(2.06)		(1.44)***		(7.59)
$\Delta_{12}y^*_{c,t}$		9.07		7.06		3.57		4.57		3.55		5.19		6.10
		(1.63)***		(1.24)***		(1.89)		(1.43)***		(2.27)		(2.01)**		(1.30)***
$\Delta_{12}y_t$		-5.77		-3.81		-3.99		-6.58		-8.12		-6.14		-6.27
		(1.61)***		(2.21)		(2.22)		(1.60)***		(1.70)***		(2.40)**		(2.54)**
DK lags	29	29	29	29	29	29	29	29	29	29	29	29	29	29
N	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512
R^2 (within)	0.14	0.27	0.31	0.38	0.12	0.15	0.09	0.19	0.14	0.20	0.27	0.34	0.22	0.26

Table A14. Varying the base currency when examining the contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term forward rates. This table presents monthly panel regressions of the form:

$$\Delta_{12}q_{c,t} = A_c + B \times \Delta_{12}(i^*_{c,t} - i_t) + D \times \Delta_{12}(f^*_{c,t} - f_t) + \Delta_{12}\varepsilon_{c,t},$$

and

$$\Delta_{12}q_{c,t} = A_c + B_1 \times \Delta_{12}i^*_{c,t} + B_2 \times \Delta_{12}i_t + D_1 \times \Delta_{12}f^*_{c,t} + D_2 \times \Delta_{12}f_t + \Delta_{12}\varepsilon_{c,t}.$$

We regress 12-month changes in the foreign exchange rate on 12-month changes in short-term interest rates and in distant forward rates in both the foreign currency and in the base (domestic) currency. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and the currencies in our sample are the USD, AUD, CAD, CHF, EUR, GBP, JPY. Moving across the columns shows the effect of varying the base (domestic) currency—a.k.a., the numeraire—in our regressions. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

Base currency:	USD		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$\Delta_{12}(i^*_{c,t} - i_t)$	2.70		5.30		3.42		2.61		3.47		5.76		4.48	
	(1.42)*		(1.15)***		(1.52)*		(1.68)		(1.60)*		(1.58)***		(2.24)*	
$\Delta_{12}(f^*_{c,t} - f_t)$	2.50		2.93		0.01		1.51		3.79		1.66		4.11	
	(1.13)*		(1.40)*		(1.63)		(1.07)		(1.32)**		(1.03)		(1.05)***	
$\Delta_{12}i^*_{c,t}$		5.54		3.78		2.58		3.19		3.29		3.96		3.56
		(1.12)***		(1.29)**		(1.53)		(1.15)**		(1.55)*		(1.19)***		(2.29)
$\Delta_{12}i_t$		-1.72		-6.45		-4.05		0.52		-3.13		-6.83		7.36
		(0.87)*		(0.62)***		(1.78)*		(1.81)		(1.74)		(1.12)***		(8.39)
$\Delta_{12}f^*_{c,t}$		5.38		3.76		0.53		2.02		1.93		1.56		4.11
		(0.84)***		(1.06)***		(1.38)		(1.10)		(1.40)		(1.09)		(1.07)***
$\Delta_{12}f_t$		-2.79		-1.62		-0.65		-3.39		-5.56		-1.91		-3.13
		(0.83)***		(1.87)		(1.63)		(0.93)***		(0.94)***		(1.43)		(2.74)
DK lags	29	29	29	29	29	29	29	29	29	29	29	29	29	29
N	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512
R^2 (within)	0.11	0.25	0.30	0.36	0.11	0.13	0.07	0.16	0.15	0.21	0.23	0.30	0.22	0.25

Table A15. Varying the base currency when forecasting foreign minus domestic bond excess return using short-term interest rates and long-term forward rates. This table presents monthly panel forecasting regressions of the form:

$$rx^{y*}_{c,t\to t+12} - rx^{y}_{c,t\to t+12} = A_c + B\times(i^*_{c,t} - i_t) + D\times(f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+12},$$

and

$$rx^{y*}_{c,t\to t+12} - rx^{y}_{c,t\to t+12} = A_c + B_1\times i^*_{c,t} + B_2\times i_t + D_1\times f^*_{c,t} + D_2\times f_t + \varepsilon_{c,t\to t+12}.$$

We forecast the difference between foreign and domestic 12-month bond returns using short-term interest rates and distant forward rates in both the foreign currency and in the base (domestic) currency. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and the currencies in our sample are the USD, AUD, CAD, CHF, EUR, GBP, JPY. Moving across the columns shows the effect of varying the base (domestic) currency—a.k.a., the numeraire—in our regressions. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

Base currency:	USD		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$i^*_{c,t} - i_t$	-0.27		-0.68		-0.23		-0.21		-0.76		-0.46		-1.10	
	(0.39)		(0.42)		(0.27)		(0.29)		(0.26)**		(0.29)		(0.35)***	
$f^*_{c,t} - f_t$	4.31		3.87		2.47		3.21		2.34		3.50		3.32	
	(0.33)***		(0.62)***		(0.52)***		(0.59)***		(0.42)***		(0.45)***		(0.70)***	
$i^*_{c,t}$		-0.51		-0.12		-0.25		-0.08		-0.17		-0.60		-0.86
		(0.42)		(0.35)		(0.27)		(0.26)		(0.26)		(0.28)*		(0.48)
i_t		-0.21		-0.07		-0.74		1.18		1.31		0.36		0.43
		(0.37)		(0.80)		(0.38)*		(0.41)**		(0.41)***		(0.28)		(2.04)
$f^*_{c,t}$		3.94		3.88		2.14		3.26		3.97		3.84		3.58
		(0.55)***		(0.69)***		(0.40)***		(0.61)***		(0.54)***		(0.51)***		(0.60)***
f_t		-4.12		-2.81		-2.13		-4.40		-3.85		-3.52		-4.31
		(0.38)***		(0.81)***		(0.56)***		(0.59)***		(0.35)***		(0.51)***		(1.33)***
DK lags	28	28	28	28	28	28	28	28	28	28	28	28	28	28
N	1,440	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512
R^2 (within)	0.22	0.28	0.17	0.22	0.12	0.22	0.17	0.23	0.14	0.30	0.27	0.28	0.21	0.22

Table A16. Varying the base currency when forecasting foreign minus domestic bond excess return using short-term interest rates and long-term forward rates. This table presents monthly panel forecasting regressions of the form:

$$rx^q_{c,t\to t+12} = A_c + B\times(i^*_{c,t} - i_t) + D\times(f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+12},$$

and

$$rx^q_{c,t\to t+12} = A_c + B_1\times i^*_{c,t} + B_2\times i_t + D_1\times f^*_{c,t} + D_2\times f_t + \varepsilon_{c,t\to t+12}.$$

In words, we forecast 12-month foreign exchange excess returns using short-term interest rates and distant forward rates in both the foreign currency and in the base (domestic) currency. All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and the currencies in our sample are the USD, AUD, CAD, CHF, EUR, GBP, JPY. Moving across the columns shows the effect of varying the base (domestic) currency—a.k.a., the numeraire—in our regressions. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

Base currency:	USD		AUD		CAD		CHF		EUR		GBP		JPY	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$i^*_{c,t} - i_t$	0.96		1.06		0.23		1.48		1.06		-0.07		0.83	
	(1.34)		(1.13)		(0.96)		(1.04)		(1.26)		(1.19)		(1.32)	
$f^*_{c,t} - f_t$	-3.59		-3.41		-0.17		-1.77		-0.41		-2.14		-1.37	
	(1.03)***		(1.20)**		(1.08)		(1.03)		(1.17)		(1.30)		(1.26)	
$i^*_{c,t}$		0.11		0.37		0.49		1.51		0.41		1.55		0.90
		(1.76)		(1.33)		(1.02)		(0.94)		(1.30)		(0.95)		(0.99)
i_t		-0.02		2.19		0.33		-4.01		-2.48		0.02		-21.02
		(1.22)		(2.12)		(1.15)		(0.83)***		(1.45)		(1.10)		(5.37)***
$f^*_{c,t}$		-1.86		-4.12		-0.29		-2.15		-0.04		-3.68		-2.82
		(0.84)*		(1.43)**		(0.97)		(1.10)*		(1.65)		(1.22)**		(1.18)**
f_t		3.51		-0.50		-0.27		3.30		1.50		3.27		8.08
		(1.09)***		(1.76)		(1.08)		(1.08)**		(1.46)		(1.44)**		(2.10)***
DK lags	28	28	28	28	28	28	28	28	28	28	28	28	28	28
N	1,440	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512	1,512
R^2 (within)	0.04	0.09	0.03	0.13	0.00	0.01	0.04	0.09	0.01	0.07	0.02	0.05	0.01	0.15

Table A17. Contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term interest rates from 1994 to 2021. This table presents monthly panel regressions of the form:

$$\Delta_H q_{c,t} = A_c + B \times \Delta_H (i^*_{c,t} - i_t) + D \times \Delta_H (y^*_{c,t} - y_t) + \Delta_H \varepsilon_{c,t},$$

and

$$\Delta_H q_{c,t} = A_c + B_1 \times \Delta_H i^*_{c,t} + B_2 \times \Delta_H i_t + D_1 \times \Delta_H y^*_{c,t} + D_2 \times \Delta_H y_t + \Delta_H \varepsilon_{c,t}.$$

We regress H-month changes in the foreign exchange rate on H-month changes in short-term interest rates and in long-term yields in both the foreign currency and in U.S. dollars. All regressions include currency fixed effects. The sample runs from 1994m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	$H=$ 3-month changes				$H=$ 12-month changes			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta_H(i^*_{c,t} - i_t)$	2.84	2.01			1.53	0.27		
	(1.20)**	(1.25)			(1.37)	(1.44)		
$\Delta_H(y^*_{c,t} - y_t)$		2.21				4.06		
		(0.80)**				(1.04)***		
$\Delta_H i^*_{c,t}$			3.84	2.67			2.95	0.78
			(1.62)**	(1.61)			(1.91)	(2.11)
$\Delta_H i_t$			-2.13	-1.31			-1.04	-0.15
			(0.85)**	(0.84)			(1.08)	(1.07)
$\Delta_H y^*_{c,t}$				3.13				5.93
				(0.84)***				(1.15)***
$\Delta_H y_t$				-2.25				-3.27
				(0.89)**				(1.19)**
DK lags	20	20	20	20	33	33	33	33
N	2,016	2,016	2,016	2,016	2,016	2,016	2,016	2,016
R^2 (within)	0.06	0.08	0.08	0.10	0.03	0.06	0.06	0.12

Table A18. Contemporaneous relationship between movements in foreign exchange, short-term interest rates, and long-term forward rates from 1994 to 2021. This table presents monthly panel regressions of the form:

$$\Delta_H q_{c,t} = A_c + B \times \Delta_H (i^*_{c,t} - i_t) + D \times \Delta_H (f^*_{c,t} - f_t) + \Delta_H \varepsilon_{c,t},$$

and

$$\Delta_H q_{c,t} = A_c + B_1 \times \Delta_H i^*_{c,t} + B_2 \times \Delta_H i_t + D_1 \times \Delta_H f^*_{c,t} + D_2 \times \Delta_H f_t + \Delta_H \varepsilon_{c,t}.$$

We regress H-month changes in the foreign exchange rate on H-month changes in short-term interest rates and in distant forward rates in both the foreign currency and in U.S. dollars. All regressions include currency fixed effects. The sample runs from 1994m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	$H=$ 3-month changes				$H=$ 12-month changes			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta_H(i^*_{c,t} - i_t)$	2.84	2.72			1.53	1.42		
	(1.20)**	(1.19)**			(1.37)	(1.38)		
$\Delta_H(f^*_{c,t} - f_t)$		1.07				1.80		
		(0.79)				(0.65)**		
$\Delta_H i^*_{c,t}$			3.84	3.59			2.95	2.41
			(1.62)**	(1.65)**			(1.91)	(2.02)
$\Delta_H i_t$			-2.13	-2.05			-1.04	-0.95
			(0.85)**	(0.82)**			(1.08)	(0.98)
$\Delta_H f^*_{c,t}$				1.64				3.21
				(0.61)**				(0.76)***
$\Delta_H f_t$				-0.96				-0.96
				(0.87)				(0.74)
DK lags	20	20	20	20	33	33	33	33
N	2,016	2,016	2,016	2,016	2,016	2,016	2,016	2,016
R^2 (within)	0.06	0.07	0.08	0.09	0.03	0.04	0.06	0.10

Table A19. Forecasting foreign minus domestic bond excess return using short-term interest rates and long-term forward rates from 1994 to 2021. This table presents monthly panel forecasting regressions of the form:

$$rx^{y*}_{c,t\to t+H} - rx^{y}_{c,t\to t+H} = A_c + B\times(i^*_{c,t} - i_t) + D\times(f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+H},$$

and

$$rx^{y*}_{c,t\to t+H} - rx^{y}_{c,t\to t+H} = A_c + B_1\times i^*_{c,t} + B_2\times i_t + D_1\times f^*_{c,t} + D_2\times f_t + \varepsilon_{c,t\to t+H}.$$

We forecast the difference between foreign and domestic H-month bond returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollars. All regressions include currency fixed effects. The sample runs from 1994m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	H = 3-month excess returns				H = 12-month excess returns			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$i^*_{c,t} - i_t$	-0.10	-0.19			-0.31	-0.59		
	(0.11)	(0.09)*			(0.39)	(0.28)*		
$f^*_{c,t} - f_t$		1.24				3.74		
		(0.26)***				(0.45)***		
$i^*_{c,t}$			-0.12	-0.29			-0.30	-1.03
			(0.11)	(0.13)**			(0.37)	(0.35)**
i_t			0.09	0.18			0.32	0.53
			(0.12)	(0.11)			(0.43)	(0.35)
$f^*_{c,t}$				1.36				4.12
				(0.22)***				(0.48)***
f_t				-1.46				-4.06
				(0.25)***				(0.49)***
DK lags	20	20	20	20	33	33	33	33
N	1,998	1,998	1,998	1,998	1,944	1,944	1,944	1,944
R^2 (within)	0.00	0.08	0.00	0.10	0.01	0.26	0.01	0.28

Table A20. Forecasting foreign exchange excess return using short-term interest rates and long-term forward rates from 1994 to 2021. This table presents monthly panel forecasting regressions of the form:

$$rx^{q}_{c,t\to t+H} = A_c + B\times(i^{*}_{c,t} - i_t) + D\times(f^{*}_{c,t} - f_t) + \varepsilon_{c,t\to t+H},$$

and

$$rx^{q}_{c,t\to t+H} = A_c + B_1\times i^{*}_{c,t} + B_2\times i_t + D_1\times f^{*}_{c,t} + D_2\times f_t + \varepsilon_{c,t\to t+H}.$$

In words, we forecast H-month foreign exchange excess returns using short-term interest rates and distant forward rates in both the foreign currency and in U.S. dollar. All regressions include currency fixed effects. The sample runs from 1994m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

	$H=$ 3-month excess returns				$H=$ 12-month excess returns			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$i^{*}_{c,t} - i_t$	0.49	0.52			1.98	2.12		
	(0.27)*	(0.27)*			(1.06)*	(1.04)*		
$f^{*}_{c,t} - f_t$		-0.46				-1.95		
		(0.24)*				(0.83)**		
$i^{*}_{c,t}$			0.50	0.38			1.96	2.03
			(0.30)	(0.40)			(1.03)*	(1.15)
i_t			-0.48	-0.56			-1.99	-2.15
			(0.27)*	(0.27)*			(1.09)	(1.08)*
$f^{*}_{c,t}$				-0.41				-1.98
				(0.29)				(0.86)**
f_t				0.76				2.44
				(0.36)*				(1.27)*
DK lags	20	20	20	20	33	33	33	33
N	1,998	1,998	1,998	1,998	1,944	1,944	1,944	1,944
R^2 (within)	0.02	0.02	0.02	0.03	0.08	0.10	0.08	0.10

Table A21. Subsample contemporaneous co-movement results. As in Table 1, columns (1) to (6) present country-level monthly forecasting regressions of the form:

$$\Delta_{12}q_{c,t} = A_c + B_1 \times \Delta_{12}i^*_{c,t} + B_2 \times \Delta_{12}i_t + D_1 \times \Delta_{12}y^*_{c,t} + D_2 \times \Delta_{12}y_t + \Delta_{12}\varepsilon_{c,t}.$$

for different subsamples. As in Table 2, columns (7) to (12) present country-level monthly forecasting regressions of the form:

$$\Delta_{12}q_{c,t} = A_c + B_1 \times \Delta_{12}i^*_{c,t} + B_2 \times \Delta_{12}i_t + D_1 \times \Delta_{12}f^*_{c,t} + D_2 \times \Delta_{12}f_t + \Delta_{12}\varepsilon_{c,t}.$$

for different subsamples. All regressions include currency fixed effects. The full sample runs from 2001m1 to 2021m12. We also show results for the 2001m1 to 2007m12 subsample and the 2008m1 to 2021m12 sample. The panel includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

Sample:	2001–2021		2001–2007		2008–2021		2001–2021		2001–2007		2008–2021	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_{12}(i^*_{c,t} - i_t)$	1.26		-1.93		4.26		2.70		-0.19		5.01	
	(1.64)		(1.47)		(1.10)***		(1.42)*		(1.66)		(1.03)***	
$\Delta_{12}(y^*_{c,t} - y_t)$	5.13		6.95		2.42							
	(1.88)**		(1.78)***		(1.99)							
$\Delta_{12}i^*_{c,t}$		3.10		-0.27		4.63		5.54		1.64		6.84
		(1.40)*		(1.26)		(0.83)***		(1.12)***		(1.50)		(0.66)***
$\Delta_{12}i_t$		-0.21		1.87		-2.25		-1.72		0.06		-3.31
		(1.08)		(1.27)		(0.75)**		(0.87)*		(1.42)		(0.69)***
$\Delta_{12}y^*_{c,t}$		9.07		7.56		8.17						
		(1.63)***		(1.76)***		(1.74)***						
$\Delta_{12}y_t$		-5.77		-7.17		-3.95						
		(1.61)***		(1.75)***		(1.63)**						
$\Delta_{12}(f^*_{c,t} - f_t)$							2.50		3.06		1.01	
							(1.13)*		(1.03)**		(1.31)	
$\Delta_{12}f^*_{c,t}$								5.38		3.46		5.03
								(0.84)***		(2.30)		(1.05)***
$\Delta_{12}f_t$								-2.79		-3.25		-1.95
								(0.83)***		(1.17)**		(1.08)
DK lags	29	29	17	17	24	24	29	29	17	17	24	24
N	1,512	1,512	504	504	1,008	1,008	1,512	1,512	504	504	1,008	1,008
R^2 (within)	0.14	0.27	0.08	0.10	0.26	0.39	0.11	0.25	0.03	0.05	0.25	0.37

Table A22. Subsample return forecasting results. As in Table 3, columns (1) to (6) present country-level monthly forecasting regressions of the form:

$$rx^{y*}_{c,t\to t+12} - rx^{y}_{c,t\to t+12} = A_c + B_1 \times i^*_{c,t} + B_2 \times i_t + D_1 \times f^*_{c,t} + D_2 \times f_t + \varepsilon_{c,t\to t+12}.$$

for different subsamples. As in Table 4, columns (7) to (12) present country-level monthly forecasting regressions of the form:

$$rx^{q}_{c,t\to t+12} = A_c + B_1 \times i^*_{c,t} + B_2 \times i_t + D_1 \times f^*_{c,t} + D_2 \times f_t + \varepsilon_{c,t\to t+h},$$

for different subsamples. All regressions include currency fixed effects. The full sample runs from 2001m1 to 2021m12. We also show results for the 2001m1 to 2007m12 subsample and the 2008m1 to 2021m12 sample. The panel includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-b asymptotic theory of Kiefer and Vogelsang (2005).

Dependent variable:	$rx^{y*}_{c,t\to t+12} - rx^{y}_{c,t\to t+12}$						$rx^{q}_{c,t\to t+12}$					
Sample:	2001–2021		2001–2007		2008–2021		2001–2021		2001–2007		2008–2021	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$(i^*_{c,t} - i_t)$	-0.27		0.75		-0.31		0.96		1.70		-0.04	
	(0.39)		(0.55)		(0.66)		(1.34)		(1.35)		(1.38)	
$(f^*_{c,t} - f_t)$	4.31		5.25		5.15		-3.59		-4.41		-2.90	
	(0.33)***		(1.17)***		(0.31)***		(1.03)***		(1.08)***		(1.70)	
$i^*_{c,t}$		-0.51		-0.72		0.27		0.11		0.32		-2.08
		(0.42)		(0.59)		(0.48)		(1.76)		(2.29)		(1.62)
i_t		-0.21		-1.45		-0.59		-0.02		-0.80		-0.94
		(0.37)		(0.42)**		(0.94)		(1.22)		(1.79)		(1.96)
$f^*_{c,t}$		3.94		3.96		4.07		-1.86		-2.87		-2.34
		(0.55)***		(0.84)***		(0.34)***		(0.84)*		(1.17)**		(1.67)
f_t		-4.12		-6.87		-4.88		3.51		5.55		3.72
		(0.38)***		(1.27)***		(0.43)***		(1.09)***		(1.96)**		(1.82)*
DK lags	28	28	16	16	23	23	28	28	16	16	23	23
N	1,440	1,440	504	504	936	936	1,440	1,440	504	504	936	936
R^2 (within)	0.22	0.28	0.22	0.40	0.28	0.31	0.04	0.09	0.10	0.12	0.02	0.07

Table A23. The role of the short-rate correlation. Columns (1) and (2) present monthly panel regressions of the form:

$$\Delta_{12}q_{c,t} = A_c + B_1 \cdot \Delta_{12}(i^*_{c,t} - i_t) + B_2 \cdot \hat{\rho}_c \times \Delta_{12}(i^*_{c,t} - i_t) + D_1 \cdot \Delta_{12}(f^*_{c,t} - f_t) + D_2 \cdot \hat{\rho}_c \times \Delta_{12}(f^*_{c,t} - f_t) + \Delta_{12}\varepsilon_{c,t},$$

where $\hat{\rho}_c$ is the estimated in-sample correlation between monthly changes in 1-year bond yields in USD and 1-year yields in currency *c*. Columns (3) and (4) present regressions of the form:

$$rx^q_{c,t\to t+12} = A_c + B_1 \cdot (i^*_{c,t} - i_t) + B_2 \cdot \hat{\rho}_c \times (i^*_{c,t} - i_t) + D_1 \cdot (f^*_{c,t} - f_t) + D_2 \cdot \hat{\rho}_c \times (f^*_{c,t} - f_t) + \varepsilon_{c,t\to t+12}.$$

All regressions include currency fixed effects. The sample runs from 2001m1 to 2021m12 and includes six currency pairs: AUD-USD, CAD-USD, CHF-USD, EUR-USD, GBP-USD, and JPY-USD. We report Driscoll-Kraay (1998) standard errors allowing for serial correlation up to a lag parameter that is chosen using a data-dependent approach based on Lazarus, Lewis, Stock, and Watson (2018). *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Statistical significance is computed using the fixed-*b* asymptotic theory of Kiefer and Vogelsang (2005).

Dependent variable:	$\Delta_{12}q_{c,t}$		$rx^q_{c,t\to t+12}$	
	(1)	(2)	(3)	(4)
$\Delta_{12}(i^*_{c,t} - i_t)$	2.70	2.13		
	(1.42)*	(2.66)		
$\hat{\rho}_c \times \Delta_{12}(i^*_{c,t} - i_t)$		1.35		
		(4.58)		
$\Delta_{12}(f^*_{c,t} - f_t)$	2.50	8.77		
	(1.13)*	(2.79)***		
$\hat{\rho}_c \times \Delta_{12}(f^*_{c,t} - f_t)$		-14.52		
		(6.42)*		
$(i^*_{c,t} - i_t)$			0.96	1.67
			(1.34)	(1.75)
$\hat{\rho}_c \times (i^*_{c,t} - i_t)$				-1.48
				(4.98)
$(f^*_{c,t} - f_t)$			-3.59	-10.74
			(1.03)***	(3.74)**
$\hat{\rho}_c \times (f^*_{c,t} - f_t)$				16.43
				(8.12)*
DK lags	29	29	28	28
N	1,512	1,512	1,440	1,440
R^2 (within)	0.11	0.12	0.04	0.06

Table A24: Comparison of our segmented-markets, quantity-driven model of foreign exchange (FX) with leading consumption-based models. This table compares our model's predictions with those of leading consumption-based models. We write "Yes/No" or "No/Yes" when the prediction depends on the model calibration.

	(1) FX rates respond to supply and demand for assets in different currencies	(2) Real short rates fall in recessions	(3) Real short rates fall in "bad times" for bond investors	(4) Real term premia can be positive: $E_t[rx^y_{t+1}]$ >0	(5) Shocks to $i^*_{t+1} - i_{t+1}$ associated with foreign currency appreciation: $Cov_t[rx^q_{t+1}, i^*_{t+1} - i_{t+1}]$>0	(6) FX trade loses (makes) money when foreign (domestic) yield-curve trade does: $Cov_t[rx^q_{t+1}, rx^{y*}_{t+1} - rx^y_{t+1}]$<0.	(7) $E_t[rx^q_{t+1}]$ negatively related to $E_t[rx^{y*}_{t+1} - rx^y_{t+1}]$	(8) Fama ('84) FX carry trade: $E_t[rx^q_{t+1}]$ increasing in $(i^*_t - i_t)$	(9) Campbell-Shiller ('91) yield curve carry trade: $E_t[rx^y_{t+1}]$ is increasing in $(y_t - i_t)$	(10) Real yield curve steep when short rates low: $(y_t - i_t)$ decreasing in i_t	(11) Lustig et al ('19): Long-term FX carry trade less profitable than short-term trade
Data	**Yes**	**Yes**	**N/A**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
Our model	**Yes**	**Yes**	**No**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
Consumption-based models:											
Textbook C-CAPM model: Power utility, homoskedastic growth shocks, positive autocorrelation of growth[i]	No	Yes	Yes	No	No	No	No	No	No	Yes	No
Non-standard C-CAPM: Power utility, homoskedastic growth shocks, negative autocorrelation of growth[ii]	No	No	No	Yes	Yes	Yes	No	No	No	Yes	No
Long-run risks: News about long-run growth, stochastic volatility, EZ-W utility, CRRA (γ) exceeds inverse-EIS (ψ^{-1}).[iii]	No	Yes	Yes	No	No	No	No	Yes	No/Yes	Yes/No	No
Long-run risks: News about long-run growth, stochastic volatility, EZ-W utility, inverse-EIS (ψ^{-1}) exceeds CRRA (γ).[iv]	No	Yes	No	Yes	Yes	Yes	Yes	No	No/Yes	Yes/No	Yes
Time-varying probability of rare consumption disasters[v]	No	Yes	Yes	No	No	No	No	Yes	No/Yes	Yes/No	No
Habit formation: Short rate rises when surplus-consumption ratio rises[vi]	No	Yes	Yes	No	No	No	No	Yes	No/Yes	Yes/No	No
Habit formation: Short rate falls when surplus-consumption ratio rises[vii]	No	No	No	Yes	Yes	Yes	Yes	No	No/Yes	Yes/No	Yes

[i] See Campbell (1986), Campbell (2003), Campbell (2018). These models cannot speak to return predictability by construction.
[ii] See Campbell (1986), Campbell (2003), Campbell (2018). These models cannot speak to return predictability by construction.
[iii] See Campbell (2003), Bansal and Yaron (2004), Colacito and Croce (2011), Bansal and Shaliastovich (2013), Campbell (2018).
[iv] See Campbell (2003), Bansal and Yaron (2004), Colacito and Croce (2011), Bansal and Shaliastovich (2013), Campbell (2018).
[v] See Wachter (2013) and Campbell (2018).
[vi] See Campbell and Cochrane (1999), Wachter (2006), Verdelhan (2010), and Campbell (2018).
[vii] See Campbell and Cochrane (1999), Wachter (2006), Verdelhan (2010), and Campbell (2018).